Dark dimension cosmology

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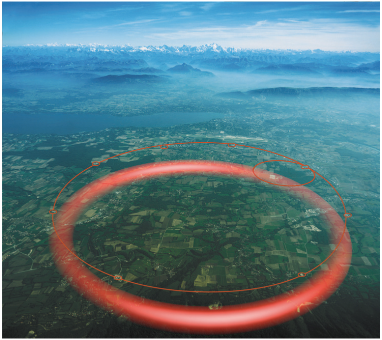
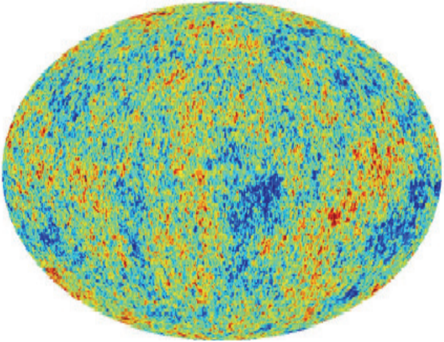
Challenge for a fundamental theory of Nature

describe both particle physics and cosmology



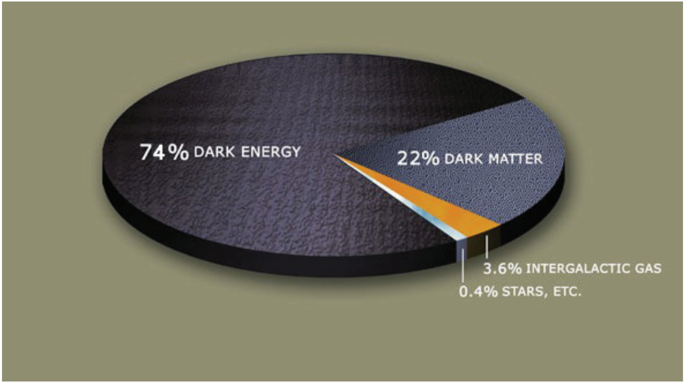


Accelerator experiments and cosmological observations:

complementary information for the same fundamental theory

Content of the Universe vs Standard Model

- Ordinary matter: only a tiny fraction $\lesssim 5\%$
- Non-luminous (dark) matter: $\sim 25\%$



Relativistic dark energy 70-75% of the observable universe

negative pressure: $p = -\rho \;\Rightarrow$ cosmological constant

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab} \;\Rightarrow\; \rho_\Lambda = \frac{c^4 \Lambda}{8\pi G} = -p_\Lambda$$

Two length scales:

- $[\Lambda] = L^{-2} \leftarrow$ size of the observable Universe

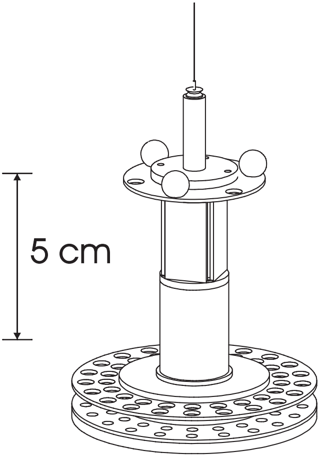
  $\Lambda_{obs} \simeq 0.74 \times 3H_0^2/c^2 \simeq 1.4 \times (10^{26}\,\mathrm{m})^{-2}$

  ↖ Hubble parameter $\simeq 73\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$

- $[\frac{\Lambda}{G} \times \frac{c^3}{\hbar}] = L^{-4} \leftarrow$ dark energy length $\simeq 85\mu\mathrm{m}$

Newton's law is valid down to distances 30 μm

Adelberger et al. '06



Strings and extra dimensions

- consistency of the theory ⇒ extra dimensions

  string coupling g_s can be treated as en extra dimension in M-theory

- matter and gauge interactions may be localized on lower dim branes

  transverse dimensions can be large [10]

⇒ **string scale M_s can be lower than the 4d Planck mass!**

opening a new way to address physics problems and scales

M_s low (multi-TeV) ⇒ electroweak hierarchy

M_s at intermediate energies $\sim 10^{11}$ GeV ($M_s^2/M_P \sim$ TeV)

⇒ SUSY breaking, strong CP axion, see-saw neutrino scale

- compactification ⇒ parameters: moduli fields + discrete fluxes
- moduli stabilization ⇒ huge landscape of vacua [11]

⇒ **need an extra input of guidance principle**

how they escape observation?

finite size R

energy cost to send a signal:

$E > R^{-1} \leftarrow$ compactification scale

experimental limits on their size

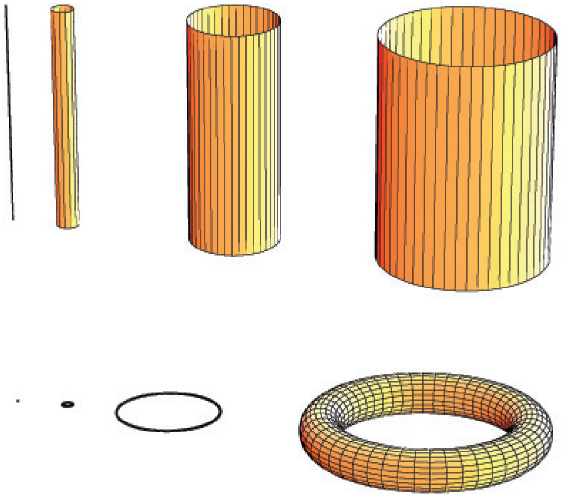
light signal $\Rightarrow$ $E \gtrsim 1$ TeV

$R \lesssim 10^{-16}$ cm

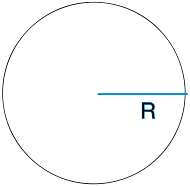
how to detect their existence?

motion in the internal space $\Rightarrow$ mass spectrum in 3d

Dimensions D=??



example: - one internal circular dimension
- light signal



plane waves e^{ipy} periodic under $y \to y + 2\pi R$

$\Rightarrow$ quantization of internal momenta: $p = \frac{n}{R}$; $n = 0, \pm 1, \pm 2, \ldots$

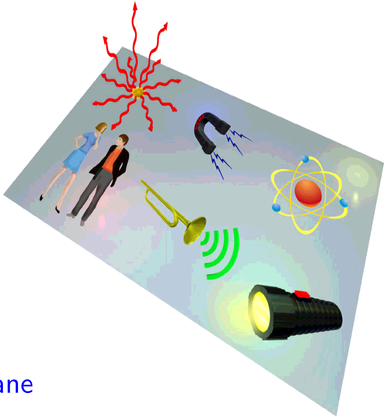
$\Rightarrow$ 3d: tower of Kaluza Klein particles with masses $M_n = |n|/R$

$$p_0^2 - \vec{p}^2 - p_5^2 = 0 \;\Rightarrow\; p^2 = p_5^2 = \frac{n^2}{R^2}$$

$E >> R^{-1}$: emission of many massive photons

$\Leftrightarrow$ propagation in the internal space [6]

Our universe on a membrane



Two types of new dimensions:

- longitudinal: along the membrane
- transverse: “hidden” dimensions

  only gravitational signal $\Rightarrow R_\perp \lesssim 0.1$ mm !

Swampland Program

Vafa '05, Ooguri-Vafa '06

Not all effective field theories can consistently coupled to gravity

- anomaly cancellation is not sufficient

- consistent ultraviolet completion can bring non-trivial constraints

those which do not, form the 'swampland'

criteria $\Rightarrow$ conjectures

supported by arguments based on string theory and black-hole physics

Some well established examples:

- No exact global symmetries in Nature
- Weak Gravity Conjecture: gravity is the weakest force

  $\Rightarrow$ minimal non-trivial charge: $q \geq m$ in Planck units $8\pi G = \kappa^2 = 1$

  Arkani-Hamed, Motl, Nicolis, Vafa '06

Distance/duality conjecture

At large distance in field space $\phi \Rightarrow$ tower of exponentially light states

$m \sim e^{-\alpha\phi}$ with $\alpha \sim \mathcal{O}(1)$ parameter in Planck units

- provides a weakly coupled dual description up to the species scale

$$M_* = M_P/\sqrt{N}$$

Dvali '07

- tower can be either

1. a Kaluza-Klein tower (decompactification of d extra dimensions)

$$m \sim 1/R\,, \quad \phi = \ln R; \quad M_* = M_P^{(4+d)} = (m^d M_P^2)^{1/(d+2)}$$

$$N = (M_* R)^d$$

2. a tower of string excitations

$$M_* = m \sim \text{the string scale} = g_s M_P\ ; \quad \phi = -\ln g_s\,, \quad N = 1/g_s^2$$

emergent string conjecture

Lee-Lerche-Weigand '19

smallness of physical scales : large distance corner of landscape?

An early example

I.A.-Bachas-Lewellen-Tomaras '88

Theorem:

assuming a light gravitino (or gaugino) present in the string spectrum

$$M_{3/2} << M_P$$

$\Rightarrow \exists$ a tower of states with the same quantum numbers and masses

$$M_k = (2Nk+1)M_{3/2}\,; \quad k = 1, 2, \ldots\,; \quad N \text{ integer (not too large)}$$

Proof:

2D free-fermionic constructions $\Rightarrow N \lesssim 10$

2D bosonic lattices $\Rightarrow N \lesssim 10^3$

$\Rightarrow$ compactification scale $m = \lambda_{3/2}^{-1} M_{3/2}$ with $\lambda_{3/2} = 1/2N$

$\Rightarrow$ large extra dimension at the TeV scale I.A. '90

Dark dimension proposal for the dark energy

$m = \lambda^{-1}\Lambda^a \quad (M_P = 1) \quad ; \quad 1/4 \le a \le 1/2$ Montero-Vafa-Valenzuela '22

- distance $\phi = -\ln\Lambda$ Lust-Palti-Vafa '19
- $a \le 1/2$: unitarity bound $m^2_{\rm spin-2} \ge 2H^2 \sim \Lambda$ Higuchi '87
- $a \ge 1/4$: estimate of 1-loop contribution $\Lambda \gtrsim m^4$

observations: $\Lambda \sim 10^{-120}$ and $m \gtrsim 0.01$ eV (Newton's law) $\Rightarrow a = 1/4$

astro/cosmo constraints $\Rightarrow d = 1^\star$ 'dark' dimension of $\sim$ micron size

species scale (5d Planck mass): $M_* \simeq \lambda^{-1/3}\, 10^8$ GeV $10^{-4} \lesssim \lambda \lesssim 10^{-1}$

*also $d = 2$ with $M_* \sim 10$ TeV Anchordoqui-I.A.-Lust '25

Our observable universe should be localised on a '3-brane' $\perp$ to the DD

I.A.-Arkani Hamed-Dimopoulos-Dvali '98

Physics implications of the dark dimension

See Review article 2405.04427 Anchordoqui-I.A.-Lust

Neutrino masses

- natural explanation of neutrino masses introducing ν_R in the bulk

  ν-oscillation data with 3 bulk neutrinos $\Rightarrow$ $m \gtrsim 2.5$ eV ($R \lesssim 0.4\,\mu$m)

  $\Rightarrow \lambda \lesssim 10^{-3}$ and $M_* \sim 10^9$ GeV

  the bound can be relaxed in the presence of bulk ν_R-neutrino masses

  Lukas-Ramond-Romanino-Ross '00, Carena-Li-Machado^2-Wagner '17

- support on Dirac neutrinos by the sharpened WGC

  non-SUSY AdS vacua (flux supported) are unstable Ooguri-Vafa '16

  avoid 3d AdS vacuum of the Standard Model with Majorana neutrinos

  Arkani-Hamed, Dubovsky, Nicolis, Villadoro '07

  lightest Dirac neutrino $\lesssim$ few eV or light gravitino $\mathcal{O}$(meV)

  Ibanez-Martin Lozano-Valenzuela '17; Anchordoqui-I.A.-Cunat '23

Dark matter candidates

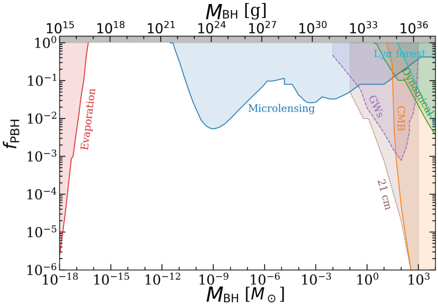
- 2 main new DM candidates:
  1. 5D primordial black holes in the mass range $10^{15} - 10^{21}\text{g}^{\star}$
     with Schwarzschild radius in the range $10^{-4} - 10^{-2}\ \mu\text{m}$
     Anchordoqui-I.A.-Lust '22
  2. KK-gravitons of decreasing mass due to internal decays (dynamical DM)
     from $\sim$ MeV at matter/radiation equality ($T \sim$ eV) to $\sim$ 50 keV today
     Gonzalo-Montero-Obied-Vafa '22

possible equivalence between the two Anchordoqui-I.A.-Lust '22

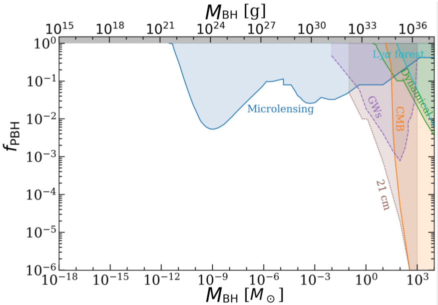
* $10^{8} - 10^{21}\text{g}$ for 6D Anchordoqui-I.A.-Lust '25

Primordial Black Holes as Dark Matter

4d PBH



5d PBH



5D BHs live longer than 4D BHs of the same mass

Dark Dimension Radion stabilization and inflation

If 4d inflation occurs with fixed DD radius $\Rightarrow$ too low inflation scale

Higuchi bound: $H_I \lesssim m \sim \text{eV}$

Interesting possibility: the extra dimension expands with time

$R_0 \sim 1/M_*$ to $R \sim \mu\text{m}$ requires ~ 40 efolds! Anchordoqui-I.A.-Lust '22

$$ds_5^2 = a_5^2(-d\tau^2 + d\vec{x}^2 + R_0^2\, dy^2) \quad R_0 : \text{initial size prior to inflation}$$

$$= \frac{ds_4^2}{R} + R^2 dy^2 \quad ; \quad ds_4^2 = a^2(-d\tau^2 + d\vec{x}^2) \quad \Rightarrow a^2 = R^3$$

After 5d inflation of $N = 40$-efolds $\Rightarrow$ 60 e-folds in 4d with $a = e^{3N/2}$

Large extra dimensions from inflation in higher dimensions

Anchordoqui-IA '23

Large hierarchies in particle physics and cosmology

Particle physics: why gravity appears so weak compared to other forces?

$$M_p/M_w \sim 10^{16}$$

Cosmology: why the Universe is so large compared to our causal horizon?

at least 10^{26} larger

Question: can uniform $(4+d)$ inflation relate the 2 hierarchies?

size of the observable universe to the observed weakness of gravity

compared to the fundamental (gravity/string) scale M_*

Extra dimensions should expand from the fundamental length

to the size required for the present strength of gravity

while at the same time the horizon problem is solved in our universe

Answer: yes for any d [22]

Large extra dimensions from higher-dim inflation

Anchordoqui-IA '23

$$ds^2_{4+d} = \left(\frac{r}{R}\right)^d ds^2_4 + R^2 dy^2 \qquad ; \qquad ds^2_4 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$$

$$= \hat{a}^2_{4+d}(\tau)(-d\tau^2 + d\vec{x}^2 + R_0^2\ dy^2) \qquad r \equiv \langle R \rangle_{\text{end of inflation}}$$

- exponential expansion in higher-dims $\Rightarrow$ power low inflation in 4D

  FRW coordinates: $e^{H\hat{t}} \sim (Ht)^{2/d} \Rightarrow R(t) \sim t^{2/d},\ a(t) \sim t^{1+2/d}$

- $\hat{N}$ e-folds in $(4+d)$-dims $\Rightarrow N = (1 + d/2)\hat{N}$ e-folds in 4D

Impose $M_* = M_p e^{-dN/(2+d)} \gtrsim 10$ TeV

$\gtrsim 10^8$ GeV for $d = 1$ ($r \lesssim 30\mu$m)

$\Rightarrow$ the horizon problem is solved for any d $\quad N \gtrsim 30 - 60$ ($N \gtrsim \ln \frac{M_I}{\text{eV}}$)

Precision of CMB power spectrum measurement

Physical distances change from higher to 4 dims

equal time distance between two points in 3-space

$$d^{\tau}_{\text{phys}}(x,x') = d(x,x')\,a(\tau) = d(x,x')\,\hat{a}(\tau)\left(\frac{R}{R_0}\right)^{d/2} = \hat{d}^{\tau}_{\text{phys}}(x,x')\frac{M_p(\tau)}{M_*}$$

co-moving distance

precision of CMB data: angles $\lesssim$ 10 degrees, distances $\lesssim$ Mpc (Gpc today)

Mpc $\to$ Mkm at $M_I \sim$ TeV with radiation dominated expansion

$\times$TeV$/M_I$ at a higher inflation scale $M_I \sim M_*$

$\times M_*/M_P$ conversion to higher-dim distances

$\Big\}\times$ TeV$/M_p$

$\simeq$ micron scale!

$d > 2$: needs a period of 4D inflation for generating scale invariant density perturbations

Density perturbations from 5D inflation

inflaton (during inflation) $\simeq$ massless minimally coupled scalar in dS space

$\Rightarrow$ logarithmic growth at large distances (compared to the horizon H^{-1})

equal time 2-point function in momentum space at late cosmic time

$$\langle \Phi^2(\hat{k},\tau)\rangle_{\tau\to 0} \simeq \frac{4}{\pi}\frac{H^3}{(\hat{k}^2)^2} \quad ; \quad \hat{k}^2 = k^2 + n^2/R^2$$

2-point function on the Standard Model brane (located at $y=0$):

$$\sum_n \langle \Phi^2(\hat{k},\tau)\rangle_{\tau\to 0} \simeq \frac{2RH^3}{k^2}\left(\frac{1}{k}\coth(\pi kR) + \frac{\pi R}{\sinh^2(\pi kR)}\right) \; ; \quad k = 2\pi/\lambda$$

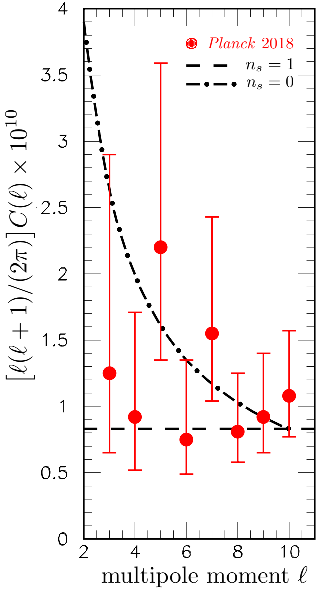
Amplitude of the power spectrum: $\mathcal{A} = \frac{k^3}{2\pi^2}\langle \Phi^2(k,\tau)\rangle_{y=0}$

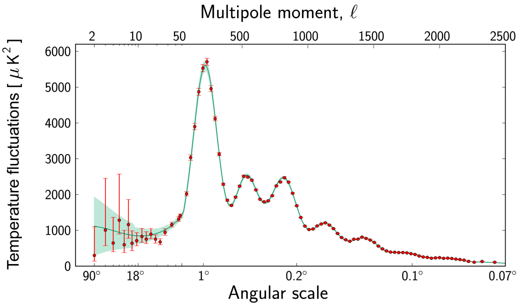
- $\pi kR > 1$ ('small' wave lengths) $\Rightarrow \mathcal{A} \sim \frac{H^2}{\pi^2}$ $\quad n_s \simeq 1$

  summation over n crucial for scale invariance: 'tower' of 4D inflatons

- $\pi kR < 1$ ('large' wave lengths) $\Rightarrow \mathcal{A} \simeq \frac{2H^3}{\pi^3 k}$ $\quad n_s \simeq 0$

Large-angle CMB power spectrum





Detailed computation of primordial perturbations:

IA-Cunat-Guillen '23

5D: inflaton + metric (5 gauge invariant modes) $\Rightarrow$

4D: 2 scalar modes (inflaton + radion), 2 tensor modes, 2 vector modes

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{1}{3\varepsilon}\mathcal{A}\left[\left(\frac{k}{\hat{a}H}\right)^{2\delta-5\varepsilon} + \varepsilon\left(\frac{k}{\hat{a}H}\right)^{-3\varepsilon} \times \begin{cases} \frac{5}{24} & R_0 k >> 1 \\ \frac{1}{3} & R_0 k << 1 \end{cases}\right]$$

$$\mathcal{P}_{\mathcal{T}} \simeq \frac{4H^2}{\pi^2}\left(\frac{k}{\hat{a}H}\right)^{-3\varepsilon} \times \begin{cases} R_0 H & R_0 k >> 1 \\ \frac{2H}{\pi k} & R_0 k << 1 \end{cases} \qquad r = 24\varepsilon$$

$$\mathcal{P}_{\mathcal{V}} \simeq \frac{4R_0 H^3}{\pi^2}\left(\frac{k}{\hat{a}H}\right)^{-3\varepsilon} \times \begin{cases} 1 & R_0 k >> 1 \\ \frac{\pi^3}{45}(R_0 k)^3 & R_0 k << 1 \quad S^1/Z_2\,(n \neq 0) \end{cases}$$

$$\mathcal{P}_{\mathcal{S}} \simeq \frac{9\varepsilon^2}{16}\mathcal{P}_{\mathcal{R}} \quad \text{entropy} \Rightarrow \beta_{\text{isocurvature}} = \frac{\mathcal{P}_{\mathcal{S}}}{\mathcal{P}_{\mathcal{R}} + \mathcal{P}_{\mathcal{S}}} \simeq \frac{9\varepsilon^2}{16} < 0.038\,\text{exp}$$

slow-roll parameters: $\varepsilon = -\frac{\dot{H}}{H^2}$; $\delta = \varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon} \simeq \eta - \varepsilon$

Bispectrum from 5D inflation IA-Chatrabhuti-Cunat-Isono '25

$$f_{\rm NL} \underset{\substack{R_0 k_1 \ll 1 \\ R_0 k_2 \ll 1 \\ R_0 k_3 \ll 1}}{\simeq} \frac{5}{12}\frac{8}{R_0^3 k_1 k_2 k_3}\left\{(\varepsilon_2-\varepsilon_1)\frac{k_1^4+k_2^4+k_3^4}{k_1^3+k_2^3+k_3^3}+8\varepsilon_1\frac{k_1^2k_2^2+k_1^2k_3^2+k_2^2k_3^2}{k_1^3+k_2^3+k_3^3}\right\}$$

$$f_{\rm NL} \underset{\substack{R_0 k_1 \ll 1 \\ R_0 k_2 \gg 1 \\ R_0 k_3 \gg 1}}{\simeq} \frac{5}{12}\frac{2}{R_0 k_1}\left\{(\varepsilon_2-\varepsilon_1)+16\varepsilon_1\frac{k_2^2k_3^2}{(k_2+k_3)\left(k_2^3+k_3^3\right)}\right\} \quad k_t =: \sum_a k_a$$

$$f_{\rm NL} \underset{\substack{R_0 k_1 \gg 1 \\ R_0 k_2 \gg 1 \\ R_0 k_3 \gg 1}}{\simeq} \frac{5}{12}\left\{(\varepsilon_2-\varepsilon_1)+\frac{16\varepsilon_1}{k_1^3+k_2^3+k_3^3}\sum_{1\le a<b\le 3}\left(\frac{k_a^2k_b^2}{k_t}+k_1k_2k_3\frac{k_ak_b}{k_t^2}\right)\right\}$$

$$f_{\rm NL}^{\rm 4D} = \frac{5}{12}\left\{(\varepsilon_2-\varepsilon_1)+\frac{\varepsilon_1}{k_1^3+k_2^3+k_3^3}\sum_{1\le a<b\le 3}\left(k_ak_b^2+k_a^2k_b+\frac{8}{k_t}k_a^2k_b^2\right)\right\}$$

$f_{\rm NL}|_{R_0 k_a \gg 1} \neq f_{\rm NL}^{\rm 4D}$: global conformal invariance is broken

End of inflation

Inflaton: 5D field φ with a coupling to the brane to produce SM matter

e.g. via a 'Yukawa' coupling suppressed by the bulk volume $y \sim 1/(RM_*)^{1/2}$

Its decay to KK gravitons should be suppressed to ensure $\Delta N_{\rm eff} < 0.2$

Anchordoqui '20

$$\left(\Gamma^{\varphi}_{\rm SM} \sim \frac{m}{M_*} m_\varphi\right) > \left(\Gamma^{\varphi}_{\rm grav} \sim \frac{m_\varphi^4}{M_*^3}\right) \Rightarrow m_\varphi < 1\,\mathrm{TeV}$$

5D cosmological constant at the minimum of the inflaton potential

$\Rightarrow$ runaway radion potential:

$$V_0 \sim \frac{\Lambda_5^{\rm min}}{R}\,; \qquad (\Lambda_5^{\rm min})^{1/5} \lesssim 100\,\mathrm{GeV} \quad \text{(Higuchi bound)}$$

canonically normalised radion: $\phi = \sqrt{3/2}\,\ln(R/r)$ $\quad r \equiv \langle R\rangle_{\text{end of inflation}}$

$\Rightarrow$ exponential quintessence-like form $V_0 \sim e^{-\alpha\phi}$ with $\alpha \simeq 0.8$

just at the allowed upper bound: Barreiro-Copeland-Nunes '00

Radion stabilisation at the end of 5D inflation

Anchordoqui-IA '23

Potential contributions stabilising the radion:

$$V = \left(\frac{r}{R}\right)^2 \hat{V} + V_C \quad ; \quad \hat{V} = 2\pi R \Lambda_5^{\rm min} + T_4 + 2\pi \frac{K}{R}$$

T_4: 3-branes tension, K: kinetic gradients, V_C: Casimir energy

↑Arkani-Hamed, Hall, Tucker-Smith, Weiner '99

Radion mass m_R: $\sim$ eV (m_{KK}) to 10^{-30} eV (m_{KK}^2/M_p) depending on K

- $K \sim M_*$, all 3 terms of $\hat{V}$ of the same order, V_C negligible

  tune $\Lambda_4 \sim 0_+ \Rightarrow m_R \lesssim m_{KK} \sim$ eV

- K negligible, all 3 remaining terms of the same order

  $\Rightarrow$ minimum is driven by a +ve $V_C = \dfrac{2\pi r^2}{32\pi^7 R^6}(N_F - N_B)$

  Arkani-Hamed, Dubovsky, Nicolis, Villadoro '07

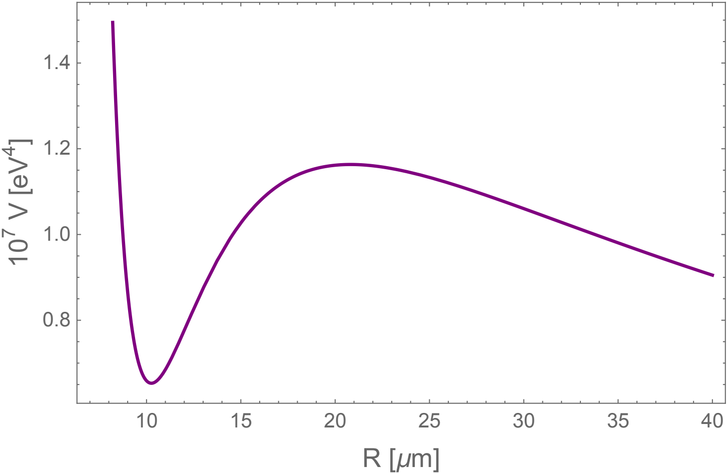
  no tuning of Λ_4 but $\Lambda_5^{\rm min}$ should be order $(\text{subeV})^5$ [30]

Casimir potential

$$V_C = 2\pi R \left(\frac{r}{R}\right)^2 \mathrm{Tr}(-)^F \rho(R,m) \quad m : 5D \text{ mass}$$

$$\rho(R,m) = -\sum_{n=1}^{\infty} \frac{2m^5}{(2\pi)^{5/2}} \frac{K_{5/2}(2\pi Rmn)}{(2\pi Rmn)^{5/2}} \begin{cases} mR \to \infty & \text{exp suppressed} \\ mR \to 0 & 1/R^5 \end{cases}$$

Example of Radion stabilisation potential



$(\Lambda_5^{\min})^{1/5} = 25\ \mathrm{meV}$, $|T_4|^{1/4} = 27\ \mathrm{meV}$, $N_F - N_B = 7$

$N_F = 12$ (3 bulk R-neutrinos) $N_B = 5$ (5D graviton)

Cosmic discrepancies and Hubble tension

Anchordoqui-I.A.-Lust '23, AAL-Noble-Soriano '24

5σ tension between global and local measurements

$H_0 = 67.4 \pm 0.5$ km/s/Mpc Planck data

$H_0 = 73.04 \pm 1.04$ km/s/Mpc SH0ES supernova

This tension can be resolved if Λ changes sign around redshift $z \simeq 2$

Akarsu-Barrow-Escamilla-Vasquez '20, AV-Di Valentino-Kumar-Nunez-Vazquez '23

AdS$\rightarrow$dS transition is hard to implement due to a swampland conjecture:

non-SUSY AdS vacua are at infinite distance in moduli space

However it could happen due to quantum tunnelling

false vacuum decay of 5D scalar

Anchordoqui-I.A.-Lüst '23, Anchordoqui-I.A.-Lüst-Noble-Soriano

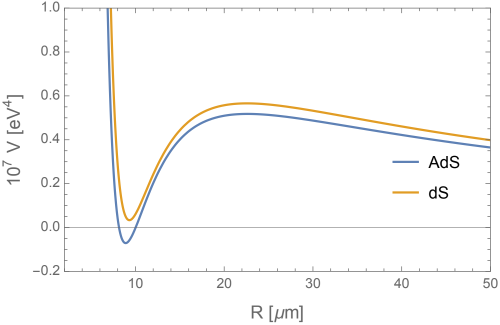
AdS→dS transition due to false vacuum decay in 5D

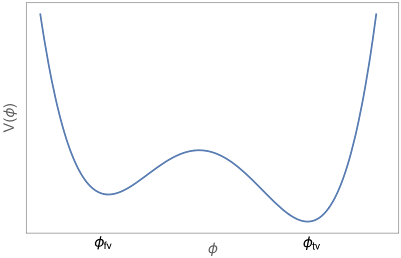
5D scalar at a false vacuum with light mass (lighter than $R_{\max}^{-1}$)

$N_F - N_B = 6 \Rightarrow$ AdS vacuum

decay to a (almost degenerate $\delta\epsilon < \Lambda$) true vacuum with heavy mass

$N_F - N_B = 7 \Rightarrow$ dS vacuum   slow transition at $z \simeq 2$





$(\Lambda_5^{\min})^{1/5} = 22.6 \text{ meV}, \; |T_4|^{1/4} = 24.2 \text{ meV}$

Conclusions

smallness of some physical parameters might signal

a large distance corner in the string landscape of vacua

such parameters can be the scales of dark energy and SUSY breaking

mesoscopic dark dimension proposal: interesting phenomenology

neutrino masses, dark matter, cosmology, SUSY breaking

Large extra dimensions from higher dim inflation

- connect the weakness of gravity to the size of the observable universe
- scale invariant density fluctuations from 5D inflation
- radion stabilization