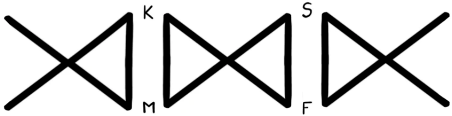
Higher geometry of charged dynamics

Study Team: Kuba Filipek, Damian Kayzer, Piotr Schmidt, Jan Turczynowicz

Supervisor: Rafał R. Suszek



Goal

- To understand which geometric structures describe dynamics of charged particles in non-contractible spacetime.

What we did already

Initial data:

- $m \in \mathbb{R}_+$, $q \in \mathbb{R}$ (mass and charge of particle)
- Riemannian manifold (M, g) (spacetime and gravitational field)
- Closed 2-form F (electromagnetic field)

What we seek for:

- Action functional S such that principle of stationary action recover Lorentz force formula:

$$m g_{\lambda\mu} \left(\ddot{x}^{\mu} + \Gamma^{\mu}_{\nu\rho} \dot{x}^{\nu} \dot{x}^{\rho} \right) = q F_{\lambda\mu} \dot{x}^{\mu}$$

What we did already

- If global potential $A \in \Omega^1(M)$: $\mathsf{d}A = F$ exists:

$$S[x] = \frac{m}{2} \int_{[t_0,t_1]} x^* g(\partial_\tau, \partial_\tau) \mathsf{d}\tau + q \int_{[t_0,t_1]} x^* A$$

  or in coordinates:

$$S[x] = \frac{m}{2} \int_{[t_0,t_1]} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \mathsf{d}\tau + q \int_{[t_0,t_1]} A_\mu \dot{x}^\mu \mathsf{d}\tau$$

- what if such potential doesn't exist, what might be the case if M is not contractible

What we did already

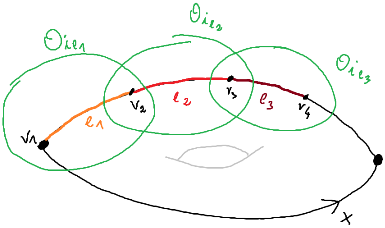
- choose good cover $\{\mathcal{O}_i\}$ (intersections $\mathcal{O}_{i_1} \cap \ldots \cap \mathcal{O}_{i_n}$ are contractible)

What we did already

- choose good cover $\{\mathcal{O}_i\}$ (intersections $\mathcal{O}_{i_1} \cap \ldots \cap \mathcal{O}_{i_n}$ are contractible)
- choose local potentials $A_i \in \Omega^1(\mathcal{O}_i)$: $\mathrm{d}A_i = F \restriction_{O_i}$

What we did already

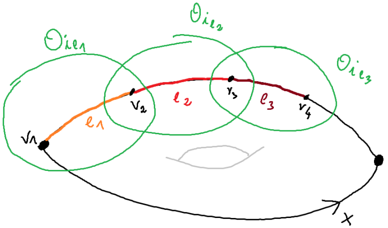
- choose good cover $\{\mathcal{O}_i\}$ (intersections $\mathcal{O}_{i_1} \cap \ldots \cap \mathcal{O}_{i_n}$ are contractible)
- choose local potentials $A_i \in \Omega^1(\mathcal{O}_i)$: $\mathrm{d}A_i = F \restriction_{O_i}$
- take loop $x \in \mathcal{C}^\infty(S^1, M)$ and choose its tesselation $\Delta = E \cup V$ (edges + vertices) subordinate to $\{\mathcal{O}_i\}$



What we did already

- choose good cover $\{\mathcal{O}_i\}$ (intersections $\mathcal{O}_{i_1} \cap \ldots \cap \mathcal{O}_{i_n}$ are contractible)
- choose local potentials $A_i \in \Omega^1(\mathcal{O}_i)$: $\mathrm{d}A_i = F \restriction_{O_i}$
- take loop $x \in \mathcal{C}^\infty(S^1, M)$ and choose its tesselation $\Delta = E \cup V$ (edges + vertices) subordinate to $\{\mathcal{O}_i\}$
- choose functions $f_{ij} \in \mathcal{C}^\infty(\mathcal{O}_i \cap \mathcal{O}_j)$: $\mathrm{d}f_{ij} = (A_j - A_i) \restriction_{\mathcal{O}_i \cap \mathcal{O}_j}$ and action functional is:

$$S[x] = \frac{m}{2} \int_{S^1} x^* g(\partial_\tau, \partial_\tau) \mathrm{d}\tau + q \sum_{e \in E} \left(\int_e (x \restriction_e)^* A_{i_e} + \sum_{v \in \partial e} \varepsilon_{ev} f_{i_e i_v} \circ x(v) \right)$$

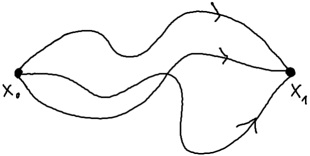


What we did already

We went through

- Dirac's heuristic argument for link between quantum amplitude and $\mathrm{e}^{\frac{\mathrm{i}}{\hbar}S}$
- Feynman's heuristic derivation of path integral formulation

$$\langle x_1|\,\mathrm{e}^{-\frac{\mathrm{i}t}{\hbar}\hat{H}}\,|x_0\rangle = \int_{\substack{x(0)=x_0\\x(t)=x_1}} \mathrm{e}^{\frac{\mathrm{i}}{\hbar}S[x]}\mathcal{D}x$$



What are we going to do

- QM requirement of $\mathrm{e}^{\frac{\mathrm{i}}{\hbar}S}$ to be well defined implies Dirac quantization condition
- circle/complex line bundle structure induced by additional term and connection (F curvature)
- geometric quantization
- Čech-De-Rham bi-complex
- characteristic classes
- S for open curves
- symmetry of S