On naturalness in supersymmetric theories.

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- Outline

I. Naturalness without prejudice: review & questions.

II. Naturalness, likelihood (χ^2) and emergent fine-tuning (Δ_q).

III. Numerical results for Δ_q in: CMSSM, NUGM, NUHM, NMSSM, GNMSSM.

IV. Conclusions.

(I). Naturalness without prejudice: review & questions

L. Susskind and K. Wilson, PRD 20 (1979) 2619.

• why SM EW scale $v \sim M_Z \ll M_P$ stable under quantum corrections. $\left[\frac{G_f h^2}{G_N c^2} = 1.7 \times 10^{33}\right]$

$\delta v^2 \sim \delta m_h^2 \sim f(\alpha_j)\,\Lambda^2$, if $\Lambda \sim M_P$: tune couplings: $1 : 10^{33}$ (!).

Hierarchy or Naturalness Problem $\Leftrightarrow$ "Fine tuning", not natural. Ways out?

A physical parameter ρ is naturally v.small if $\rho = 0$ increases symmetry.

"Naturalness dogma": 't Hooft (1979)

- Solutions:

1. A new symmetry: scale (conformal) symmetry.... see for example Bardeen 1995;

2. SUSY: $\delta m_h^2 \sim m_S^2 \ln \Lambda / m_S$, $m_S \sim$TeV... no SUSY seen, $m_S \gg \mathrm{TeV} \rightarrow$ back to SM fine-tuning

.....why worry? worse fine tunings: cosmological constant. $\left[\frac{\rho_v}{\rho} \approx \frac{(2.3\times 10^{-12}\mathrm{GeV})^4}{(10^{19}\,\mathrm{GeV})^4}\right]$

.....because softly broken TeV-scale SUSY solves the hierarchy problem!

3. Ignore it (!), use an EFT approach, enforce consistency at every loop order.

- General potential, SUSY models:

$$V = m_1^2\,|h_1|^2 + m_2^2\,|h_2|^2 - (B_0\,\mu_0\,h_1\cdot h_2 + h.c.) + \lambda_1\,|h_1|^4 + \lambda_2\,|h_2|^4 + \lambda_3\,|h_1|^2\,|h_2|^2$$
$$+\ \lambda_4\,|h_1\cdot h_2|^2 + \left[\,\lambda_5/2\,(h_1\cdot h_2)^2 + \lambda_6\,|h_1|^2\,(h_1\cdot h_2) + \lambda_7\,|h_2|^2\,(h_1\cdot h_2) + h.c.\right]$$

$$m^2 \equiv m_1^2\,\cos^2\beta + m_2^2\,\sin^2\beta - B_0\,\mu_0\,\sin 2\beta, \quad \mathrm{UV}: m_{1,2}^2 = m_0^2 + \mu_0^2$$
$$\lambda \equiv \lambda_1\,\cos^4\beta + \lambda_2\,\sin^4\beta + \lambda_{345}/4\,\sin^2 2\beta + \sin 2\beta\left(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta\right)$$

- The Problem: scales vs. couplings "tension" ("fine-tuning"):

$$v^2 = -m^2/\lambda, \ \text{ with } \ v = \mathcal{O}(100\ \mathrm{GeV}), \quad \lambda < 1, \qquad \textbf{but} \quad m_{1,2}, B_0 \text{ and } m \sim \mathcal{O}(1\,\mathrm{TeV}).$$

- $m_h > m_Z \leftrightarrow$ large loop effects $\leftrightarrow$ large $m_{1,2}, B_0$...; also a problem of couplings (λ small)

$\Rightarrow$ Solution: - increase λ by: a) quantum corrections. b) susy "new physics" beyond MSSM (tree)

- live with fine-tuning? (cl. nonlinear systems); fundamental concept? UV ignorance!

- Unanswered Questions.....

$$\Delta_{max} \equiv \max_{\gamma} \left|\Delta_\gamma\right|, \qquad \Delta_\gamma \equiv \frac{\partial \ln v^2}{\partial \ln \gamma^2}, \quad \gamma = \{m_0, m_{1/2}, \mu_0, A_0, B_0\}, \quad v = \text{EW scale}$$

Ellis, Enqvist, Nanopoulos, Zwirner (1986)
Barbieri, Giudice (1988)

a). WHY this definition? how to avoid a definition?

b). "small" Δ wanted.... but what is "small"? ≤ 20? 1000?

c). other definitions? $\Delta_q = \left[\sum_\gamma \Delta_\gamma^2\right]^{1/2}$; $\gamma \supset y_t, y_b$?

see: Anderson, Castano, hep-ph/9409419
Dreiner et al arXiv:1204.4199
Baer et al 2012.

d). different def's $\rightarrow$ different results?

e). Take 2 models: A: v.good χ^2/ndf but large $\Delta = 1000$ $\rightarrow$ model v. likely but...unnatural!

B: good χ^2/ndf but small $\Delta = 10$. $\rightarrow$ model less likely, but ...natural!

$\chi^2 = \sum_i (O_i(\gamma) - O_i^{exp})^2/\sigma_i^2$, χ^2/ndf≈ 1. What model is better?

f). Is the fine-tuning "cost" to fix m_Z related to χ^2 "cost" to fit m_Z regarded as observable?

$\Rightarrow \Delta$: tune γ_i to fix EW scale $v \sim m_Z$ (same for observables); $\Rightarrow \Delta$, likelihood (χ^2) related!?

• A closer look: Lagrangian $\mathcal{L}$ of UV parameters $\gamma_i : m_0, \mu_0, A_0, B_0, m_{1/2}, \tan\beta, \cdots$.

$$\frac{(g_1^2+g_2^2)\,v^2}{8} = -\mu^2 + \frac{m_1^2 - m_2^2\,\tan^2\beta}{\tan^2\beta - 1} + \cdots,$$
$$2\,m_3^2 = (m_1^2 + m_2^2 + 2\mu^2)\,\sin 2\beta + \cdots\,,$$

so $v = v(\gamma,\beta)$, $\beta = \beta_0(\gamma) \Rightarrow \quad m_Z = m_Z(\gamma, \beta_0(\gamma))$. Taylor expansion

$$m_Z = m_Z^0 + \left(\frac{\partial m_Z}{\partial \gamma_i}\right)_{\gamma_i=\gamma_i^0} (\gamma_i - \gamma_i^0) + \cdots\,, \qquad m_Z^0 = 91.187\,\mathrm{GeV}$$

$$\Rightarrow \frac{\delta m_Z}{m_Z^0} = \Delta_q\, n^i\, \frac{\delta\gamma_i}{\gamma_i^0} + \mathcal{O}((\delta\gamma_i)^2), \qquad \vec{n} \text{ normal to } m_Z(\gamma_i^0, \beta_0(\gamma_i^0)) = m_Z^0.$$

with notation $\Delta_q \equiv \left\{ \sum_i \left(\frac{\partial \ln m_Z}{\partial \ln \gamma_i}\right)^2_{\gamma_i=\gamma_i^0} \right\}^{1/2}$

$$\frac{\delta m_Z}{m_Z^0} = 4.6\times 10^{-5}\ (2\sigma), \text{if } \Delta_q \approx 1000 \ \rightarrow \frac{\delta\gamma_i}{\gamma_i^0} \approx 4.6\times 10^{-8} \rightarrow \gamma_i = 1\,\mathrm{TeV} \rightarrow \delta\gamma_i = 46\,\mathrm{keV}!$$

(II). Naturalness, likelihood (χ^2) and emergent fine-tuning.

D.G., G. Ross, arXiv:1208:0837, arXiv:1304.1193, arXiv:1302.5262

- Naturalness: fixing the EW scale $v \sim m_Z$ to $v_0 = 246$ GeV ($m_Z^0 = 91.187$ GeV).

- SM does not fix v. SUSY does it and predicts $v = v(\gamma_i) \Rightarrow$ regard m_Z as an observable.

- Likelihood to fit observables O_j other than m_Z ($\chi^2 \equiv -2 \ln L$):

$$L(\text{data}|\gamma_i; v, \beta) = \prod_{j \geq 1} L(O_j|\gamma_i; v, \beta), \qquad \text{UV param}: \quad \gamma_i = m_0, m_{1/2}, A_0, B_0, \mu_0, \ldots \ (y_t, y_b, \ldots)$$

$$f_1(\gamma_i, v, \beta) = v - (-m^2/\lambda)^{1/2} = 0, \qquad f_2(\gamma_i, v, \beta) = \tan\beta - \tan\beta_0(\gamma_i) = 0$$

$$\begin{aligned} L(\text{data}, m_Z|\gamma_i) &= \int dv \ d(\tan\beta) \ \delta\left(f_1(\gamma_i; v, \beta)\right) \ \delta\left(f_2(\gamma_i; v, \beta)\right) \ L(\text{data}|\gamma_i; v, \beta) \ \delta(1 - m_Z/m_Z^0) \\ &= v_0 \left[L\big(\text{data}|\gamma_i;\ v_0,\ \beta\big) \ \delta\big[f_1\big(\gamma_i;\ v_0,\ \beta\big)\big]\right]_{\beta=\beta_0(\gamma_i)} \\ &= L\big(\text{data}|\gamma_i;\ v_0,\ \beta_0(\gamma_i)\big) \ \delta\Big[1 - \frac{m_Z(\gamma_i, \beta_0(\gamma_i))}{m_Z^0}\Big], \end{aligned}$$

$$v_0 = 246 \text{ GeV}$$
$$m_Z = gv/2$$
$$m_Z^0 = gv_0/2$$
$$g^2 = g_1^2 + g_2^2$$

(Note: factorization assumes no correlations, see later)

D.G., G.G. Ross, arXiv:1208:0837.

If all γ_i vary:

$$\delta\Big[1-\frac{m_Z(\gamma_i,\beta_0(\gamma_i)}{m_Z^0}\Big]=\frac{1}{\Delta_q}\,\delta\Big[n^j\,(1-\frac{\gamma_j}{\gamma_j^0})\Big],\qquad \Delta_q\equiv\Big[\sum_i\Big(\frac{\partial\ln m_Z(\gamma_i,\beta_0(\gamma_i))}{\partial\ln\gamma_i}\Big)\Big]^{1/2}_{\gamma_i=\gamma_i^0},$$

$$\Rightarrow\quad L(\mathrm{data},m_Z|\gamma_i^0)=\frac{1}{\Delta_q}\,L(\mathrm{data}|\gamma_i;v_0,\beta_0(\gamma_i))\Big|_{\gamma_i=\gamma_i^0}$$

$\Rightarrow$ Δ_q sole result of fixing the EW scale! $\Rightarrow$ can only be interpreted as fine-tuning!

$\Rightarrow$ measure of fine-tuning Δ_q: unique, emerges automatically (not defined)!

$\Rightarrow$ Δ_q as part of total likelihood. $\Rightarrow$ maximize the ratio in the rhs! $\Rightarrow$ answers a), c), f).

$\Rightarrow$ answers e): how to compare (likely but unnatural) and (unlikely but natural) models!

Bayesian case: see Casas et al 2008, Allanach et al 2007, 2009, Fichet 2012, D.G., H.M. Lee, M. Park 2012

$$\delta(f(\vec{z}))=\frac{1}{|\nabla_z f|_o}\,\delta\Big[\vec{n}.(\vec{z}-\vec{z}^0)\Big],\quad n_i=\frac{\partial_{z_i}f}{|\nabla f|_o},\qquad \text{normal to } m_Z(\gamma_i^0,\beta_0(\gamma_i^0))=m_Z^0$$

With $\chi^2 = -2 \ln L$:

D.G., G.G. Ross, arXiv:1208:0837.

$$\chi^2_{\rm z}(\gamma_i) = \Big[\; \chi^2(\gamma_i) + \underbrace{2 \ln \Delta_q(\gamma_i)}_{\delta\chi^2} \; \Big]_{f_1=0, f_2=0}$$

- Good fit: total $\chi^2_{\rm z}$ /ndf ≈ 1. $\Rightarrow$ Model-independent naturalness bound: $\Delta_q < \exp({\rm ndf}/2) \sim 100$.

$\Rightarrow$ Good fit criterion $\rightarrow$ demands smaller EW fine tuning! (answers b)

$\Rightarrow$ Possible corrections: from correlations among observables and/or γ_i in UV (symmetries, etc)

$\Rightarrow$ Implications for SUSY models?

$$\begin{aligned} \Delta_q &\approx 10 \Rightarrow \delta\chi^2/{\rm ndf} \approx 4.6/9 \\ \Delta_q &\approx 100 \Rightarrow \delta\chi^2/{\rm ndf} \approx 9.2/9 \\ \Delta_q &\approx 500 \Rightarrow \delta\chi^2/{\rm ndf} \approx 12.5/9 \end{aligned}$$

- what is the value of Δ_q in SUSY models?

(III). Numerical results for Δ in SUSY models:

D.G., H. M. Lee, M. Park, arXiv:1203:0569

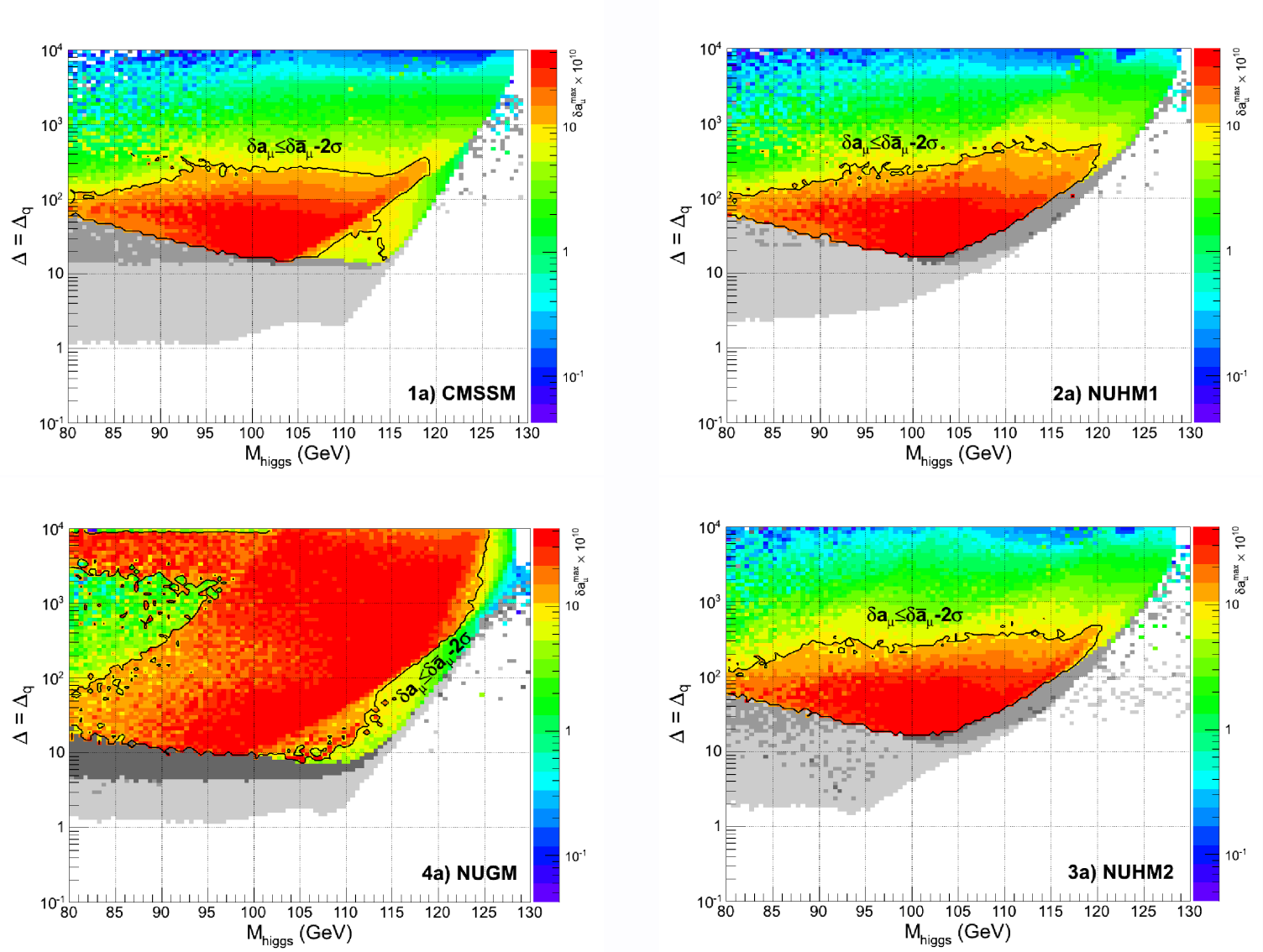
1) CMSSM (constrained MSSM) $\gamma = \{m_0, \mu_0, m_{1/2}, A_0, B_0\}$
2) NUHM1 (non universal Higgs Mass) $\gamma = \{m_0, \mu_0, m_{H_1}^{uv} = m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
3) NUHM2 (non universal Higgs Mass) $\gamma = \{m_0, \mu_0, m_{H_1}^{uv}, m_{H_2}^{uv}, m_{1/2}, A_0, B_0\}$
4) NUGM (non universal gaugino masses) $\gamma = \{m_0, \mu_0, m_{\lambda_{1,2,3}}, A_0, B_0\}$

• Experimental Constraints:

- SUSY masses: micrOmegas 2.4.5, "MSSM/masslim.c"
- muon magnetic moment: $\delta a_\mu = (25.5 \pm 2 \times 8) \times 10^{-10}$ at 2σ
- $b \to s\,\gamma$ $3.03 < 10^4$ Br$(b \to s\gamma) < 4.07$ at 2σ
- ρ-parameter $-0.0007 < \delta\rho < 0.0033$ at 2σ
- dark matter $\Omega h^2 = 0.1099 \pm 3 \times 0.0062$ at 3σ
- ATLAS/CMS: $m_h \approx 126$ GeV and δa_μ not imposed.

• Tools: micrOMEGAs 2.4.5, SoftSUSY 3.2.4. Random scan. $\Rightarrow \Delta_q$, Δ_{max} at 2-loop LL.

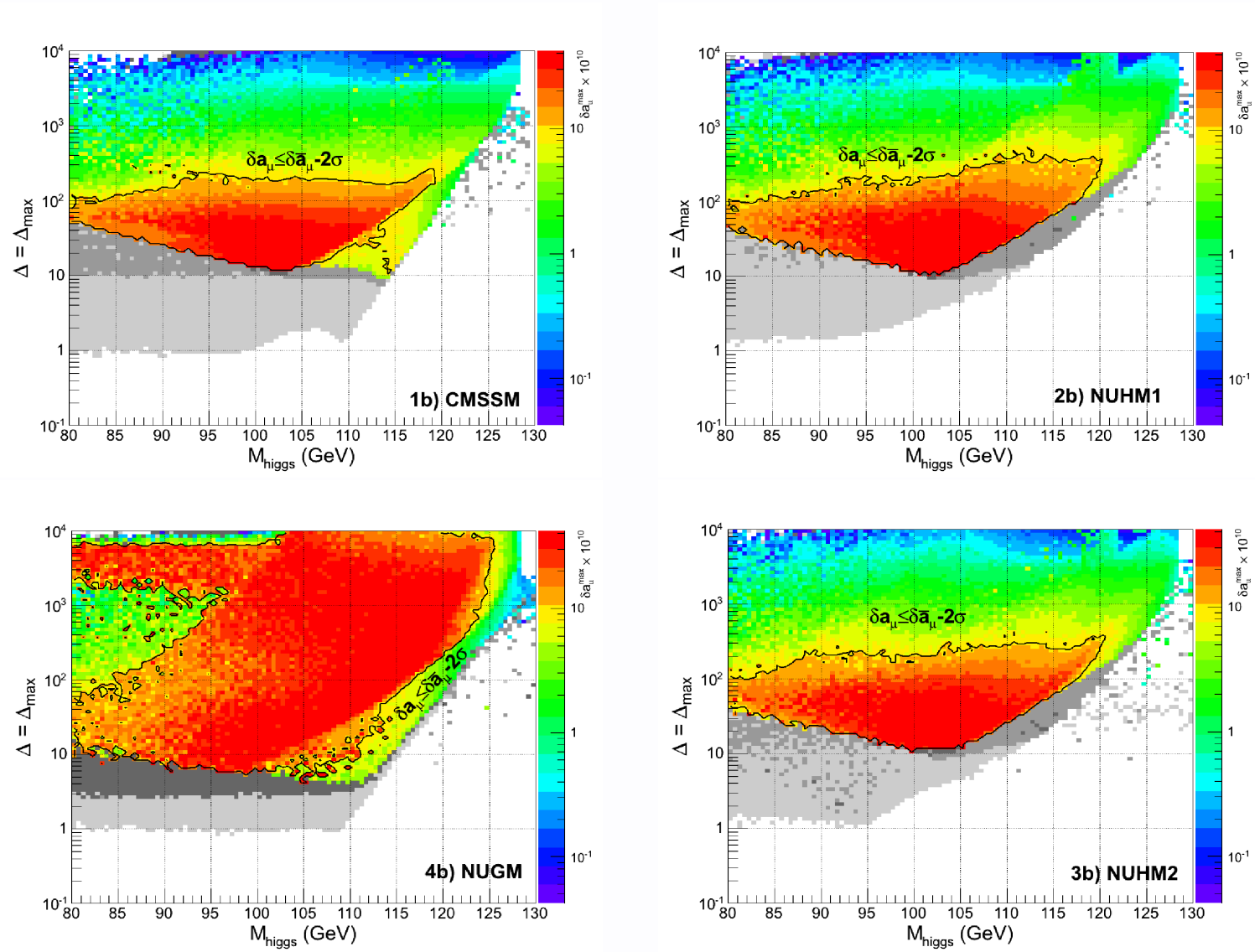
- Impact of m_h, δa_μ on Δ_q. [2-loop, all $\{\gamma, \tan\beta\}$ all values] D.G., H. M. Lee, M. Park, arXiv:1203:0569



- grey 0: excluded by SUSY; grey 1: $b \to s\gamma$, $B_s \to \mu^+\mu^-$, $\delta\rho$; grey 2: excluded by $\delta a_\mu > 0$.

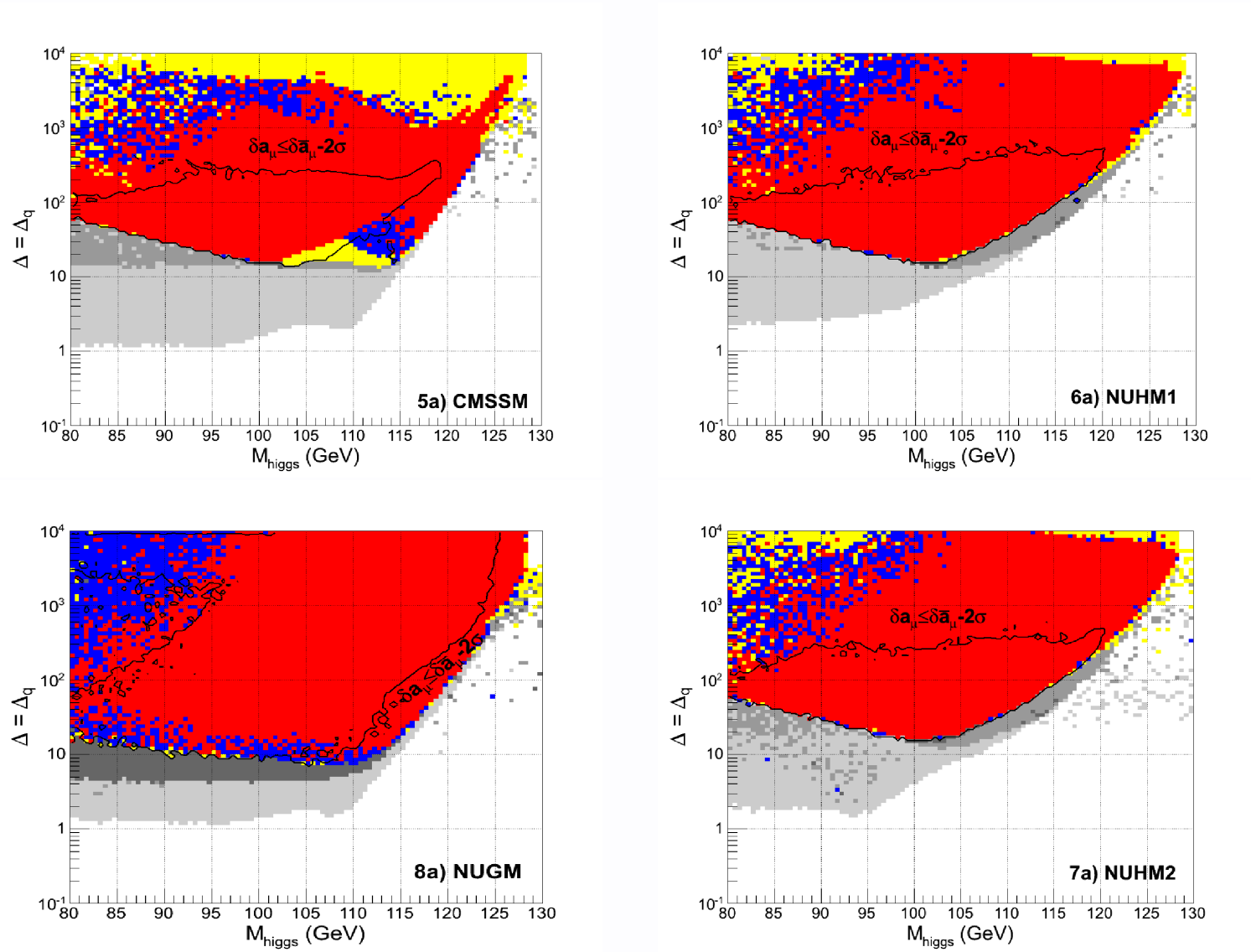
$\Rightarrow$ m_h strongest impact: $\Delta_q \sim \exp(m_h)$. $\Delta_q \sim 1000$ (!) near 125 GeV. NUGM better behaviour.

- Impact of m_h, δa_μ on Δ_{max}. [2-loop, all $\{\gamma, \tan\beta\}$ all values]. D.G., H. M. Lee, M. Park, arXiv:1203:0569



- δa_μ: 2σ contour (red) [smaller δa_μ outside]. $\Rightarrow$ Δ_{max} identical behaviour to Δ_q (marginally smaller) answers d)

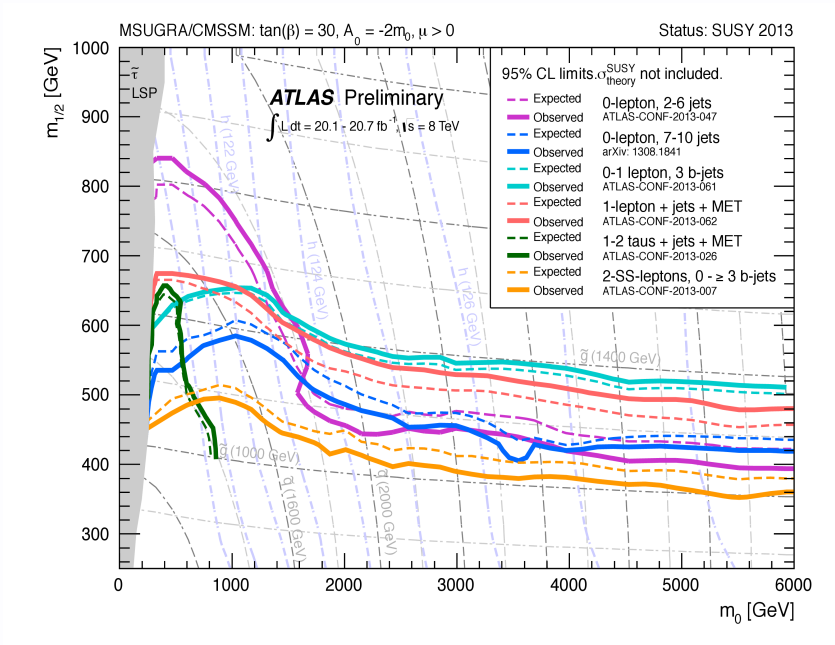
- Impact of m_h, dark matter on Δ_q: [2-loop, $\{\gamma, \tan\beta\}$ all values]. D.G., H. M. Lee, M. Park, arXiv:1203:0569



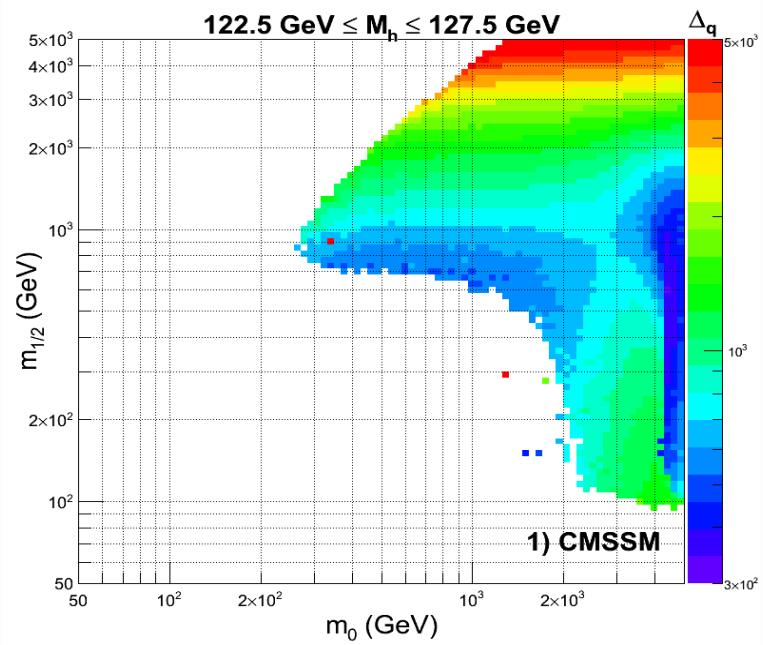
- blue: consistent with Ωh^2; Red: saturate Ωh^2 within 3σ. Grey areas ruled out by data.

$\Rightarrow$ Δ_{max} (not shown) nearly identical plots to Δ_q, (marginally smaller for same Higgs mass).

- SUSY searches from ATLAS and impact on parameter space (CMSSM only):



ATLAS-2013

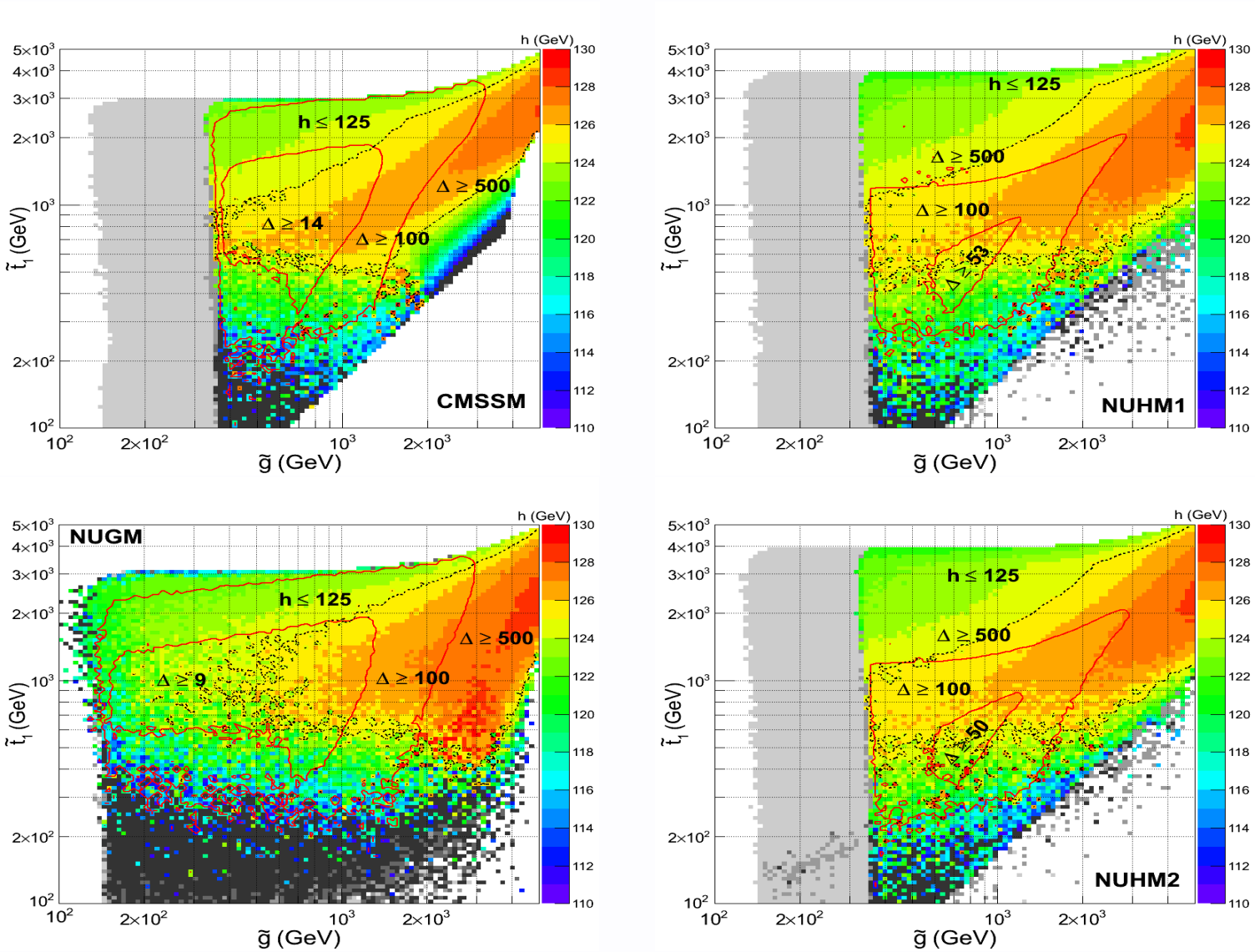


CMSSM, $122.5 \leq m_h \leq 127.5$ GeV and Δ_q.

[all values for γ, $\tan\beta$]. $\Delta_q > 500$.

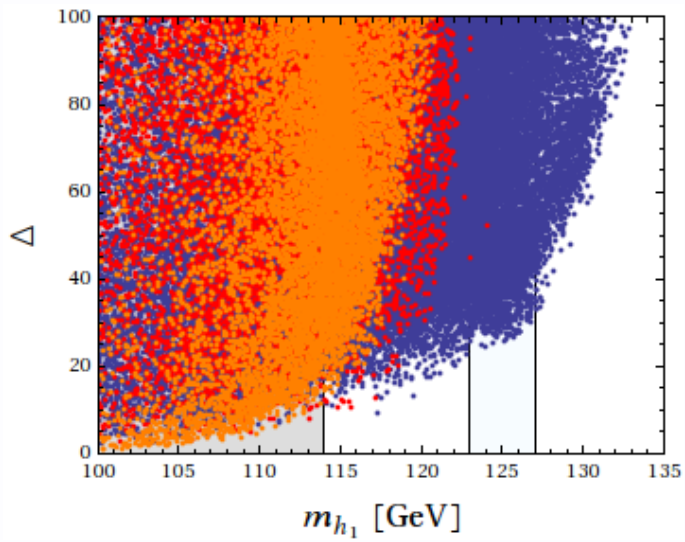
D.G., Hyun Min Lee, Myeonghun Park (2012)

- Stop vs Gluino with largest m_h and min Δ_q. $[\{\gamma, \tan\beta\}$ all values$]$ D.G., H. M. Lee, M. Park, arXiv:1203:0569



$\Rightarrow$ constraints on m_h strongly reduce the viable regions.

Other models (beyond MSSM): NMSSM, GNMSSM, ...



MSSM

NMSSM: $W = W_Y + \lambda S H_1 H_2 + \kappa S^3, \qquad \Delta \sim 200.$

GNMSSM: $W = W_{NMSSM} + m_{3/2}^2 S + m_{3/2} S^2 + m_{3/2} H_1 H_2.$

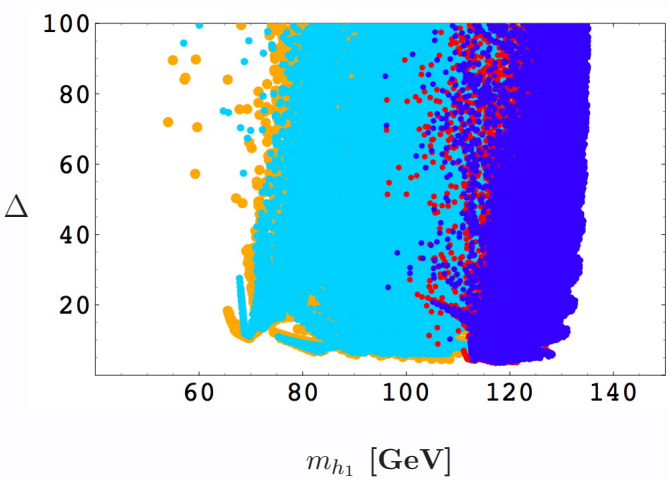
from underlying Z_4^R symmetry.

$$W = W_Y + (\mu + \lambda S)\, H_1 H_2 + M_*\, S^2 + \kappa S^3$$

$$\Delta < 30 \;\text{ for }\; m_h \leq 126\, GeV.$$

G.G. Ross et al, arXiv:1205.1509, 1102.3595

U. Ellwanger et al arXiv:1107.2472



GNMSSM: limit of decoupling S: MSSM + (d=5 operator)

$$W = W_Y + (\mu + \lambda S)\, H_1 H_2 + M_*\, S^2$$

$$\Rightarrow W = W_Y + \mu\, H_1 H_2 + \zeta\, (H_1 H_2)^2, \;\; \zeta \sim \lambda / M_*$$

$$\Delta < 30 \;\text{ for }\; m_h \leq 126\, GeV.$$

S. Cassel, D.G., G.G. Ross, NPB 825(2010)

$\Rightarrow$ massive singlet $\Rightarrow$ reduces Δ considerably. usually: $\Delta_q(m_h) \approx \exp(-\delta m_h/\text{GeV})\, \Delta_q(m_h)|_{\text{CMSSM}}$

• Corrections to Δ_q: error of the 2-loop calculation of m_h: 2-3 GeV.

- $\Delta_q \sim \exp(m_h/GeV) \quad \Rightarrow \quad \exp(3) \sim 20 \quad \Rightarrow \quad \Delta_q = 20 \; or \; 400$ are equally good (or bad....)!

• More generally, using $v = v(\gamma)$ and $\mathcal{O}_i = \mathcal{O}_i(\gamma, v(\gamma))$: D.G. arXiv:1311.6144

$$\mathcal{L}(\mathcal{O}|\gamma) \sim \exp\Big\{ -1/2\,(\mathcal{O}_i - \mathcal{O}_i^0)\,(M^{-1})_{ij}\,(\mathcal{O}_j - \mathcal{O}_j^0)\Big\}$$
$$\sim \frac{1}{\Delta}\exp\Big\{ -1/2\,(\gamma_\alpha - \gamma_\alpha^0)\,\tilde{M}^{-1}_{\alpha\beta}\,(\gamma_\beta - \gamma_\beta^0)\Big\}.$$

$$\mathcal{O}_i(\gamma) = \mathcal{O}_i(\gamma^0) + (\gamma_\alpha - \gamma_\alpha^0)\left(\frac{d\mathcal{O}_i}{d\gamma_\alpha}\right)_{\gamma=\gamma^0} + \cdots, \quad \tilde{M}^{-1} \equiv \mathcal{J}^T M^{-1}\mathcal{J}, \qquad \mathcal{J}_{i\alpha} \equiv \frac{1}{\mathcal{O}_i^0}\left[\frac{d\mathcal{O}_i}{d\ln\gamma_\alpha}\right]_{\gamma=\gamma^0}$$

where $\Delta \equiv \big[\det M \det \tilde{M}^{-1}\big]^{\frac{1}{2}}$, and covariance matrix, if $M_{ij} = \sigma_i^2\,\delta_{ij}$

$$\tilde{M}^{-1}_{\alpha\beta} = \Big\{\Big(\frac{d(\mathcal{O}_i/\sigma_i)}{d\ln\gamma_\alpha}\Big)\Big(\frac{d(\mathcal{O}_i/\sigma_i)}{d\ln\gamma_\beta}\Big)\Big\}_{\gamma=\gamma^0}, \qquad \alpha,\beta = 1,2,....s.$$

• Covariance matrix automatically contains information about Δ_q (from all $\mathcal{O}_i$!) if $v = v(\gamma)$:

D.G. arXiv:1311.6144

$$\tilde{M}^{-1}_{\alpha\beta} = \tilde{M}^{-1}_{\alpha\beta}\Big|_{v=const} + \sum_{i=1}^{s}\Big\{\Big(\frac{\partial\mathcal{O}_i/\sigma_i}{\partial\ln v}\Big)^2\Big(\frac{\partial\ln v}{\partial\ln\gamma_\alpha}\Big)\Big(\frac{\partial\ln v}{\partial\ln\gamma_\beta}\Big)\Big\}_{\gamma=\gamma^0}$$
$$+ \sum_{i=1}^{s}\Big\{\Big(\frac{\partial\mathcal{O}_i/\sigma_i}{\partial\ln v}\Big)\Big(\frac{\partial\ln v}{\partial\ln\gamma_\alpha}\Big)\Big(\frac{\partial\mathcal{O}_i/\sigma_i}{\partial\ln\gamma_\beta}\Big) + (\alpha\leftrightarrow\beta)\Big\}_{\gamma=\gamma^0}$$

- The s-standard deviation confidence interval:

$$-2\ln L \leq -2\ln L_{max} + s^2, \quad \Rightarrow \quad \sum_{i=1}^{n}\Big\{\Big(\frac{d\mathcal{O}_i/\sigma_i}{d\ln\gamma_\alpha}\Big)_{\gamma=\gamma^0}(\gamma'_\alpha/\gamma^0_\alpha - 1)\Big\}^2 \leq s^2$$

$$\Delta_q \leq \frac{s\,\sigma_z}{m_Z(\gamma^0)}\Big|\frac{n^\alpha\,(\gamma'_\alpha - \gamma^0_\alpha}{\gamma^0_\alpha}\Big|^{-1} \leq \frac{s\,\sigma_z}{m_Z(\gamma^0)}\Big|\frac{n^\alpha\,\sigma_{th,\alpha}}{\gamma^0_\alpha}\Big|^{-1}$$

$\Rightarrow$ bound on Δ_q, controlled by σ_{th} (theory loop order) and $\sigma_{\mathcal{O}_i}$ (exp).

$\Rightarrow$ Δ_q bounds: byproduct of a good fit condition, if $v = v(\gamma)$, as SUSY predicts.

$\Rightarrow$ $\tilde{M}$ more fundamental, includes fine-tuning! Precision data fits miss this effect (v is constant)

answers e)

(IV) Conclusions:

$\Rightarrow$ A fine-tuning measure, unique, model indep., Δ_q emerges from the condition of fixing EW scale.

$\Rightarrow$ total likelihood to fit the data (including m_Z) thus accounts for naturalness.

$\Rightarrow$ large $L(\mathrm{data}, m_Z|\gamma) = L(\mathrm{data}|\gamma)/\Delta_q$ required (or small $\chi_z^2 = \chi^2 + 2\ln\Delta_q$).

good fit: χ_z^2 / ndf ≈ 1 model independent bound: $\Delta_q < \exp(\mathrm{ndf}/2) \sim 100.$

$\Rightarrow$ More general: covariance matrix automatically includes fine-tuning effects if $v = v(\gamma)$.

Numerical results:

$\Rightarrow$ Δ_q in CMSSM, NUHM1,2, NUGM: of all data, m_h strongest constraint.

$\Rightarrow$ Δ_q, $\Delta_{max} \sim \exp(m_h/\mathrm{GeV}) \sim 500 - 1000$, for $m_h \approx 126$ GeV. GNMSSM has $\Delta_q \sim \mathcal{O}(20)$.

$\Rightarrow$ Significant error of Δ_q due to σ_{m_h}. $\Delta_q \sim 20$ as good (bad) as $\Delta_q \sim 400$!

$\Rightarrow$ Δ_q can be reduced to $\mathcal{O}(10)$ in MSSM + singlet, U(1)′, etc (using MSSM+effective operators).

– Backup slides –

• Naturalness in the Bayesian approach. $p(a|b)\,p(b) = p(a \cap b) = p(b|a)\,p(a)$

Bayes theorem: [initial belief + data → updated belief].

Thomas Bayes (1761), Laplace (1812)

$$p(\gamma|\text{data}) = \frac{L(\text{data}|\gamma)\;p(\gamma)}{p(\text{data})}, \qquad p(\text{data}) = \int L(\text{data}|\gamma)\,p(\gamma)d\gamma, \qquad \gamma : \{m_0, m_{1/2}, \mu_0, A_0, B_0; y_t, y_b, ...\}.$$

- $p(\text{data})$: "evidence". Models $1, 2$: $p_1(\text{data})/p_2(\text{data})$. $p(\gamma)$=priors.
- EW constraints: $f_1(\gamma; v, \beta) = f_2(\gamma; v, \beta) = 0, \;\Rightarrow\; v(\gamma, ...), \tan\beta_0(\gamma...)$.

$$p(\text{data}) = \int d\gamma\; p(\gamma)\, dv\; d(\tan\beta)\; \delta(m_Z - m_Z^0)\; \delta\left(f_1(\gamma; v, \beta)\right)\; \delta\left(f_2(\gamma; v, \beta)\right) L(\text{data}|\gamma; \beta, v),$$

$$= \int_{f_{1,2}=0} dS_\gamma\; \frac{L(\text{data}|\gamma)}{\Delta_q(\gamma)}\; p(\gamma), \qquad \Delta_q \equiv \Big[\sum_\gamma \Delta_\gamma^2\Big]^{1/2}, \;\; d\gamma \equiv \prod_i d\gamma_i, \;\; p(\gamma) = p(\gamma_1, ...\gamma_i).$$

D.G., H. M. Lee, M. Park, arXiv:1203:0569

$\Rightarrow \Delta_q$ due to fixing EW scale, in addition to/independent of priors $p(\gamma)$! large L/Δ_q needed.

answers e)?

$$\int_{R^n} h(z_1, ..., z_n)\, \delta(g(z_1, ..., z_n))\, dz_1....dz_n = \int_{S_{n-1}} dS_{n-1}\, h(z_1, ...z_n)\, \frac{1}{|\nabla_{z_i} g|},$$