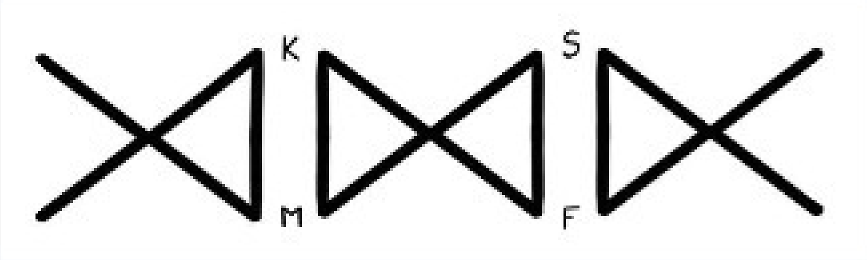
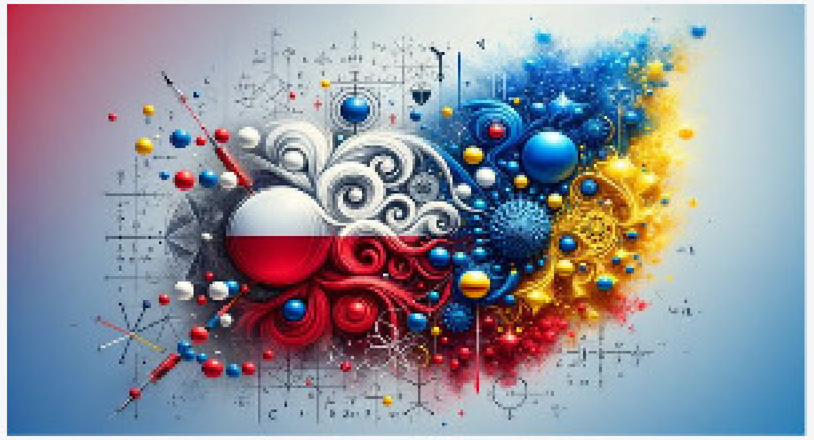
# Trans-Carpathian Student Circle

Moldoveanu

Lake Capra

# What we did?

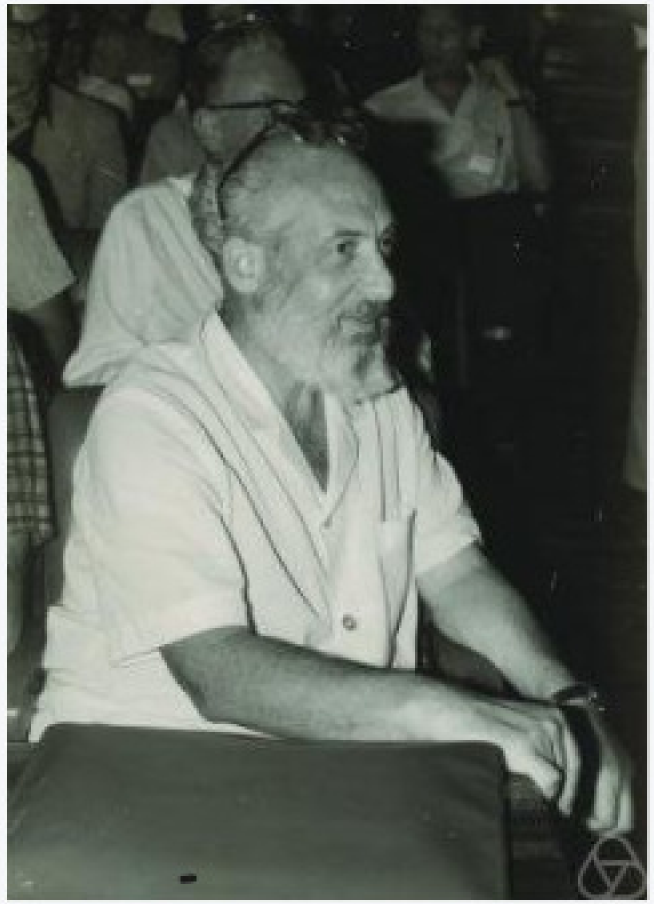


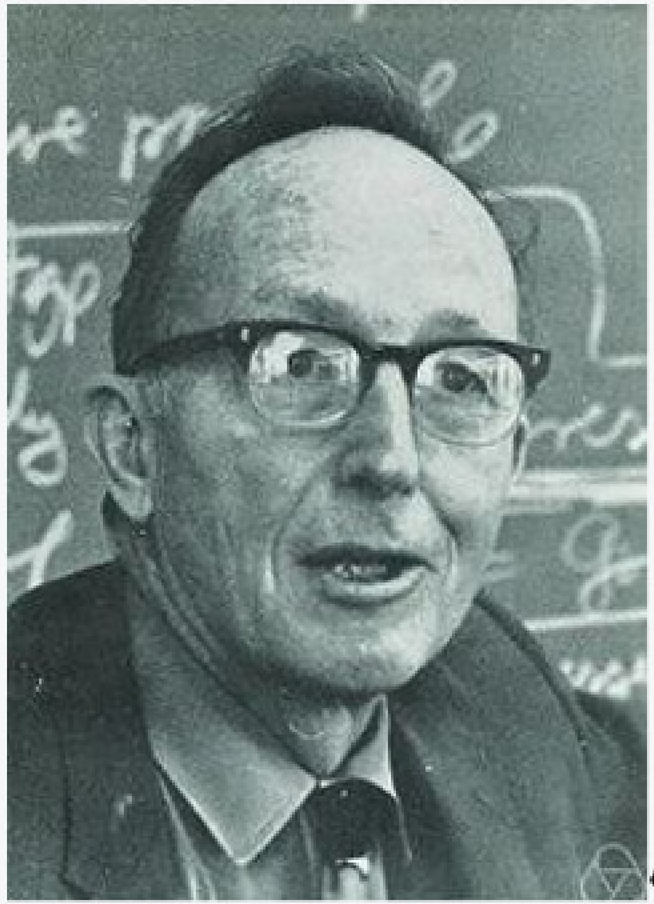
## ¡Introduction to Category Theory!

 — proving hammer

# Historical background



Samuel Eilenberg



Sanders MacLane

spacebar

Nobuo Yoneda

# Comutative diagrams

$$
\begin{tikzcd}
A \arrow[r, "\alpha"] \arrow[d, "\gamma"'] \arrow[dr, phantom, "\circlearrowright"] & B \arrow[d, "\beta"] \\
C \arrow[r, "\delta"'] & D
\end{tikzcd}
\iff
\forall_{x \in A} : \quad \beta \circ \alpha(x) = \delta \circ \gamma(x)
$$

# Comutative diagrams

$$
\begin{tikzcd}
A \arrow[r, "\alpha"] \arrow[d, "\gamma"'] \arrow[rd, bend left=60, "\beta \circ \alpha"] \arrow[rd, bend right=60, "\delta \circ \gamma"'] \arrow[rd, phantom, "\circlearrowright"] & B \arrow[d, "\beta"] \\
C \arrow[r, "\delta"'] & D
\end{tikzcd}
$$

$$\Longleftrightarrow \quad \forall_{x \in A} : \quad \beta \circ \alpha(x) = \delta \circ \gamma(x)$$

path = path

# Group $G$

i) ass.

$$(g \circ h) \circ k = g \circ (h \circ k)$$

ii) neut.

$$\exists_e : ge = eg = g$$

iii) inv.

$$\forall_g \exists_{g^{-1}} : gg^{-1} = g^{-1} \circ g = e$$

# Group $G$

i) ass.

$$(g \cdot h) \cdot k = g \cdot (h \cdot k)$$

ii) neut.

$$\exists_e : ge = eg = g$$

iii) inv.

$$\forall_g \exists_{g^{-1}} : g g^{-1} = g^{-1} \circ g = e$$

$$m : G \times G \ni (g, h) \longmapsto g \cdot h \in G$$

i)

$$\begin{tikzcd}
G \times G \times G \arrow[r, "m \times id"] \arrow[d, "id \times m"'] \arrow[dr, phantom, "\circlearrowleft"] & G \times G \arrow[d, "m"] \\
G \times G \arrow[r, "m"'] & G
\end{tikzcd}$$

ii)

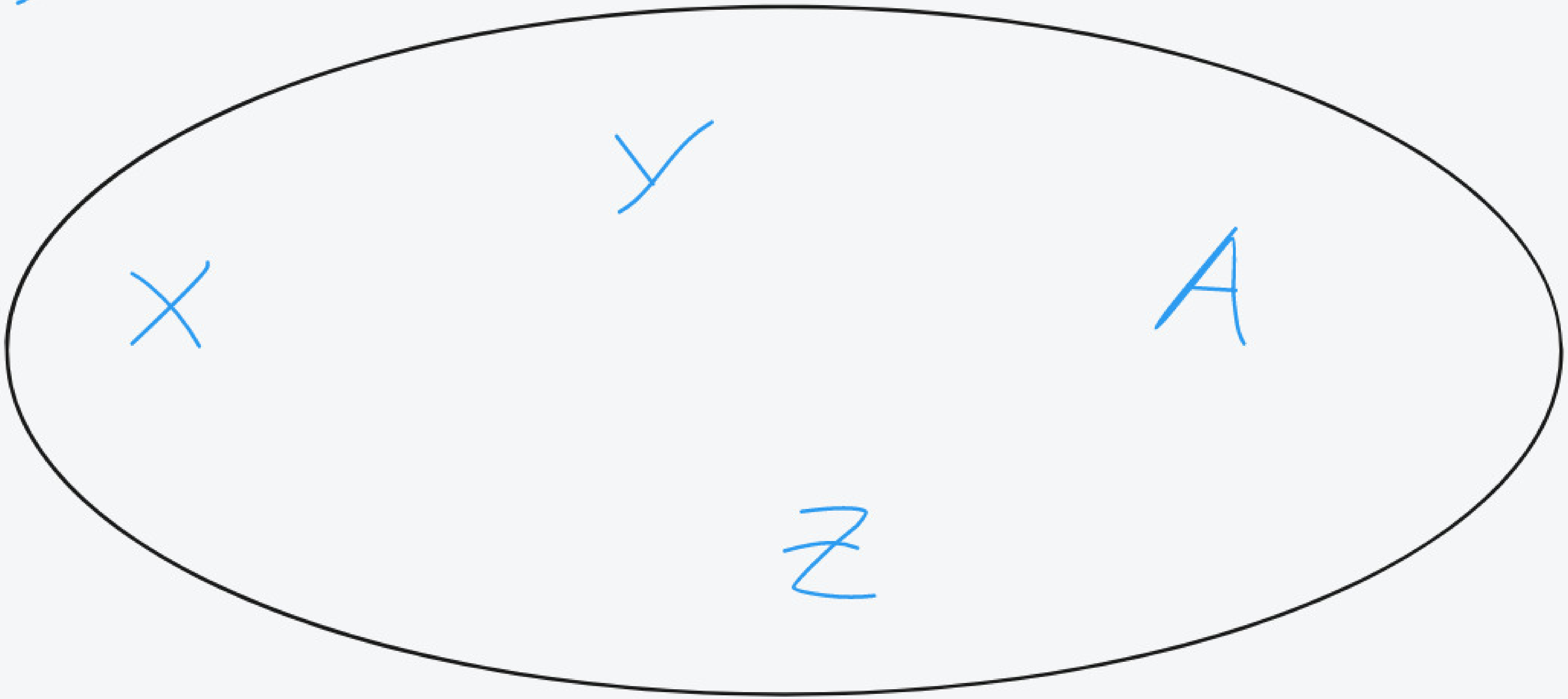
$$\begin{tikzcd}
G \times \{e\} \arrow[r, "\sim"] \arrow[dr, "pr_1"'] & G \times G \arrow[d, "m"] \arrow[dl, phantom, "\circlearrowleft"] \\
 & G
\end{tikzcd}$$

iii)

$$\begin{tikzcd}
G \times \{e\} \arrow[r, "(id, ()^{-1}) \circ pr_1"] \arrow[dr, "pr_2"'] & G \times G \arrow[d, "m"] \arrow[dl, phantom, "\circlearrowleft"] \\
 & G
\end{tikzcd}$$

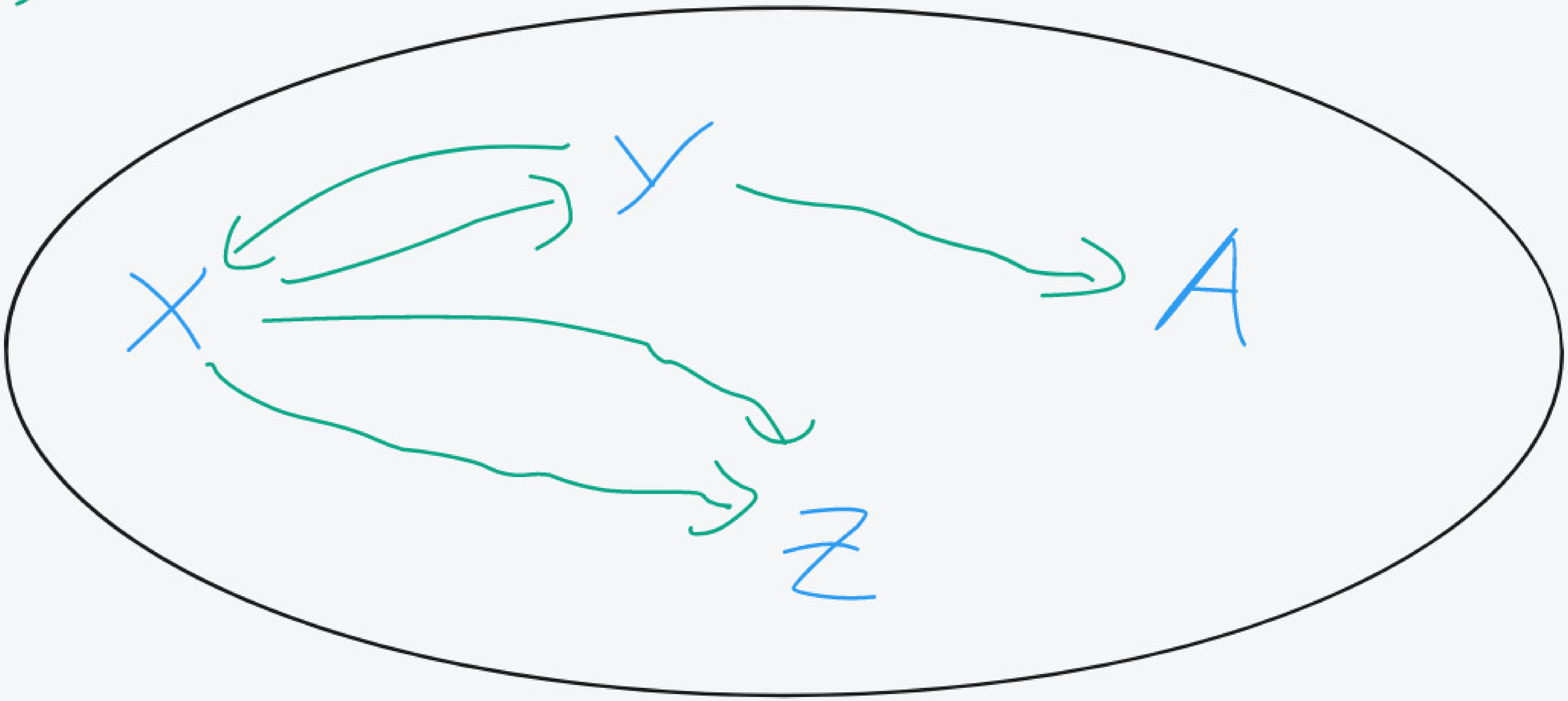
# Category

„set" of objects: Ob



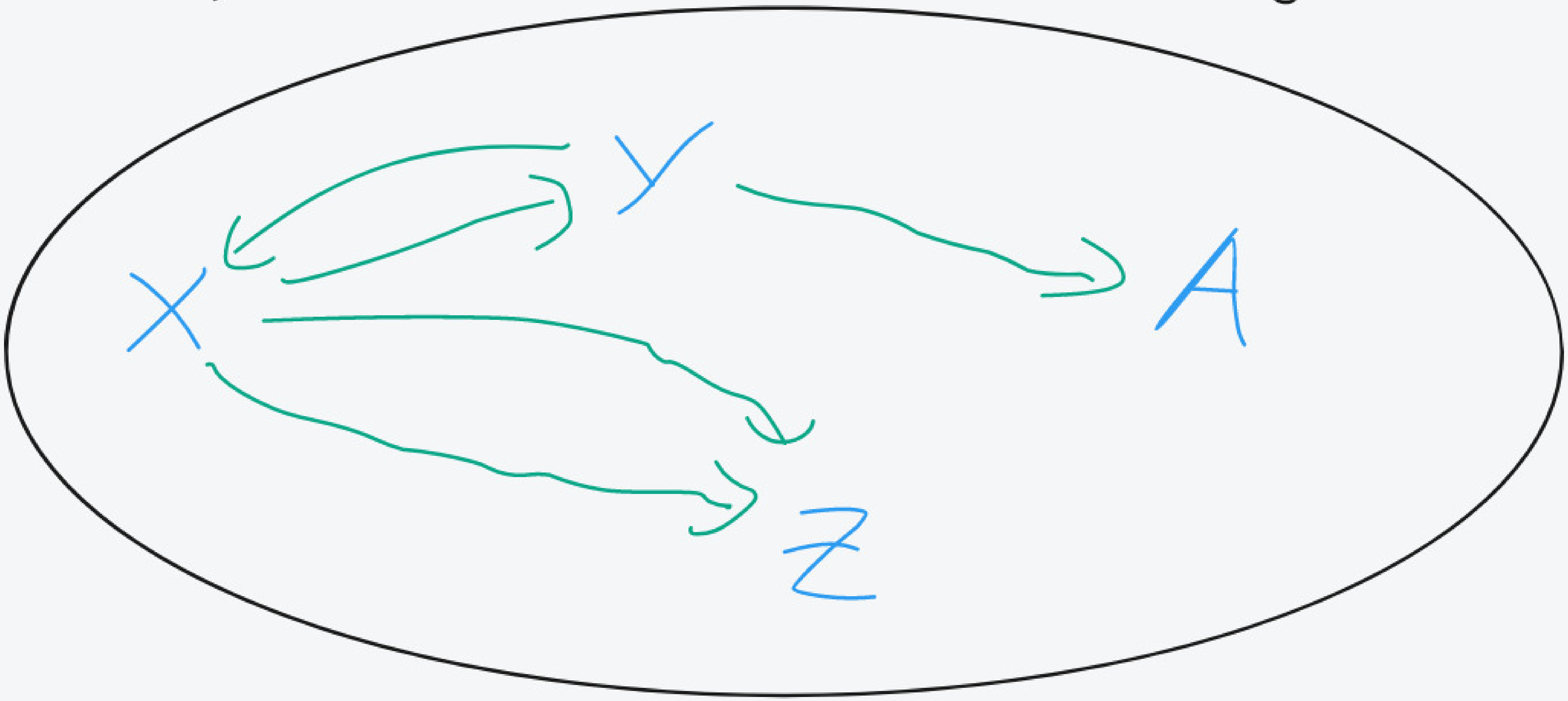
# Category

„set" of morphisms: Mor

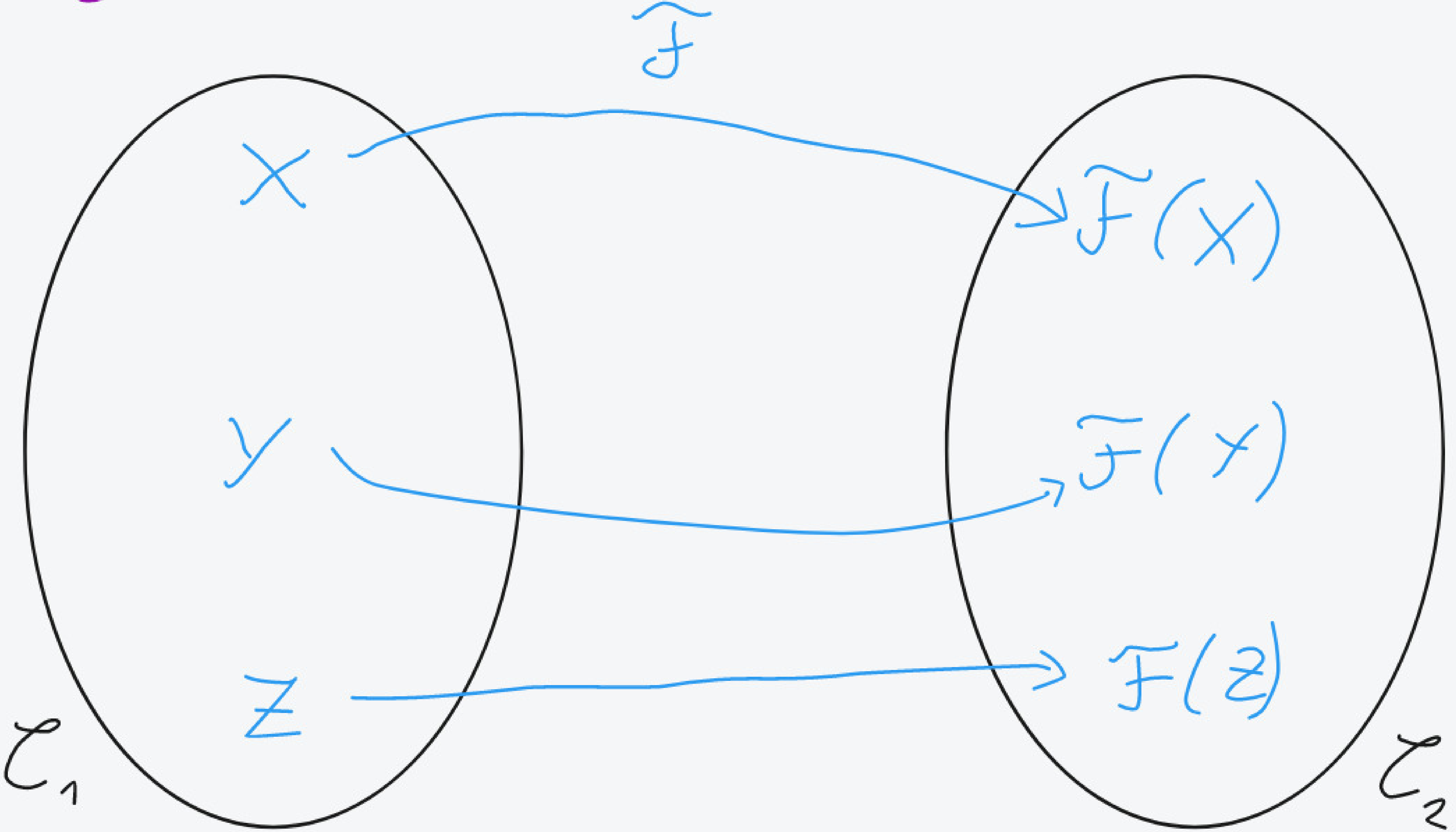


# Category $\mathcal{C}$

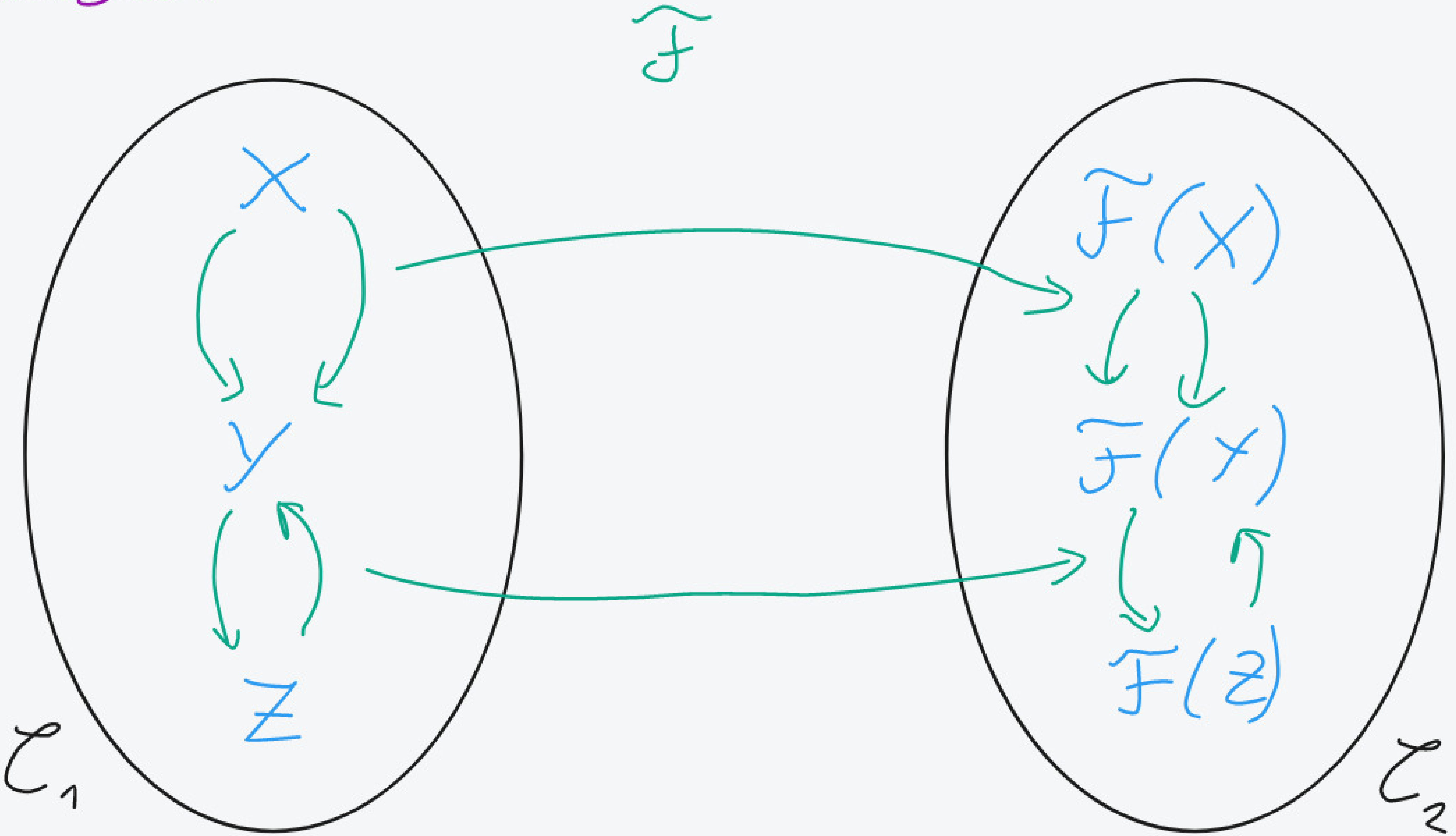
$(\mathrm{Ob}\,\mathcal{C}, \mathrm{Mor}\,\mathcal{C})$ & some relations $\Rightarrow$ Category $\mathcal{C}$



Functor $\mathcal{F}$

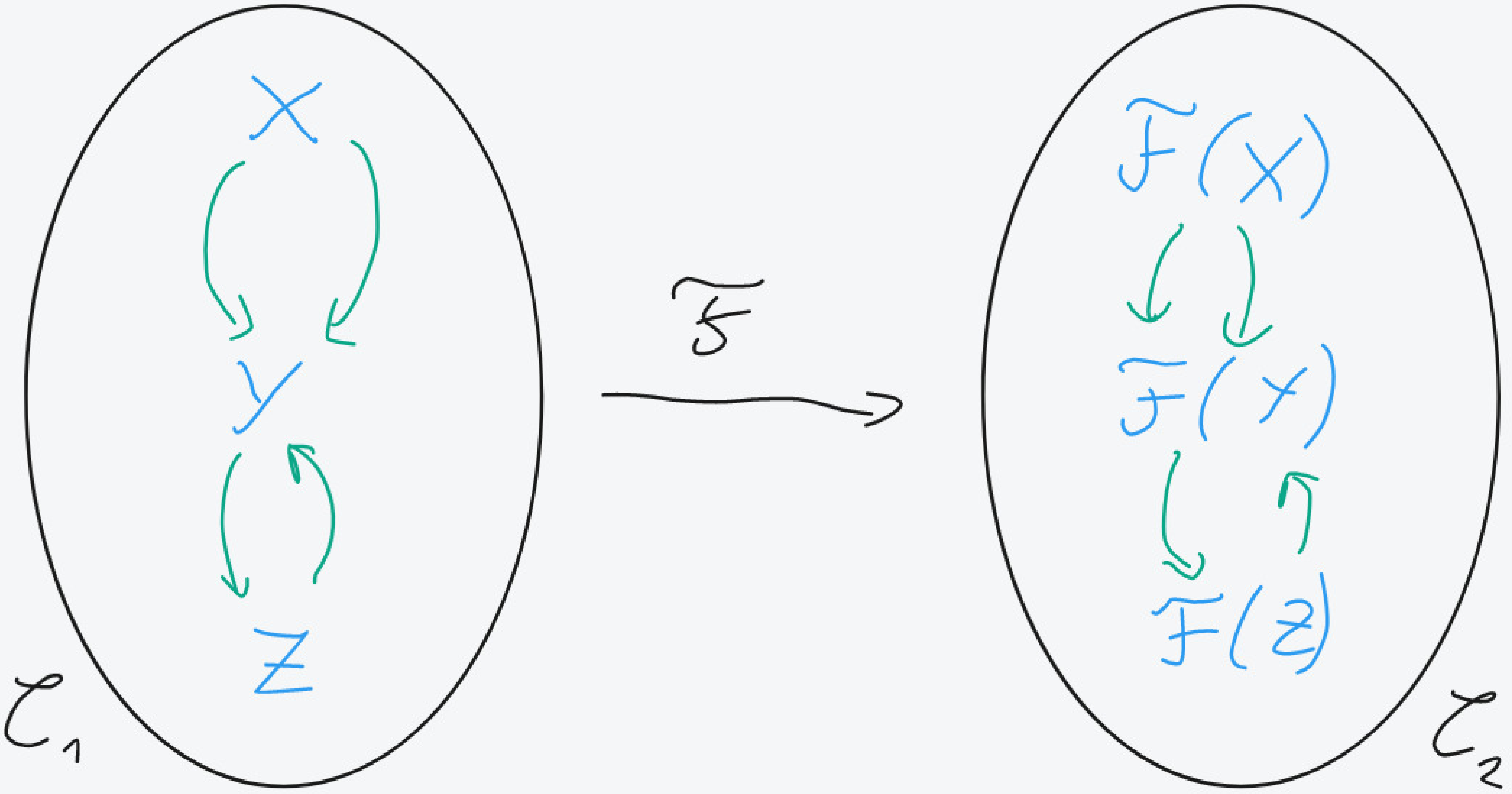


# Functor $\mathcal{F}$



# Functor $\mathcal{F}$

$\mathcal{F}$ : Ob morphisms & Mor morphisms



# One of first Functors

$$\text{Category Top:} \xrightarrow[\pi_1]{\text{Fundamental group}} \text{Category Grp}$$

Ob Top - Topological spaces

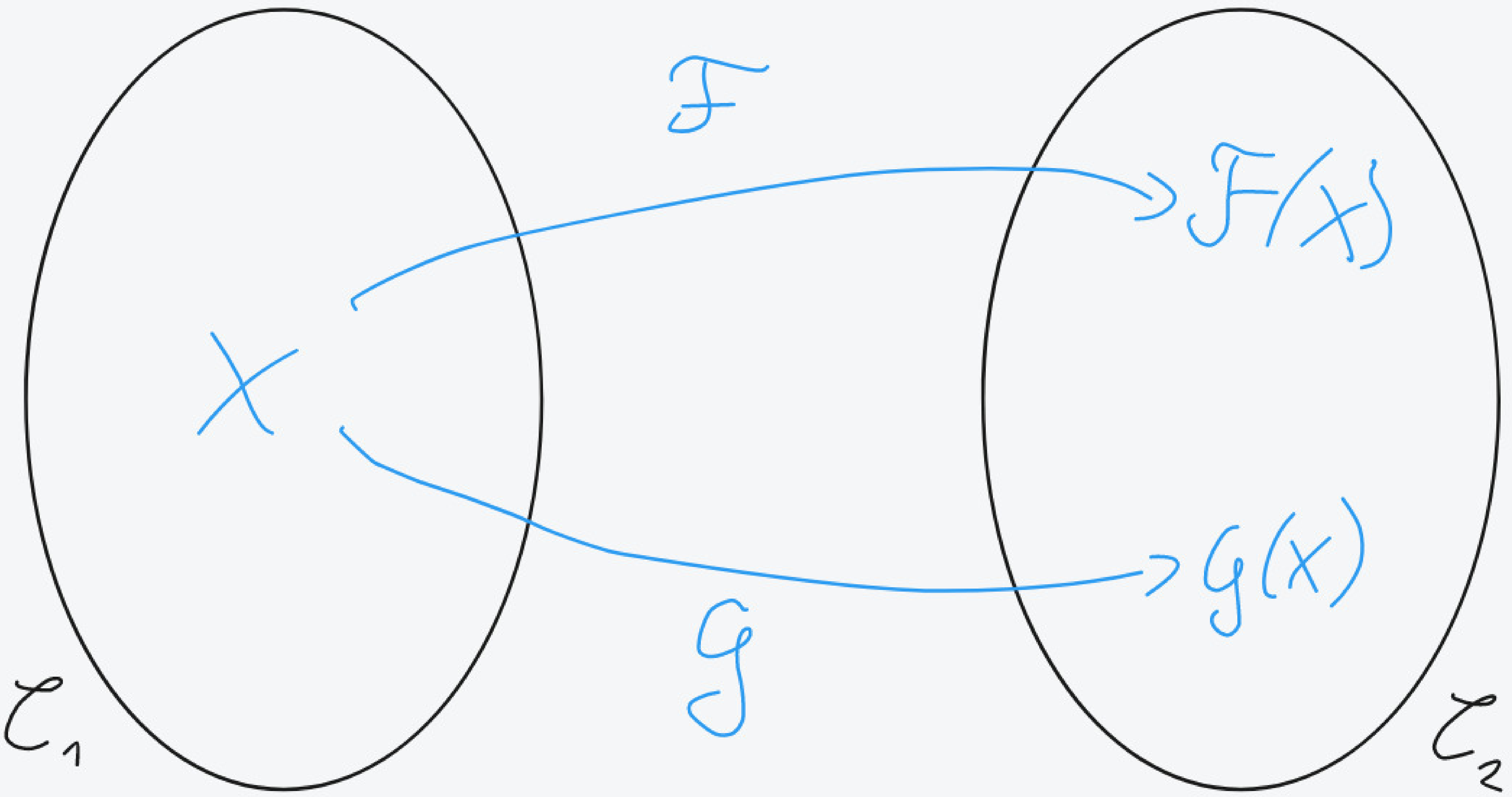
Mor Top - continuous maps

Ob Grp - groups

Mor Grp - homomorphisms

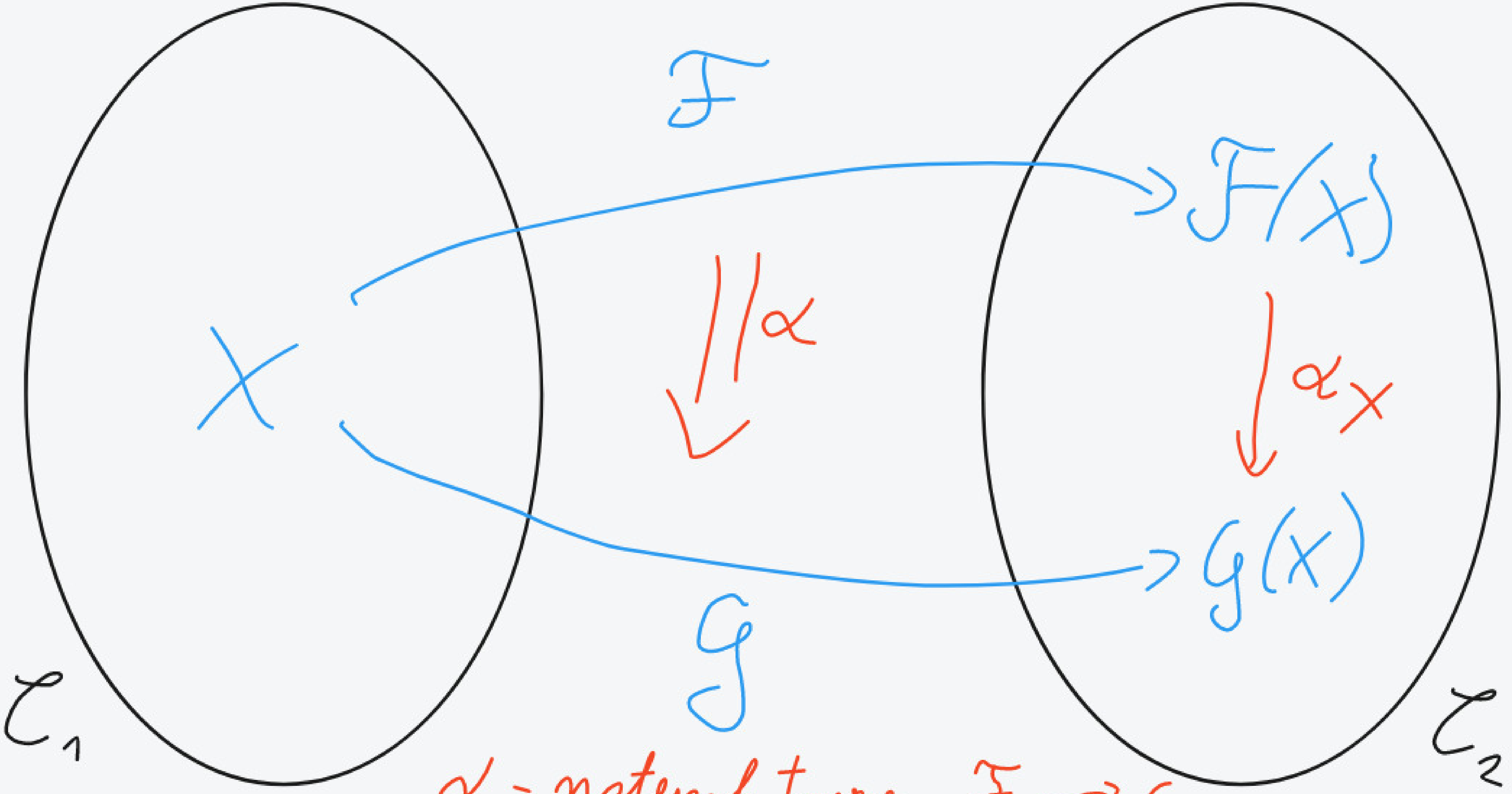
## Natural transformation

$\mathcal{F}, \mathcal{G}$ - Functors



# Natural transformation

$\mathcal{F}, \mathcal{G}$ - Functors




$\alpha$ - natural trans. $\mathcal{F} \longrightarrow \mathcal{G}$

# Composition of Nat. trans.

Vertical

$$\begin{tikzcd}[row sep=large]
\mathcal{C}_1 \arrow[r, bend left=60, "\mathcal{F}", ""{name=F, below}] \arrow[r, "\mathcal{G}" description, ""{name=G}] \arrow[r, bend right=60, "\mathcal{H}"', ""{name=H, above}] & \mathcal{C}_2
\arrow[from=F, to=G, Rightarrow, "\alpha^1"]
\arrow[from=G, to=H, Rightarrow, "\alpha^2"]
\end{tikzcd}$$

Horizontal

$$\begin{tikzcd}
\mathcal{C}_1 \arrow[r, bend left=50, "\mathcal{F}^1", ""{name=A, below}] \arrow[r, bend right=50, "\mathcal{F}_2"', ""{name=B, above}] & \mathcal{C}_2 \arrow[r, bend left=50, "\mathcal{G}^1", ""{name=C, below}] \arrow[r, bend right=50, "\mathcal{G}^2"', ""{name=D, above}] & \mathcal{C}_3
\arrow[from=A, to=B, Rightarrow, "\alpha"]
\arrow[from=C, to=D, Rightarrow, "\beta"]
\end{tikzcd}$$

# Next plans/partially fullfiled

- Yoneda Lemma

- Universal objects

$\Downarrow$

- Tensor Product as an Universal object

$\otimes$

Thank You !