Surface defects in 3d topological field theory

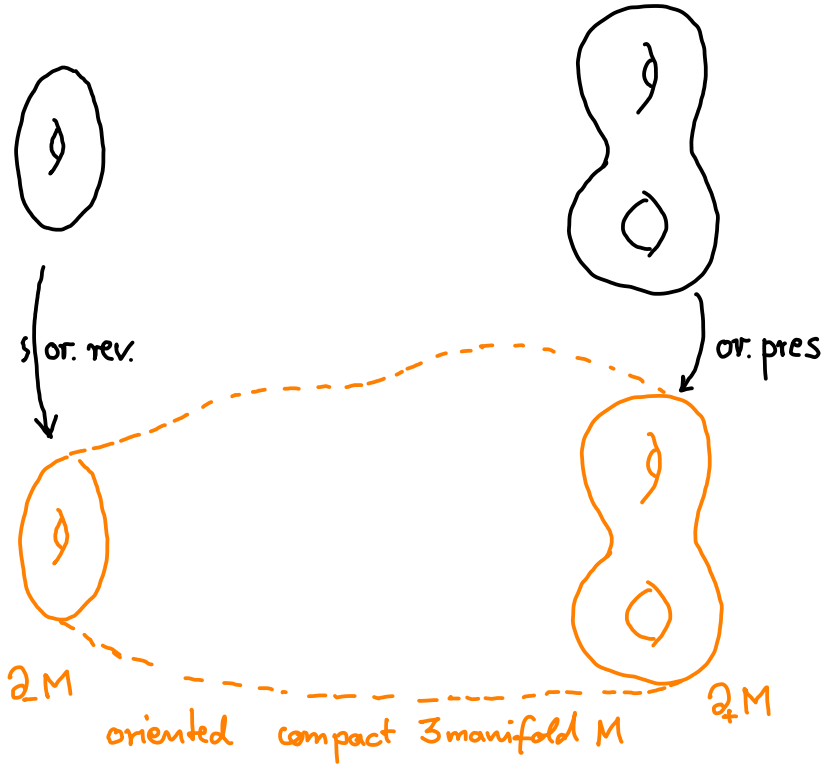
Ingo Runkel (Univ. Hamburg)

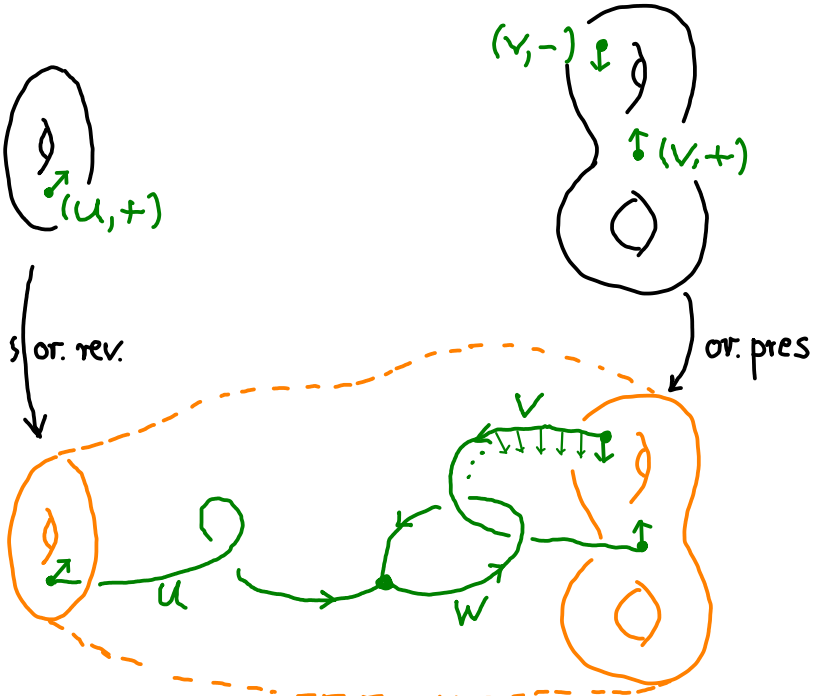
joint with

Nils Carqueville and Gregor Schaumann

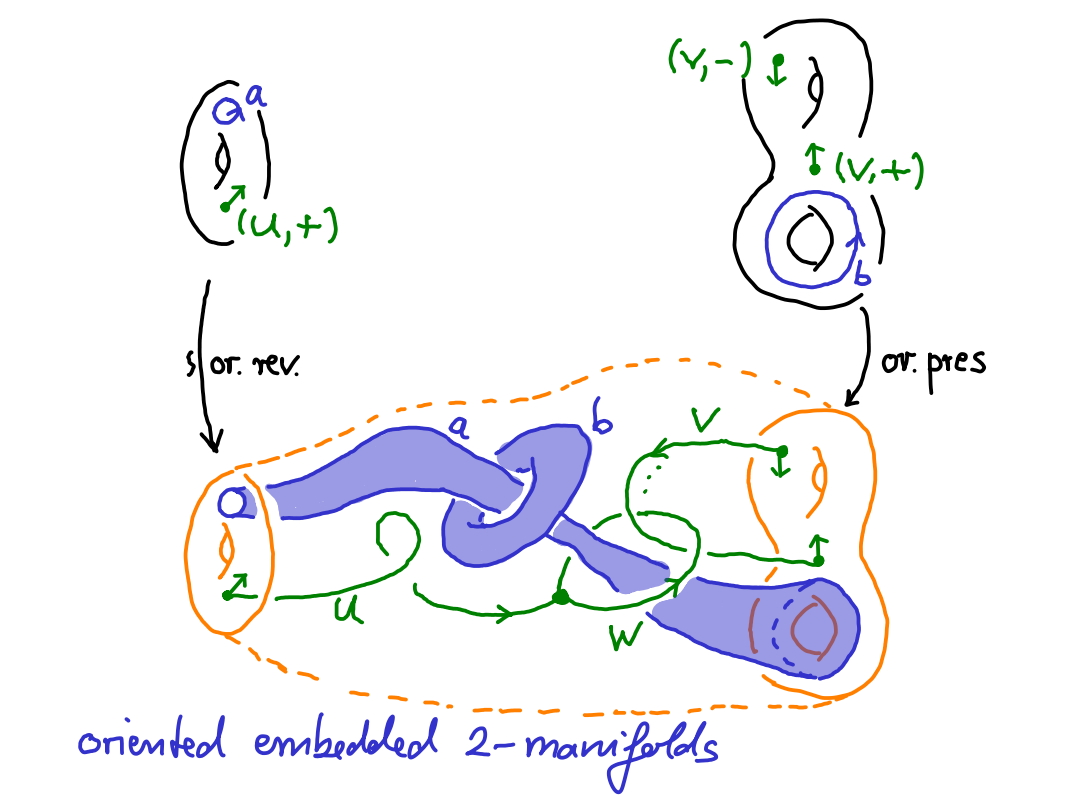
Outline:

- axiomatic description of 3d TQFT
- observables on submf. of dim. 0, 1, 2
- examples from U(1) and SU(2)


s| or. rev.
or. pres
2M
2+M
oriented compact 3manifold M



oriented & framed embedded 1-manifolds



oriented embedded 2-manifolds

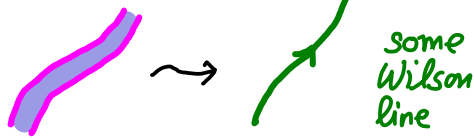
Examples of surface defects

Idea:

Kapustin, Saulina '10

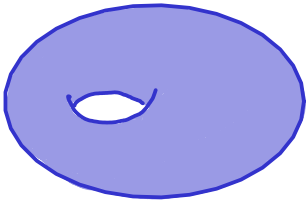
- Surface def. with bnd:

- Thin ribbon of surf. def.

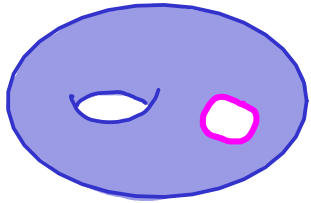


- Suppose $Z_c\left(\right) = \underset{\text{in } \mathbb{C}^\times}{\varphi} \cdot Z_c\left(\right)$

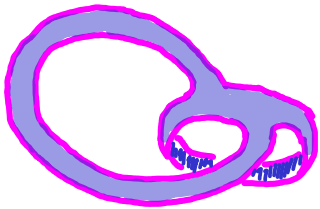
- Reduce surface defects to line defects, e.g.



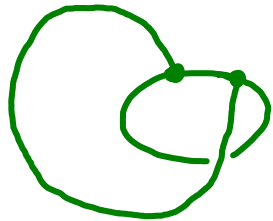
$= \varphi^{-1} \cdot$



$=$ deform



$=$ replace by line def.



Example : SU(2) level k

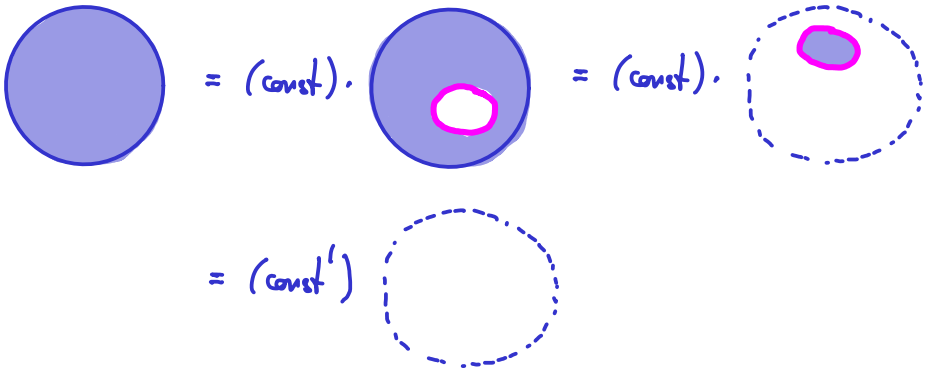
line defect labels : 0, 1, 2, ... , k

surface defect labels :

level k	# of Morita cl. of ssFa	
odd	1	label A
even, ≠ 10, 16, 28	2	A, D
	3	A, D, E_6, E_7, E_8

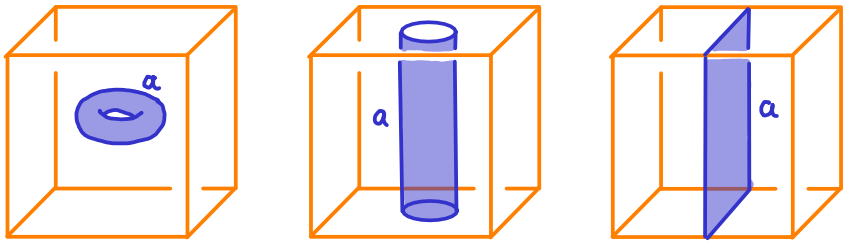
E.g. $k = 16$ surf. def. A, D, E

- embedded S^2 does not detect any topology



E.g. $k = 16$ surf. def. A, D, E

- embed T^2 into T^3 :



a	invariant $Z(-)$		
A	17	17	17
D	34	18	10
E	34	18	7

level k	# of Morita cl. of ssFa	
odd	1	label A
even, $\neq$ 10, 16, 28	2	A, D
10, 16, 28	3	A, D, E_6, E_7, E_8

fusion of surface defects

A : identity

level k	fusion
$\in 4\mathbb{Z}$	$D * D = 2D$
$\in 4\mathbb{Z}+2$	$D * D = A$
10	$D * E_6 = E_6$, $E_6 * E_6 = 2E_6$
16	$D * E_7 = 2E_7$, $E_7 * E_7 = D + E_7$
28	$D * E_8 = 2E_8$, $E_8 * E_8 = 4E_8$

level k	fusion	
$\in 4\mathbb{Z}$	$D * D = 2D$	→ not a group
$\in 4\mathbb{Z}+2$	$D * D = A$	→ $\mathbb{Z}_2$

⚠ Orbifold by $\mathbb{Z}_2$-sym. at $k \in 4\mathbb{Z}+2$ does not give $SO(3)$-Chern-Simons TQFT

→ only exists at $k \in 4\mathbb{Z}$