Supersymmetry breaking in STRING THEORY

Review based on old work in collaboration .with

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(1998-2001, 2010,2012) + work in progress with
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Content

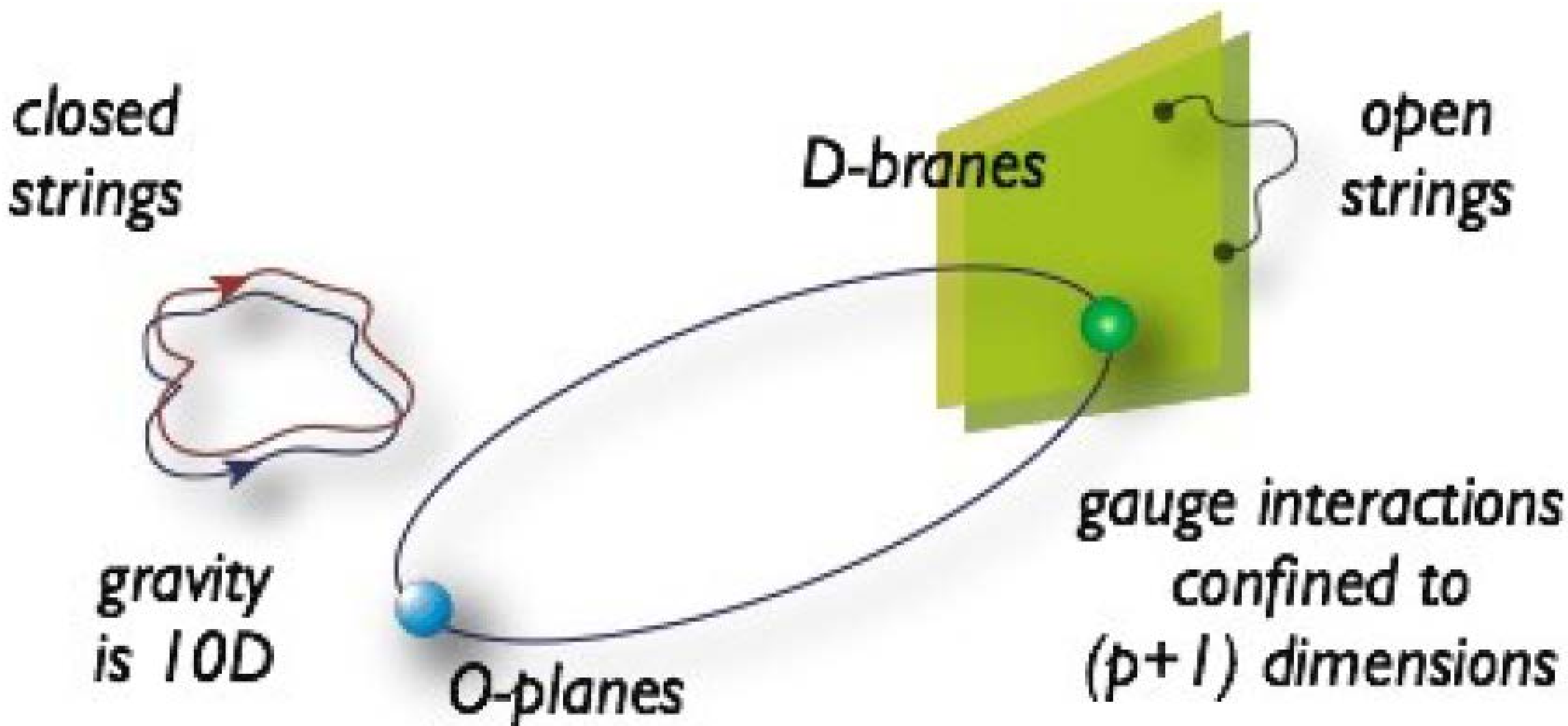
- Orientifolds
- Supersymmetry breaking by compactification
- Tachyon-free non-BPS models (Brane supersymmetry breaking)
- Internal magnetic fields $\Longleftrightarrow$ intersecting branes
- Ground state and D-brane solutions in non-SUSY strings
- Applications to Cosmology
- Conclusions

Orientifolds

Prototype: type I strings = IIB/Ω , where Ω is a left-right projection (involution).

Orientifolds have closed and open strings.

Cartoon picture of type II orientifolds (Sagnotti):
open/closed strings, Dp-branes/O-planes

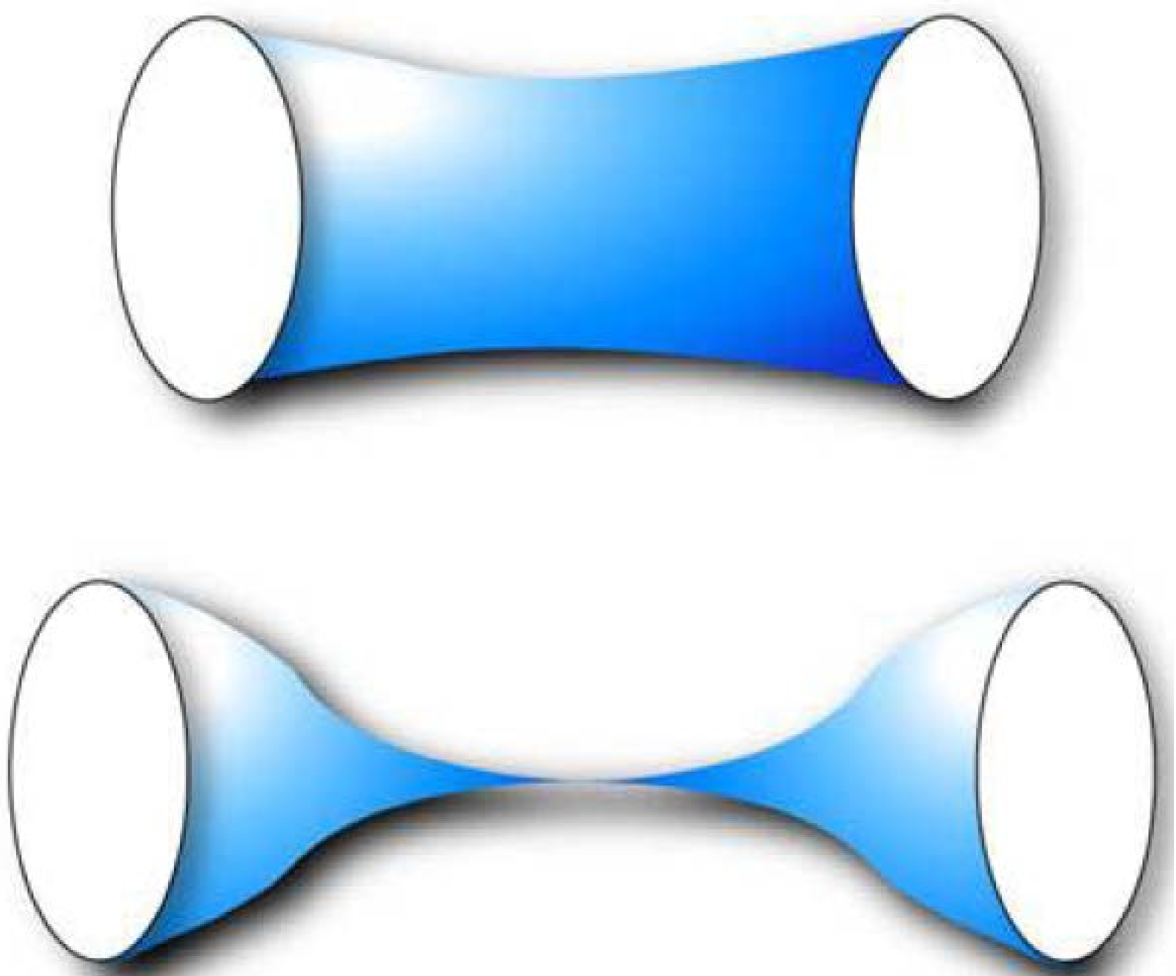


D-branes/O-planes have Tension and Charges

(T_p, q_p) Crucial constraint: RR tadpole constraints

UV finiteness $\Longleftrightarrow$ Gauss law in internal space

$$\sum_{Dp} q_{Dp}^{(n)} + \sum q_{Op}^{(n)} = 0 \quad ; \quad SUSY \quad \rightarrow \quad T_p = q_p \tag{1}$$



Supersymmetry breaking by compactification

The Scherk-Schwarz mechanism (ScherkSchwarz; Fayet; Rohm; Ferrara,Kounnas,Porrati,Zwirner)

Main idea : use symmetries S of the higher-dimensional theory which do not commute with supersymmetry :

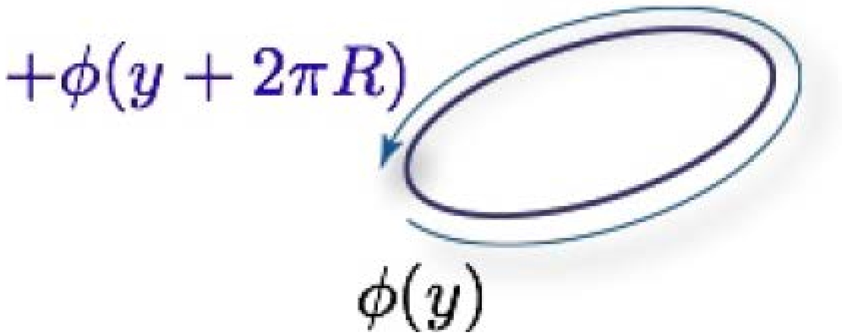
R-symmetries or the fermion number (-1)^F.

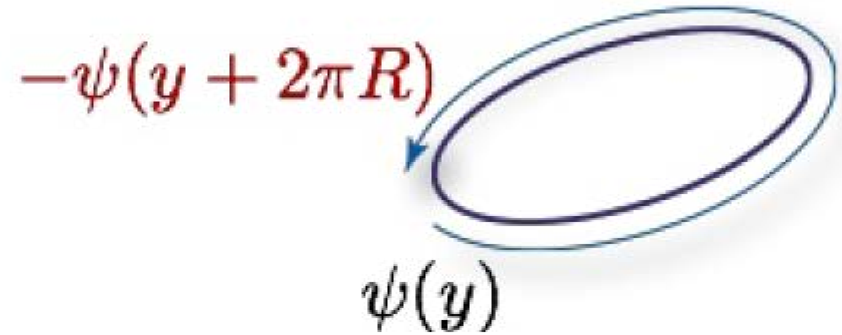
After being transported around the compact space, bosonic and fermionic fields Φ return to the initial value (at y=0)

only up to a symmetry operation

$$\Phi_i(2\pi R, x) = U_{ij}(\omega)\Phi_j(0, x)$$

where the matrix $U \in \mathcal{S}$ is different for bosons and fermions.





$$\Phi(y,\mathbf{x}) = \sum_k e^{\frac{iky}{R}} \Phi(\mathbf{x})^{(k)} \rightarrow M_k = \frac{k}{R} \,,$$

$$\Psi(y,\mathbf{x}) = \sum_k e^{\frac{i(k+1/2)y}{R}} \Phi(\mathbf{x})^{(k)} \rightarrow M_k = \frac{(k+1/2)}{R}$$

This procedure is very similar to the breaking of supersymmetry at finite temperature

The terms breaking supersymmetry generate UV finite effects, even at the field theory level.

In models with D-branes there are two different ways in which SUSY can be broken by compactification

(Antoniadis,E.D.,Sagnotti)

- The D brane is parallel to the direction of breaking ; D brane spectrum experience tree-level SUSY breaking. This is the analog of the heterotic constructions:

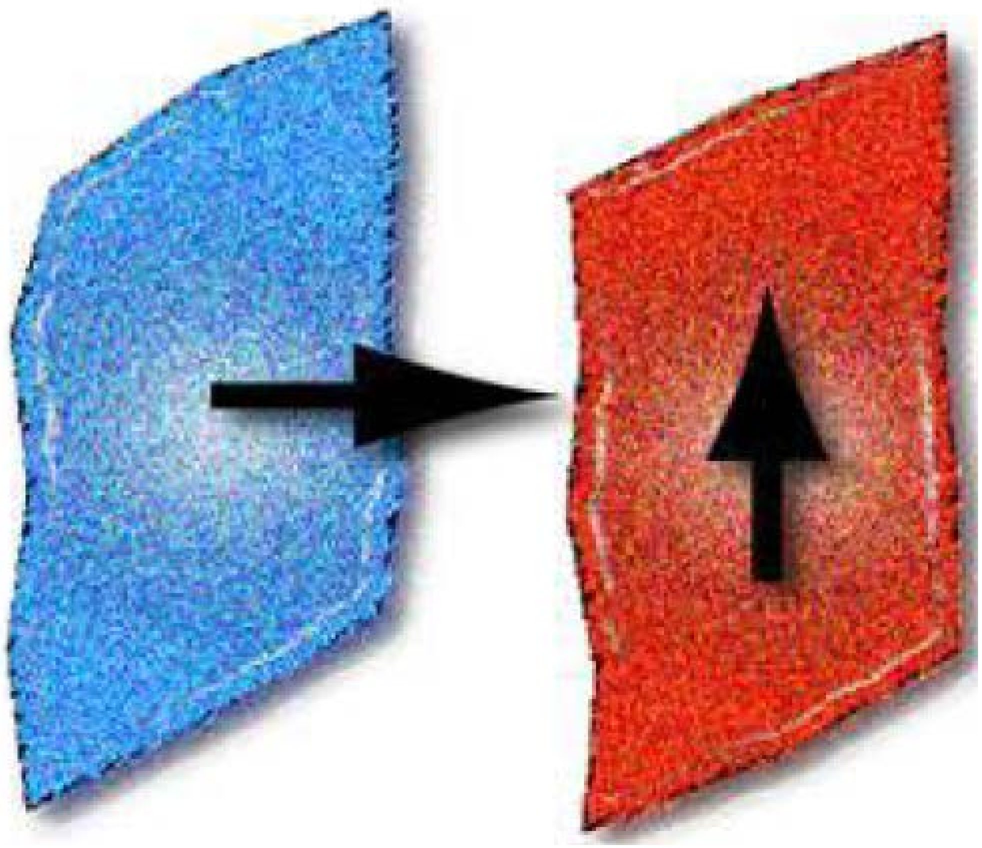
$$m_{3/2} \sim M_{1/2} \sim \frac{1}{R} \quad \text{(tree-level)}$$

- the D brane is perpendicular to the direction of the breaking; massless D brane spectrum is SUSY at tree-level. SUSY breaking transmitted by radiative corrections from the brane massive states or from the gravitational sector.

$$m_{3/2} \sim \frac{1}{R} \quad , \quad m_0 \sim \frac{1}{R^2 M_P} \quad , \quad M_{1/2} \sim \frac{1}{R^3 M_P^2}$$

(tree-level) (one-loop)

Parallel and perpendicular Scherk-Schwarz breaking



Parallel dims ⇒ TeV radii $\qquad M_{SUSY} \sim R^{-1}$

Perpendicular dims ⇒ intermediate radii $\qquad M_{SUSY} \sim \frac{R^{-2}}{M_P}$

Problem : large cosmological constant

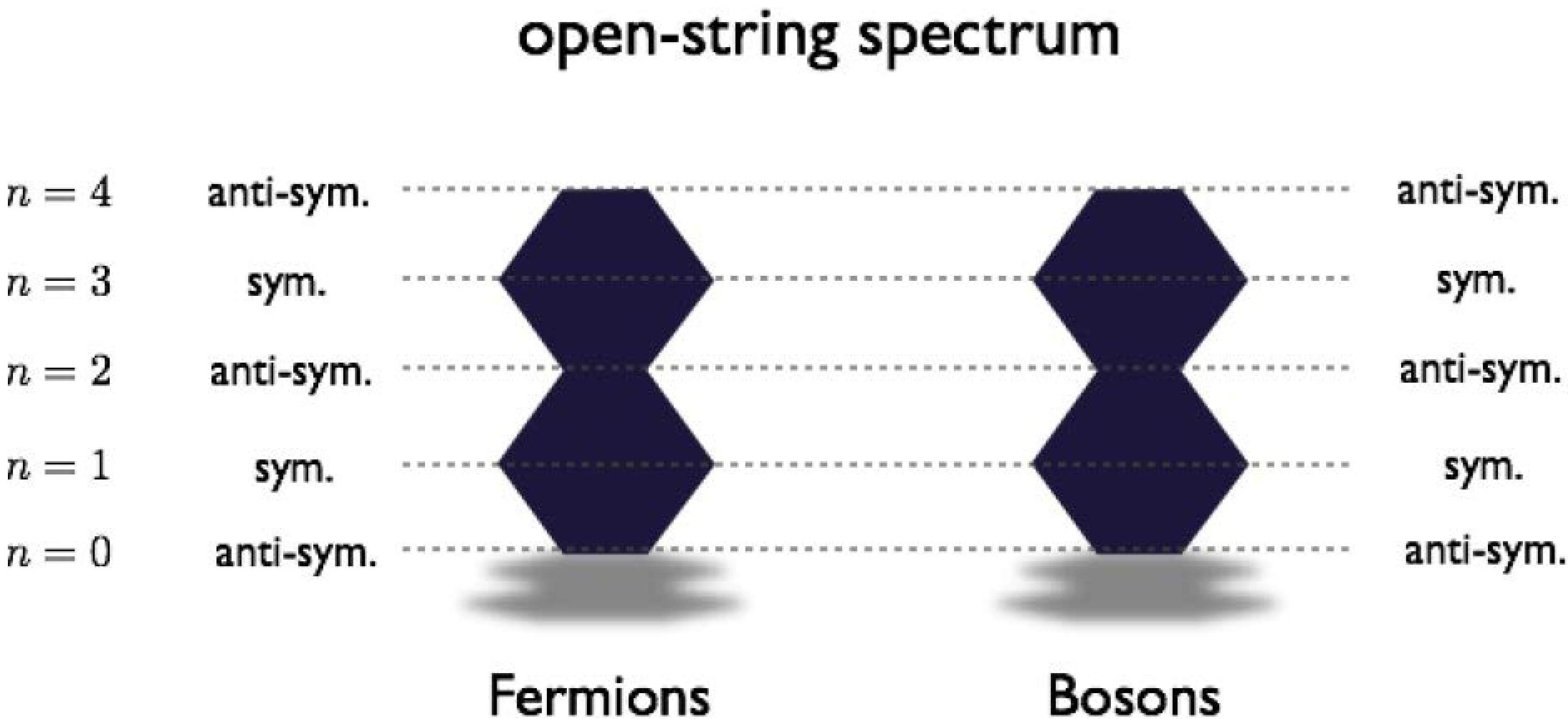
Tachyon-free non-BPS models (Brane supersymmetry breaking)

(Sugimoto; Antoniadis, E.D.,Sagnotti,1999)

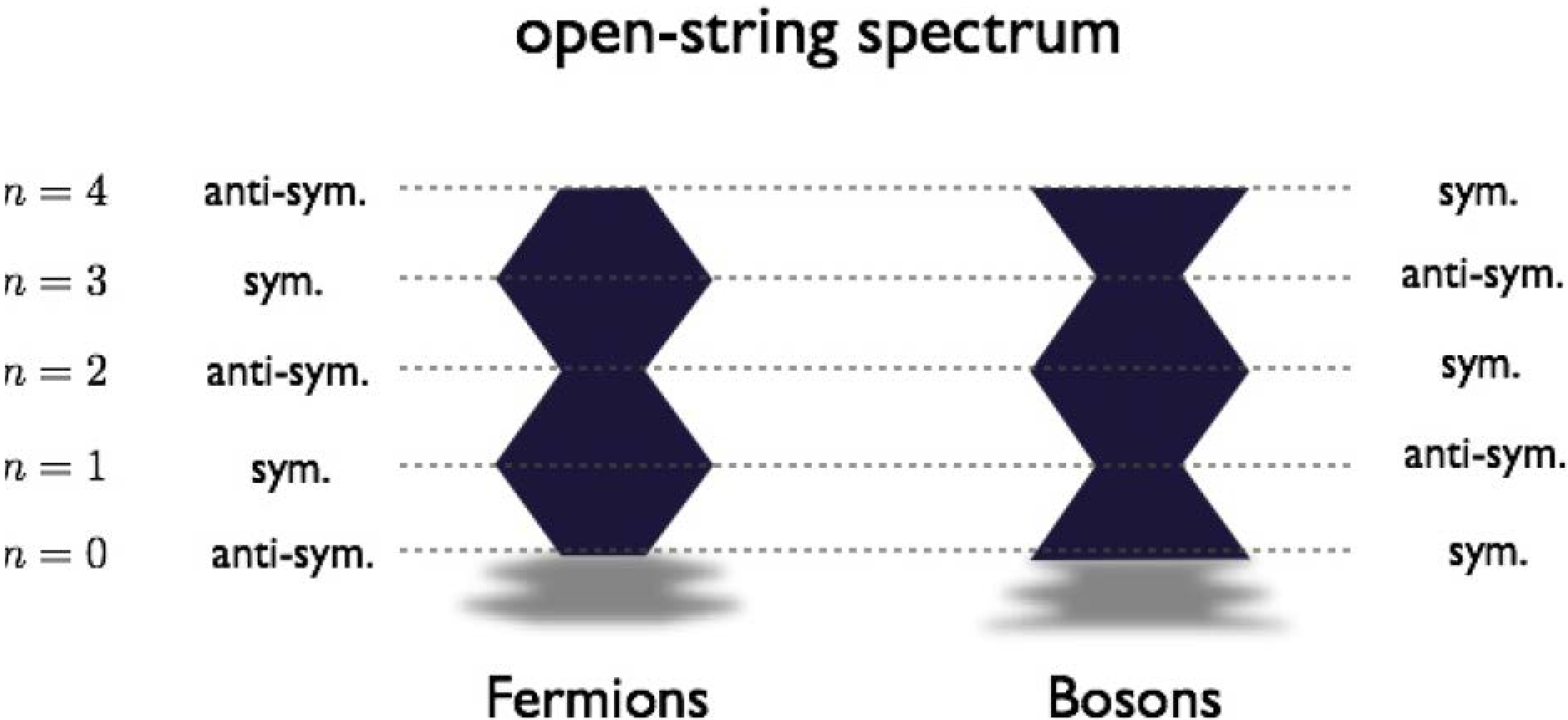
In these constructions, the closed (bulk) sector is SUSY to lowest order, whereas SUSY is broken at the string scale on some stack of (anti)branes.

- String consistency asks for the existence of exotic $O9_{+}$ planes of positive RR charge. Then charge conservation /RR tadpoles ask for antibranes in the open sector.

SUSY case (SO gauge group) : Bose-Fermi degeneracy



Brane SUSY breaking case (USp gauge group): spectrum is "misaligned"



$\overline{D}p - Op^{+}$ system is non-BPS but tachyon-free.

It breaks SUSY at string scale in the open sector, closed/gravitational sector SUSY at tree-level.

- There is a NS-NS dilaton tadpole

$$\sum_{Dp} q_{Dp}^{(n)} + \sum_{Op} q_{Op}^{(n)} \neq 0$$

- Singlet in the open string fermionic spectrum which can be correctly identified with the Goldstino realizing a nonlinear SUSY on antibranes (E.D.,Mourad; 2000).
- - No obvious candidate for decay to a SUSY vacuum
- (folklore : non-SUSY vacua decay into SUSY ones).
- Suggestion : nonperturbative instabilites (Angelantonj,E.D.;2007)

- - low-string scale ⇒ light moduli $m \sim M_s^2/M_P$

The 10d $USp(32)$ Type I model contains the gravitational multiplet with bosonic fields (g_{MN}, B_{MN}, ϕ) and fermions (Ψ_M, λ) in the closed spectrum. The open spectrum contains gauge fields A_M in the adjoint of the gauge group, fermions χ in the $\mathbf{495}$ and the fermion gauge singlet θ.

Up to terms quadratic in fermions, the effective lagrangian of the model in the Type I string metric, to be found explicitly in the following sections, reads

$$\begin{aligned} L &= L_{SUGRA} + \sqrt{-G}e^{-\hat{\phi}}\Big[-\frac{1}{4}G^{MP}G^{NQ}\; tr(F_{MN}F_{PQ}) - \Lambda\Big] + \\ & \sqrt{-g}\left\{-\frac{1}{2}tr\bar{\chi}\Gamma^M D_M\chi + \frac{1}{16}tr\bar{\chi}\Gamma^{MNP}\chi H_{MNP} + T^{MNPQ}\; tr(F_{[MN}F_{PQ]})\right\}, \end{aligned} \quad (3)$$

where G_{MN} and $\hat{\phi}$ are the modified Volkov-Akulov metric and the dilaton, respectively, defined in (26), (27) and we introduced the tensor notation

$$\begin{aligned} T^{MNPQ} &= -\frac{1}{16}e^{-\phi}\bar{\theta}\Gamma^{MNPQR}(-\Psi_R + \frac{1}{2}D_R\theta + \frac{\sqrt{2}}{6}\Gamma_R\lambda) \\ & -\frac{\bar{\theta}}{16}[(\frac{1}{16}\Gamma^{MNPQN_1N_2N_3} + \frac{9}{4}g^{[MN_1}g^{NN_2}\Gamma^{PQN_3]})H_{N_1N_2N_3} + e^{-\phi}\Gamma^{[NPQ}\partial^{M]}\phi]\theta\ . \end{aligned} \quad (4)$$

where

$$
\begin{aligned}
E^a &= e^a + \frac{1}{4}\bar{\theta}\Gamma^a D_M \theta dx^M - \frac{1}{2}\bar{\theta}\Gamma^a \Psi_M dx^M + \frac{1}{2}\bar{\theta} S^a{}_b \theta e^b \ , \ G_{MN} = E^a_M E^b_N \eta_{ab} \ , \\
\hat{\phi} &= \phi - \frac{1}{\sqrt{2}}\bar{\theta}\lambda - \frac{1}{32} e^{\phi}\bar{\theta}\Gamma^{PQR}\theta H_{PQR} \ ,
\end{aligned} \tag{26}
$$

with

$$
S^{ab} = -\frac{1}{32} e^{\phi}[\Gamma^{PQR} H_{PQR}\eta^{ab} - 6 H_{PQ}{}^{R}\Gamma^{PQ(b} e_R^{a)}] \ . \tag{27}
$$

Then, by using (17), it is easily verified that, up to terms in (fermi)^3, E^a and $\hat{\phi}$ transform under local supersymmetry as

$$
\delta\hat{\phi} = L_v \hat{\phi} \ , \quad \delta E^a = L_v E^a + \Lambda^a{}_b E^b \ , \tag{28}
$$

where $v = -\frac{1}{4}\bar{\theta}\Gamma^M \eta \ \partial_M$ implements a reparametrization

and Λ_{ab} is a local Lorentz transformation.

SUSY acts in the open sector in the standard non-linear way: reparametrization + Lorentz

So the effective action of this 10d USp(32) non-linear SUSY , tachyon-free string , seems consistent (other 6d and 4d similar examples known) .

There are however several puzzles about this BSB (Brane Supersymmetry Breaking) theory and its lower dim. cousins :

- Are they really stable ?
- Not possible to have a super-Higgs mechanism in 10d.

Degree of freedom of massless gravitino + goldstino $\theta - 64$

Degree of freedom of massive gravitino $\Psi_M - 128$

However, in 9d this seems possible:

Degree of freedom of massive 9d gravitino + Ψ_9 = 64

What does this mean ???

Internal magnetic fields intersecting branes

Particles of different spin couple differently to magnetic field, breaking supersymmetry. Define

$$\theta_i = arctan(\pi q_L^{(i)} H_i) + arctan(\pi q_R^{(i)} H_i)$$

Mass splittings of string states are

$$\delta m^2 = (2n+1)|\epsilon_i| + 2\Sigma_i \epsilon_i \ ,$$

where n are the Landau levels of the charged particles in the magnetic field and Σ_i are internal helicities.

Generic problem : NS-NS tadpoles.

Type II B/ Ω with internal magnetic fields

$\Updownarrow$ T-dual

Type II A/ $\Omega' = \Omega \times \mathcal{I}$ with intersecting branes

Simple way of partially or totally breaking SUSY is by rotating the branes in the compact space.

Type IIA orientifolds : there are three angles $\theta_1, \theta_2, \theta_3$ that D6 brane(s) can make with the horizontal axis x_4, x_6, x_8 of the three torii of the compact space. Preserved supercharge is (Berkooz,Douglas,Leigh)

$$Q \; + \; P \, \tilde{Q} \; ,$$

P is the parity in the space transverse to the D6 brane(s).

Compact space : two important additional ingredients:

- rotations of branes in the compact space are quantized, according to

$$\tan\theta_i^{(a)} = \frac{m_i^{(a)} R_{i2}}{n_i^{(a)} R_{i1}} ,$$

where $(m_i^{(a)}, n_i^{(a)})$ are the *wrapping numbers* of the brane(s) $D^{(a)}$ along the two compact directions of the compact torus T_i^2.

For two stacks of branes $D^{(a)}$ and $D^{(b)}$, the number of times they intersect in the compact torus T_i^2 is given by the *intersection number*

$$I_i^{(ab)} = m_i^{(a)} n_i^{(b)} - n_i^{(a)} m_i^{(b)} .$$

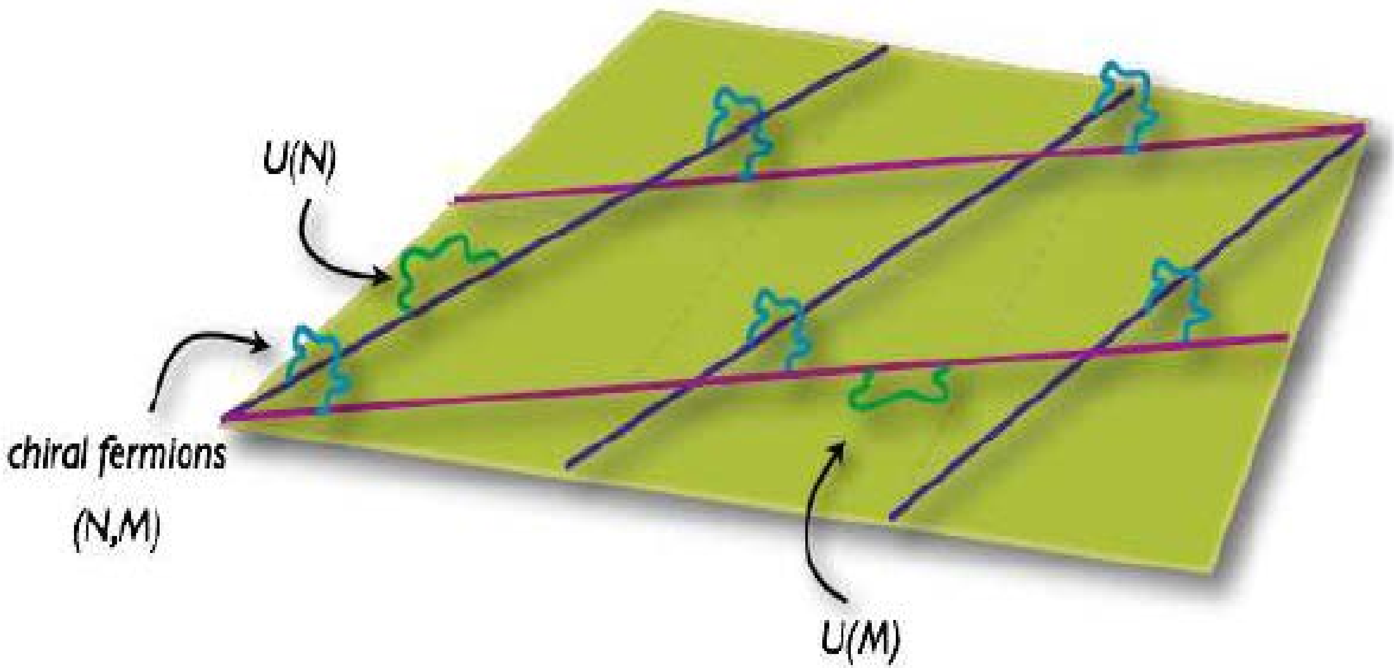
Branes at angles generate 4d chirality. Example: type IIA string with two sets of M_a coincident $D^{(a)}$ and M_b coincident $D^{(b)}$ intersecting branes in toroidal compactification :

- the gauge group is $U(M_a) \otimes U(M_b)$.

- strings stretched between the two D-branes have chiral fermions $(M_a, \bar{M}_b)$

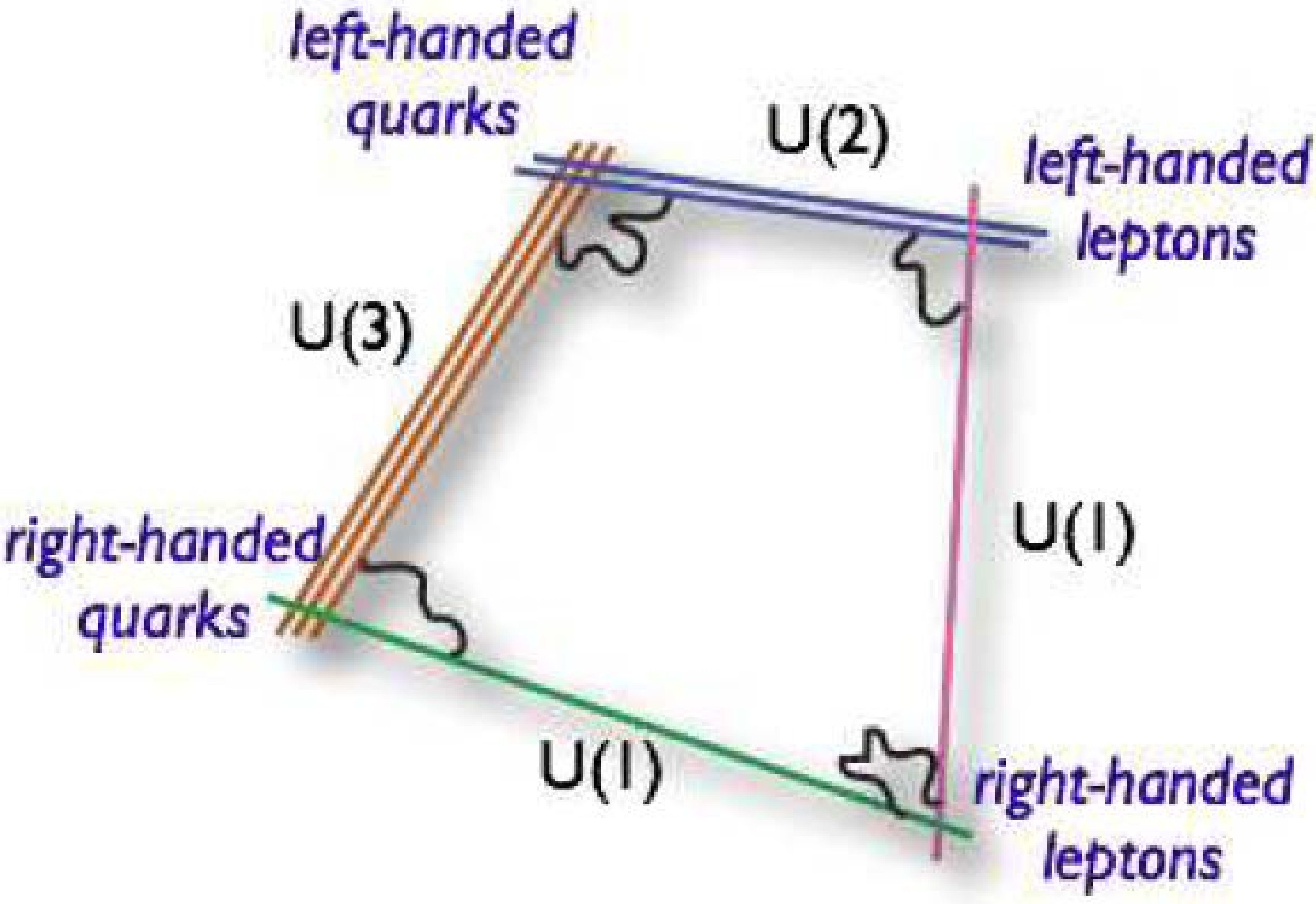
Multiplicity equal to the total number of times the branes intersect in the compact space

$$D^{(a)}\!-\!D^{(b)} \quad : \quad I^{(ab)} = \prod_{i=1}^{3} I_i^{(ab)} = \prod_{i=1}^{3} \left(m_i^{(a)} n_i^{(b)} - n_i^{(a)} m_i^{(b)}\right)$$

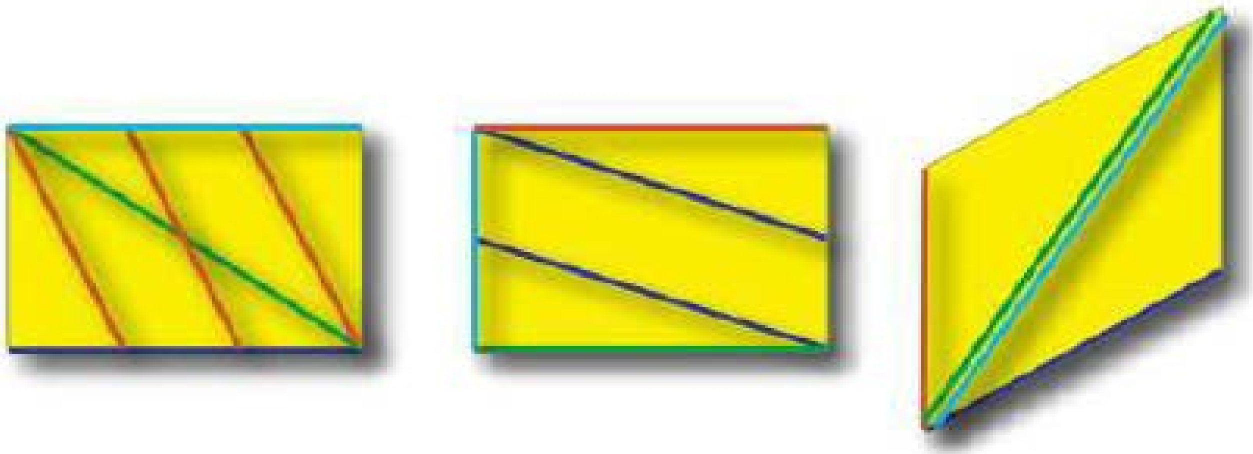


Quasi-realistic models with intersecting were constructed in the last couple of years. The generic Standard Model type construction contains four (or more) stacks, containing D-branes with a minimal gauge group $U(3) \times U(2) \times U(1)^2 = SU(3) \times SU(2) \times U(1)^4$.

"Standard Model" quiver



Intersection pattern



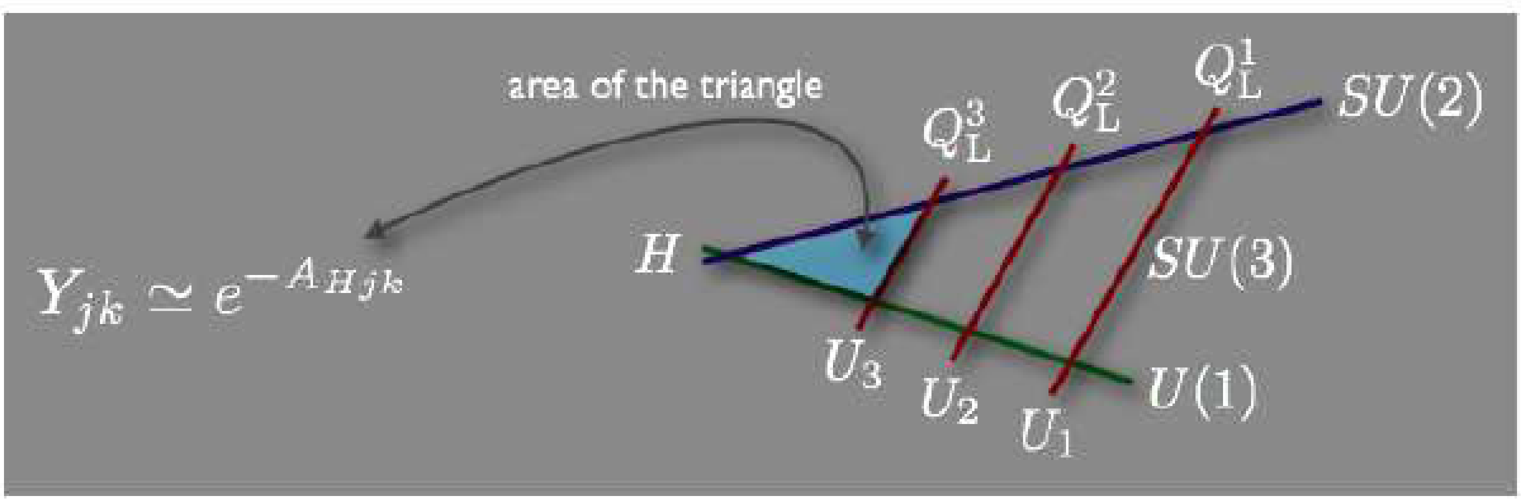
U(3)	(1,0)	(2,-1)	(1,0)
U(2)	(1,-1)	(1,0)	(1,1)
U(1)	(1,-3)	(1,0)	(0,1)
U(1)	(1,0)	(0,1)	(1,1)

Yukawa couplings

Number of Generation = number of intersections between branes.

Then Yukawa couplings have a nice geometrical intepretation

(Cremades,Ibanez,Marchesano)



Ground state and D-brane solutions in non-SUSY strings

SUSY breaking generates dilaton potential / NS-NS tadpoles.

String frame :

$$V \sim e^{-\Phi}(\mathrm{BSB}) \ , V = const \ (\mathrm{Scherk-Schwarz})$$

In both cases, in the Einstein frame they are of the type

$V \sim e^{\gamma\Phi}$ For 10d Sugimoto model $\gamma = 3/2$

The vacuum/ground state is not 10d. The maximal symmetric Solutions have

- SO(1,8) (space dependent) or
- S0(9) (time-dependent) symmetry.

The SO(1,8), space-dependent case

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2B(y)}dy^2 \ , \ \ \Phi = \Phi(y)$$

In the gauge $e^{2B}e^{\gamma\Phi} = const$ the solution can be found explicitly

$$\Phi = \frac{3}{4}\alpha_E y^2 + \frac{2}{3}\ln|\sqrt{\alpha_E}y| + \Phi_0 \ ,$$

$$ds_E^2 = |\sqrt{\alpha_E}y|^{1/9}e^{-\alpha_E y^2/8}\eta_{\mu\nu}dx^\mu dx^\nu + |\sqrt{\alpha_E}y|^{-1}e^{-3\Phi_0/2}e^{-9\alpha_E y^2/8}dy^2$$

It has naked singularities at $y=0$ and $y=\infty$; the internal coordinate y becomes compact. However, Planck mass, YM constant are finite.

In the SO(9), time-dependent case we get

$$g_s = e^\Phi = e^{\Phi_0}|\sqrt{\alpha}t|^{2/3}e^{-3\alpha t^2/4} \ ,$$

$$ds^2 = -|\sqrt{\alpha}t|^{-2/3}e^{-\Phi_0}e^{3\alpha t^2/4}dt^2 + |\sqrt{\alpha}t|^{4/9}e^{\Phi_0/2}e^{-\alpha t^2/4}\delta_{\mu\nu}dx^\mu dx^\nu$$

t=0 is a big-bang singularity

Applications to Cosmology

- Consider the action for gravity and a scalar ϕ :

$$S = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) + \ldots \right]$$

- Look for **cosmological solutions** of the type

$$ds^2 = -e^{2\mathcal{B}(t)} dt^2 + e^{2A(t)} d\mathbf{x} \cdot d\mathbf{x}$$

(Halliwell, 1987)
……………..
(E.D,Mourad, 2000)
(Russo, 2004)
……………….

- Make the convenient **gauge choice** $V(\phi)\, e^{2\mathcal{B}} = M^2$

- Let :

$$\beta = \sqrt{\frac{d-1}{d-2}}\,, \qquad \tau = M\,\beta\, t\,, \qquad \varphi = \frac{\beta\,\phi}{\sqrt{2}}\,, \qquad \mathcal{A} = (d-1)\,A$$

- In expanding phase :

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^{\,2}} + \left(1 + \dot{\varphi}^{\,2}\right) \frac{1}{2V} \frac{\partial V}{\partial \varphi} = 0$$

- **OUR CASE :**

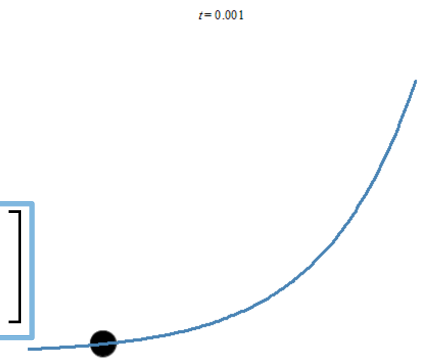
$$V = \exp(2\,\gamma\,\varphi) \longrightarrow \frac{1}{2\,V} \frac{\partial V}{\partial\,\varphi} = \gamma$$

A climbing scalar in d dim's

- $\gamma < 1$? **Both signs of speed**

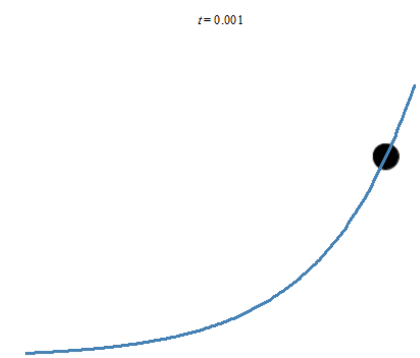
a. **"Climbing" solution** (φ climbs, then descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$



b. **"Descending" solution** (φ only descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$



NOTE : only φ_o . Early speed → singularity time !

Limiting τ- speed (LM attractor):

$$v_l = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

$\gamma \to 1$: LM attractor & descending solution **disappear**

- $\gamma \geq 1$? **Climbing !** **E.g. for $\gamma=1$:**

$$\dot{\varphi} = \frac{1}{2\,\tau} - \frac{\tau}{2}$$

CLIMBING : in ALL asymptotically exponential potentials with **$\gamma \geq 1$** !

Climbing and Inflation

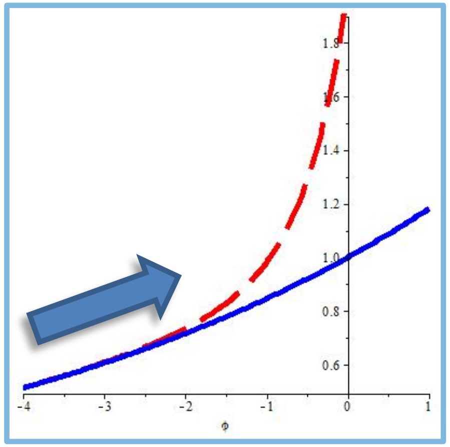
a. **"Hard"** exponential of **Brane SUSY Breaking**

b. **"Soft"** exponential ($\gamma < 1/\sqrt{3}$):

Would need : $\gamma \approx \frac{1}{12}$

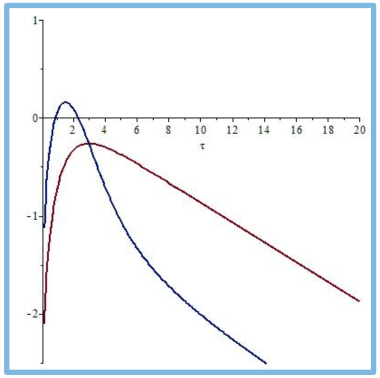
$$\boxed{V(\phi) = \overline{M}^{\,4}\left(e^{2\varphi} + e^{2\gamma\varphi}\right)}$$

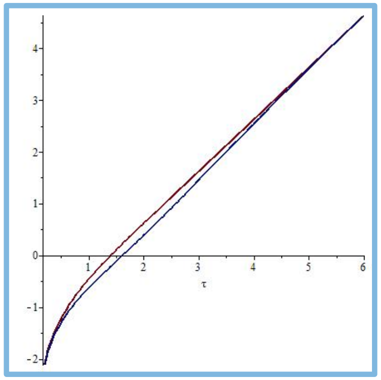
Non-BPS D3 brane gives $\gamma = 1/2$
[+ stabilization of Φ_s]



(Sen , 1998)
(E.D .J.Mourad, A.Sagnotti 2001)

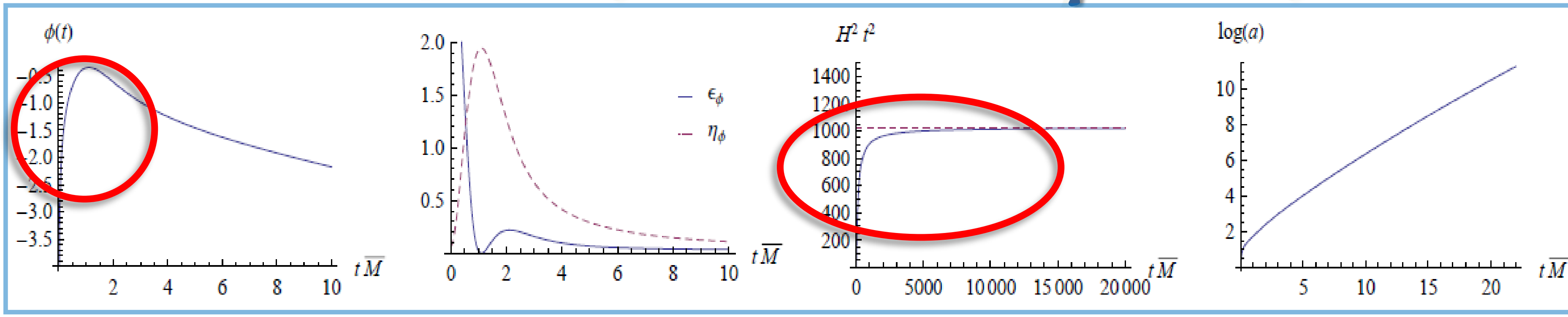
- **BSB "Hard exponential"** → makes initial **climbing** phase **inevitable**
- **"Soft exponential"** → drives **inflation** during subsequent descent





φ_o : "hardness" of kick !

Numerical Power Spectra



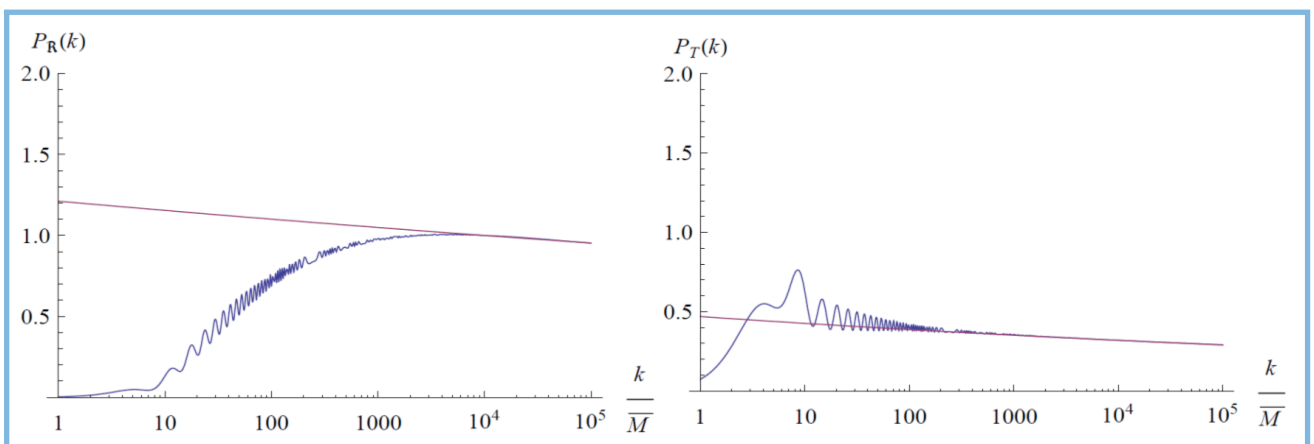
Key features:

1. **Harder "kicks"** make φ reach **later** the attractor
2. Even with mild kicks the **time scale** is 10^3- 10^4 in t M !
3. **η re-equilibrates slowly**

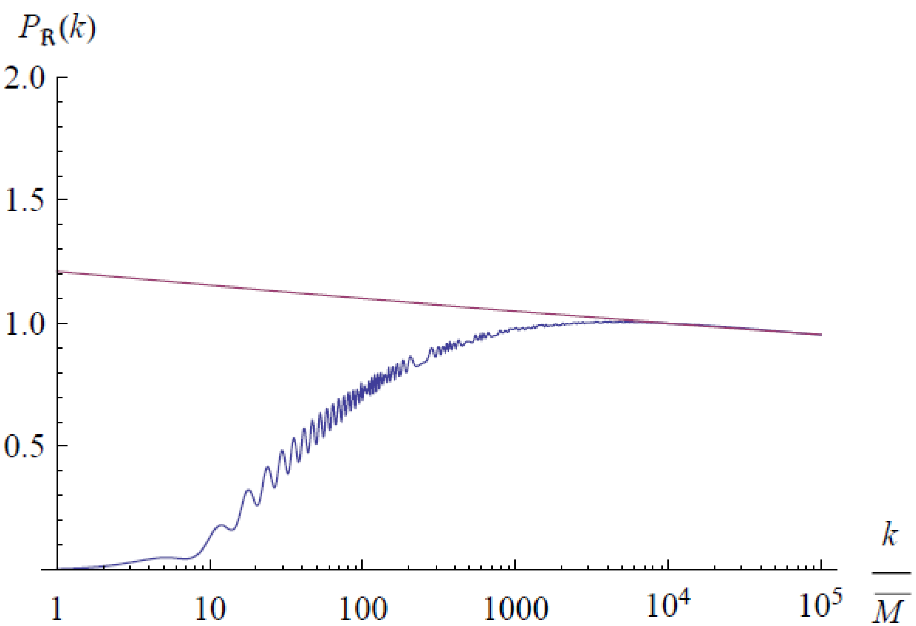
$$\epsilon_\phi \equiv - \frac{\dot{H}}{H^2} \,, \qquad \eta_\phi \equiv \frac{V_{\phi\phi}}{V}$$

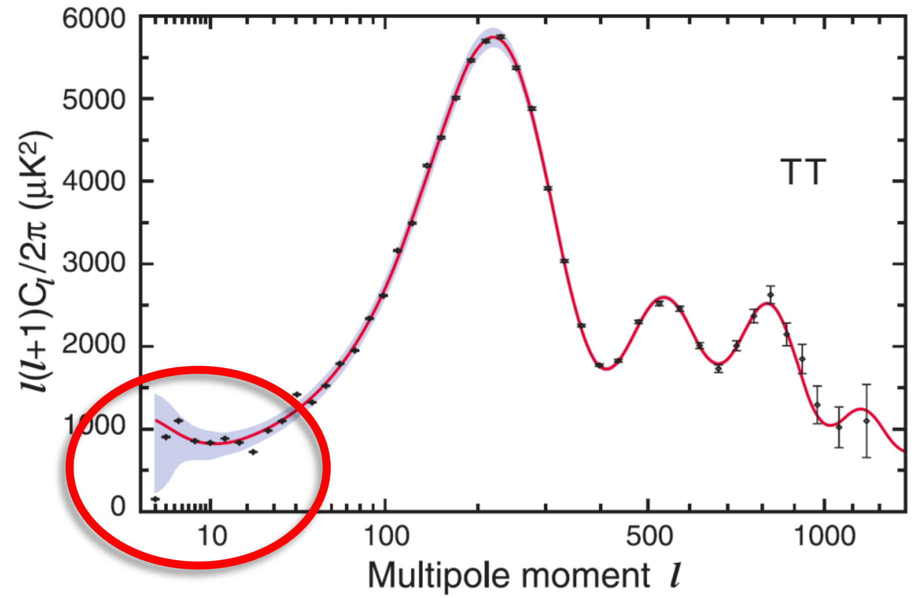
$$P_{S,T} \ \sim \int \frac{dk}{k} \ k^{n_{S,T}-1}$$

$$\begin{aligned} n_S - 1 &= 2(\eta_\phi - 3\,\epsilon_\phi) \,, \\ n_T - 1 &= -\,2\,\epsilon_\phi \end{aligned}$$



WMAP9/Planck power spectrum :





NOTE :

$$\left(\left| \frac{\Delta C_\ell}{C_\ell} \right| = \sqrt{\frac{2}{2\ell+1}} \right)$$

Qualitatively the low-k tail

Conclusions

- There are several known ways to break SUSY in string theory:
- by compactification: parallel and perpendicular to branes
- by non-BPS configurations (BSB)
- by internal magnetic fields/brane rotations
- by closed-string fluxes

- Few 4d « realistic » models with broken SUSY constructed along these lines .

- Conceptual problems in defining the ground state of the theory. To date, all static solutions I know of have naked singularities.

- Time-depedent solutions seem to have interesting early-time cosmological interpretations. A realistic setting should combine SUSY breaking with moduli stabilization.