Canonical quantum gravity and cosmological applications

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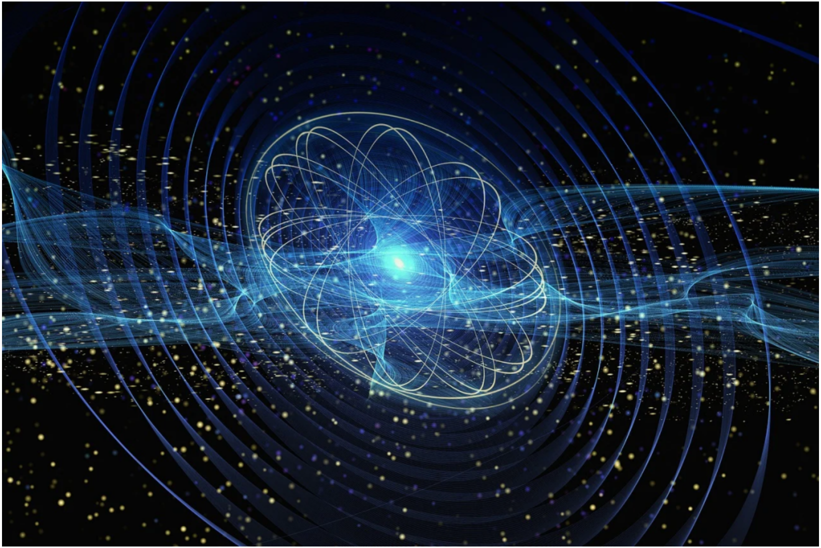
Outline

1. What is “quantum gravity”
2. Quantum cosmology

What is “quantum gravity” ?



“Stat rosa pristina nomine; nomina nuda tenemus.” – Umberto Eco, *The name of the rose.*

Talking is not the same as understanding

It has become fashionable to talk about "Quantum gravity" without stating what one intends that expression to mean.

- Einstein gravity is not power-counting renormalizable in spacetime dimension $d \geq 4$. It is exceedingly unlikely that it could be made into a renormalizable field theory without changing its microscopic structure in some fundamental manner.
- The only remotely plausible counter-argument to this point (proposed by Steven Weinberg) is the idea of "asymptotic safety". It is entirely unclear if that truly means that one could somehow hope to quantize Einstein gravity by standard field theory methods.
- There currently exist only two even remotely serious proposals for an attempt to quantize Einstein gravity, namely String Theory (ST) and Loop Quantum Gravity (LQG) – the latter being based on an ad-hoc rewriting of the constrained Hamiltonian description of gravity due to ADM and Wheeler-deWitt.
- Anyone who talks about "quantum gravity" without specifying which of these two approaches they have in mind is engaged in speculation rather than science.
- Both of these theories are subjects in mathematical phsyics, which has much higher mathematical standards of rigor, clarity and precision than ordinary theoretical physics. In mathematical physics, it is pointless to talk about a model that has not been specified to full mathematical precision.

What does it mean to understand GR ?

- It is excceedingly unlikely that anyone who fails to have an excellent understanding of the mathematical foundations of General Relativity would have any chance of making progress in the very hard problem of quantizing that theory.
- It is impossible to understand General Relativity without having an excellent understanding of the following subjects in pure mathematics:
  1. Differential Geometry and some differential topology
  2. Riemannian and pseudo-Riemannian geometry
  3. Fiber bundles and characteristic classes
  4. Spin geometry and index theory
  5. The geometry of jet bundles and the geometric theory of (nonlinear) PDEs defined on manifolds
- You do not understand General Relativity if you haven't carefully studied basic references such as the standard textbooks by Wald and Hawking and Ellis. These references are not advanced, but standard requirements for anyone who wants to talk about classical gravity in the first place.
- It is exceedingly unlikely that any person who doesn't have an excellent understanding of General Relativity would have anything of value to contribute to the very hard problem of quantizing that theory or any of its interesting and meaningful modifications (or even have any idea what such modifications might be).

What is mathematical cosmology

- Mathematical cosmology is a subject within mathematical physics which sits at the intersection of General Relativity and Quantum Field Theory on Curved Spacetimes (book by Birrell and Davies).
- The most basic tool of cosmology is Cosmological Perturbation Theory (CPT) [Bardeen 1980, Kodama & Sasaki 1984 followed by many others].
- One of the most important developments in mathematical cosmology during the past 30 years has been the application of ideas and technique from Algebraic Quantum Field Theory (the Kastler-Haag approach) to Quantum Field Theory on Curved Spacetimes and to cosmology (Klaus Fredenhagen and collaborators).

What is “quantum cosmology” ?

- “Quantum cosmology” is an attempt to understand the Plack epoch by combing the Hartle-Hawking “no boundary proposal” with various ideas originating from String Theory, Loop Quantum Gravity, AQFT on Curved Spacetimes and various advanced areas of mathematics and mathematical physics.
- It is a topic within mathematical physics and hence subject to the standards of mathematical precision and rigor of that area of science.
- It requires advanced knowlegde of general relativity, Quantum Field Theory, Functional Analysis and Operator Theory.
- It is closely related to matematical subjects such as (Geometric) Scattering Theory (Melrose) and Geometric Analysis
- Its appliication to multifield models also requires a good grasp of Geometric Dynamical Systems theory (Katok and Hasselblatt).
- Its application to two-field cosmological models requires a good understanding of the theory of Riemann surfaces, including Poincare's uniformization theorem and the theory of Fuchsian and Surface Groups (which is a subset of the theory of Arithmetic Groups).

Geometric scattering theory

Figure: Prof. Melrose

Main idea:

- The Hartle-Hawking wavefunction proposal for the minisuperspace approximation of mutifield cosmological models turns out to be equivalent to a problem in the geometric scattering theory of a geometric PDE defined on a (generally non-comact) complete Riemannian manifold $(\mathcal{M}, \mathcal{G})$ by the positive time propagation of a certain perturbation $\Box + P$ of the Klein-Gordon operator $\Box$ of the Lorentzian half-cylinder $C_+(\mathcal{M}) \stackrel{\mathrm{def.}}{=} \mathbb{R}_{\geq 0} \times \mathcal{M}$ over $(\mathcal{M}, \mathcal{G})$.
- Here P is a (generally non-compact) scalar perturbation of $\Box$ which depends on the choice of the scalar potential $V \in \mathcal{C}^\infty(M, \mathbb{R}_{>0})$ of the multifield cosmological model.
- The problem can be approached with the tools of Geometric Scattering Theory developed by Melrose and collaborators. This requires a good understanding of the (Freudenthal) ends of $(\mathcal{M}, \mathcal{G})$.
- For hyperbolic two field models (i.e. when $(\mathcal{M}, \mathcal{G})$ is a hyperbolic surface), the problem can be approached using Poincare's uniformization theorem. ' In this case, it relates via uniformization to a similar problem for the Poincare upper half plane, which can be studied using classical results of Lax, Philips and others.

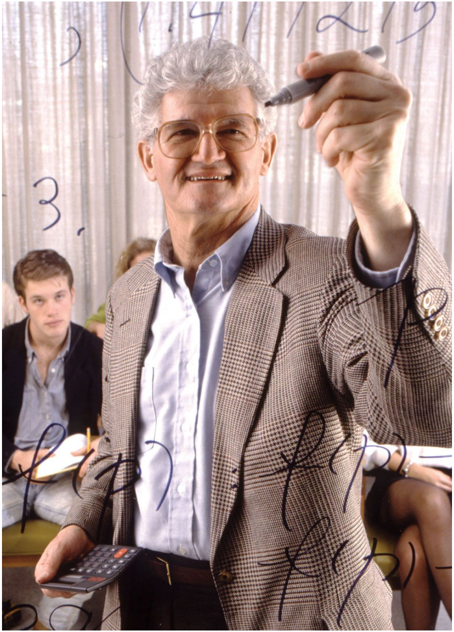


Figure: Peter Lax