Numerical Cosmology 3

Teleparallel Gravity – Observational constraints

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Modified Teleparallel Gravity

- What modifications of GR/TEGR are physically meaningful?

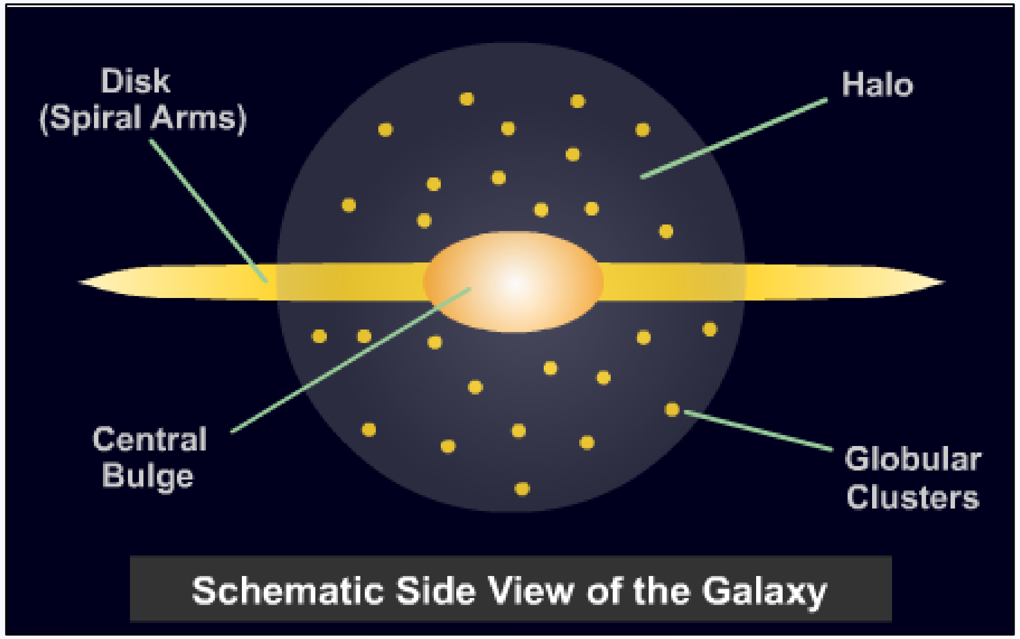
$$S = -\frac{1}{16\pi G}\int d^4x\, e\, T$$

- Take the following
  - $\boldsymbol{f(T)}$ **gravity**: $S = \frac{1}{16\pi G}\int d^4x\, e[-T + f(T)] + S_{\text{mat}}$
  - $\boldsymbol{f(T,B)}$ **gravity**: $S = \frac{1}{16\pi G}\int d^4x\, e[-T + f(T,B)] + S_{\text{mat}}$

Flat FLRW Cosmology	$T = 6H^2$ $B = 6(3H^2 + \dot{H})$ $\mathcal{R} \equiv -T + B = 6(2H^2 + \dot{H})$

Galactic Scale Physics

Can cosmologically motivated models produce new galactic physics?



- **Disk surface brightness distribution**

$$\Sigma(R) = \Sigma_0 \mathrm{e}^{-R/\beta_d}$$

where Σ_0 is the central surface brightness and β_d is the disk scale length

- **Bulge surface brightness spherical distribution** (de Vaucouleurs)

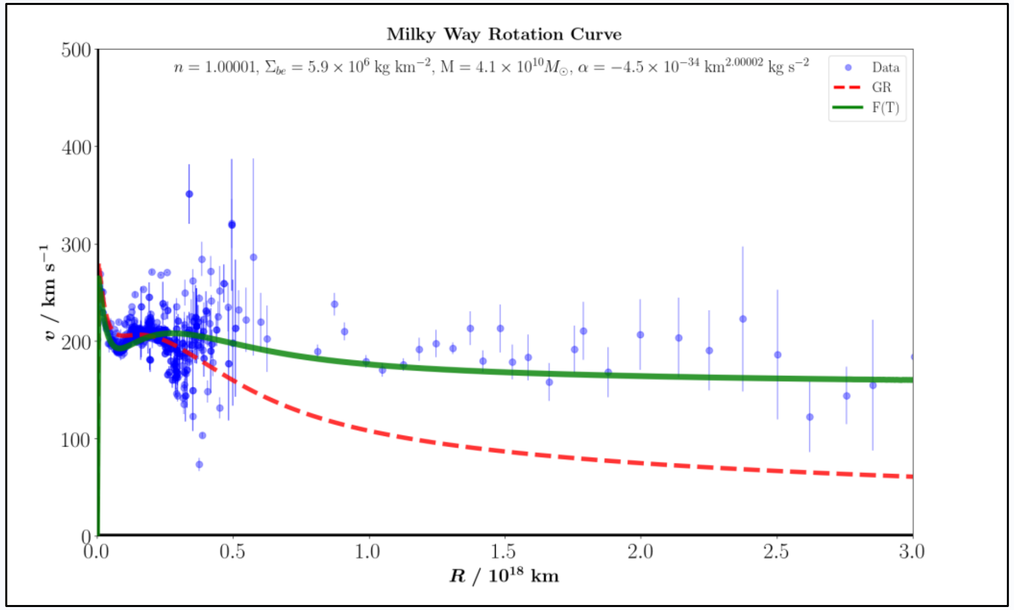
$$\Sigma_b(R) = I_e \operatorname{Exp}\left[-7.6695\left(\left(\frac{R}{R_e}\right)^{1/4} - 1\right)\right]$$

where I_e is the surface brightness at R_e, and R_e is the isophote radius

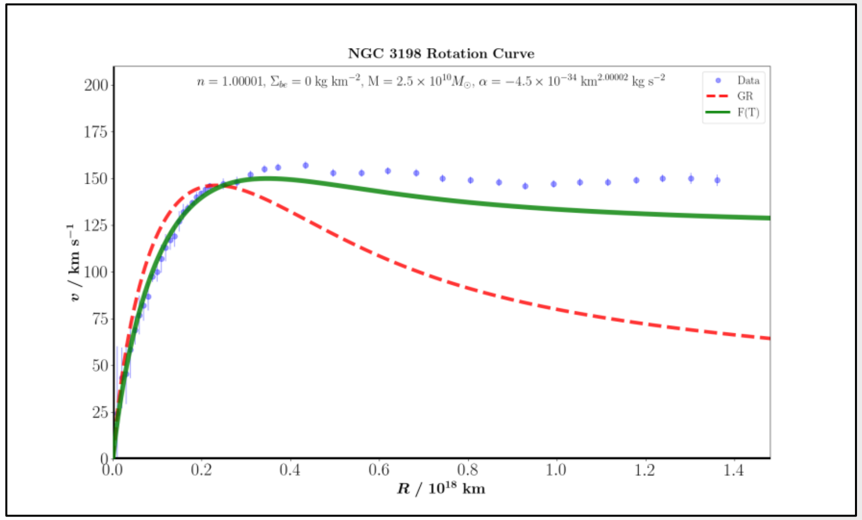
arXiv:1806.09677

Galactic Scale Physics

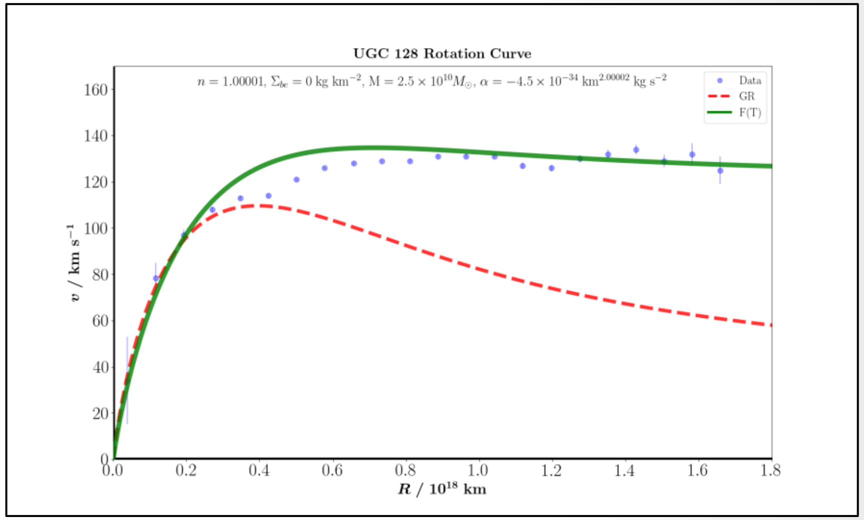
$$f(T) = \alpha T^n$$



Barred Spiral Galaxy



Low Surface Brightness Galaxy



arXiv:1806.09677

How do these theories beyond general relativity fare in cosmology?

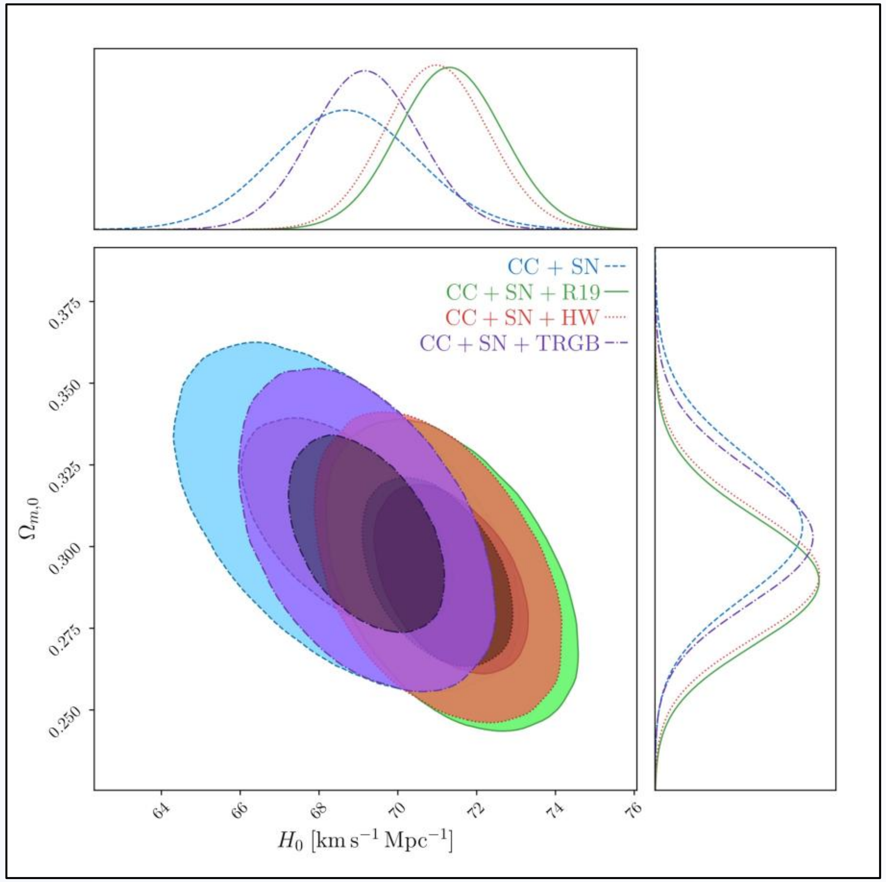
Local Expansion Data

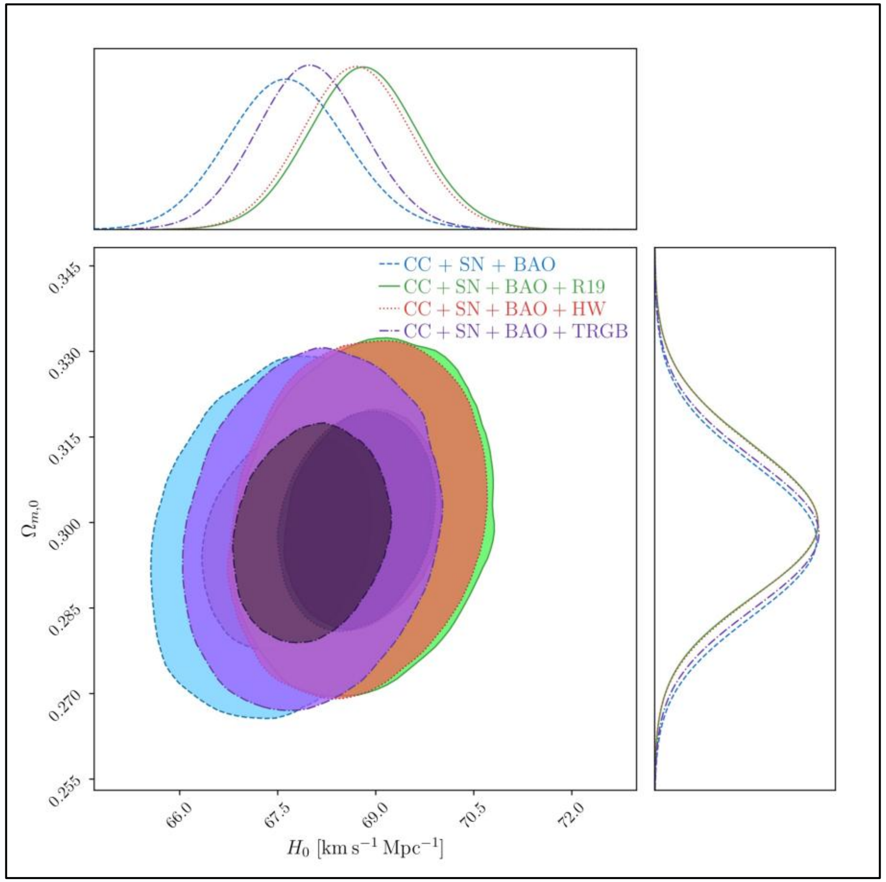
- Cosmic Chronometers (CC): Spectroscopic dating and **independent of cosmological models** ($z \sim 2$)
- Type Ia supernovae (SN): **Pantheon+ Sample** - 1701 light curves of 1550 Type Ia supernovae
- Baryonic Acoustic Oscillations (BAO): 2D data from **SDSS** for $z < 2.4$

H_0 Priors

- Planck Collaboration (18): **ΛCDM Model dependent** $\rightarrow H_0^{\mathrm{P18}} = 67.4 \pm 0.5\ \mathrm{km\ s^{-1} Mpc^{-1}}$
- SH0ES Survey [H_0^{R}]: Based on **geometry and Cepheid variables** $\rightarrow H_0^{\mathrm{R}} = 74.03 \pm 1.42\ \mathrm{km\ s^{-1} Mpc^{-1}}$
- Tip of the Red Giant Branch [TRGB]: Freedman et al. (2019) reports $H_0^{\mathrm{TRGB}} = 69.8 \pm 1.9\ \mathrm{km\ s^{-1} Mpc^{-1}}$
- H0licow [HW]: Based on strong lensing $\rightarrow H_0^{\mathrm{HW}} = 73.3 \pm 1.8\ \mathrm{km\ s^{-1} Mpc^{-1}}$

ΛCDM Cosmology





ΛCDM Eur.Phys.J.Plus 137 (2022) 5, 532

ΛCDM Constraints

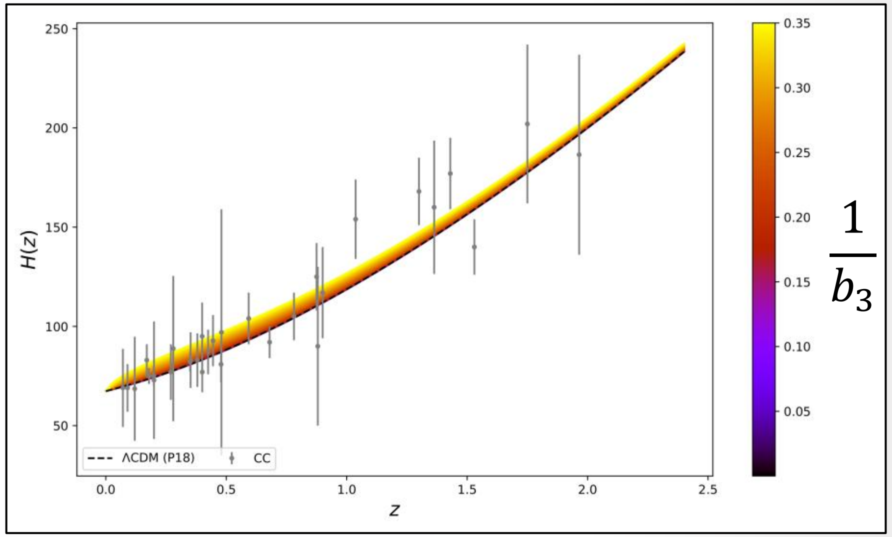
Data Sets	H_0 (km/s/Mpc)	$\Omega_{m,0}$	χ^2_{min}	AIC	BIC
CC+ SN	68.7 ± 1.8	$0.306^{+0.022}_{-0.021}$	1041.49	1047.49	1062.44
CC + SN + R19	$71.3^{+1.4}_{-1.3}$	$0.290^{+0.019}_{-0.020}$	1046.32	1052.32	1067.27
CC + SN + HW	71.0 ± 1.3	$0.291^{+0.020}_{-0.019}$	1044.99	1050.99	1065.94
CC + SN + TRGB	69.2 ± 1.4	$0.303^{+0.021}_{-0.020}$	1041.69	1047.69	1062.64
CC + SN + BAO	67.63 ± 0.90	0.297 ± 0.013	1057.46	1063.46	1078.45
CC + SN + BAO + R19	$68.81^{+0.82}_{-0.84}$	0.300 ± 0.013	1068.30	1074.30	1089.30
CC + SN + BAO + HW	68.70 ± 0.83	$0.300^{+0.014}_{-0.013}$	1066.03	1072.03	1087.03
CC + SN + BAO + TRGB	$67.98^{+0.85}_{-081}$	0.298 ± 0.013	1058.56	1064.56	1079.56

ΛCDM Eur.Phys.J.Plus 137 (2022) 5, 532

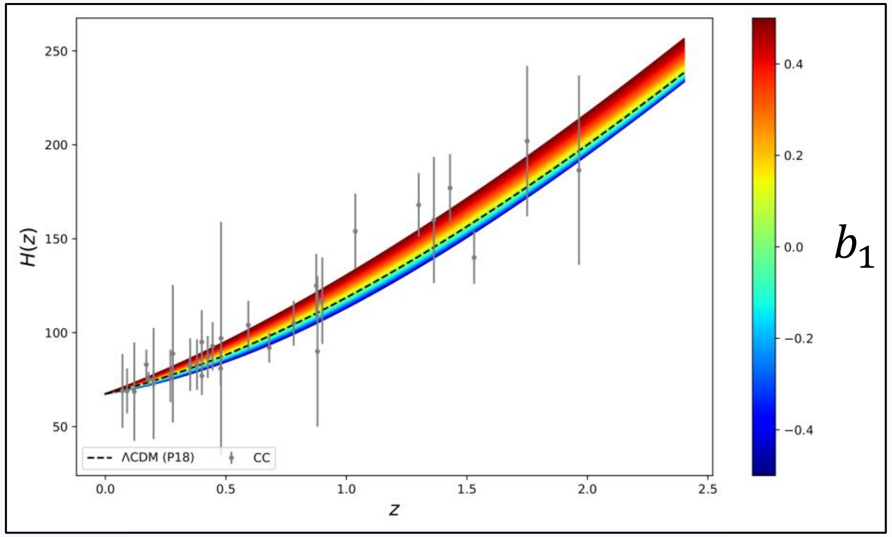
$f(T)$ Cosmological Models

$f(T)$ models:

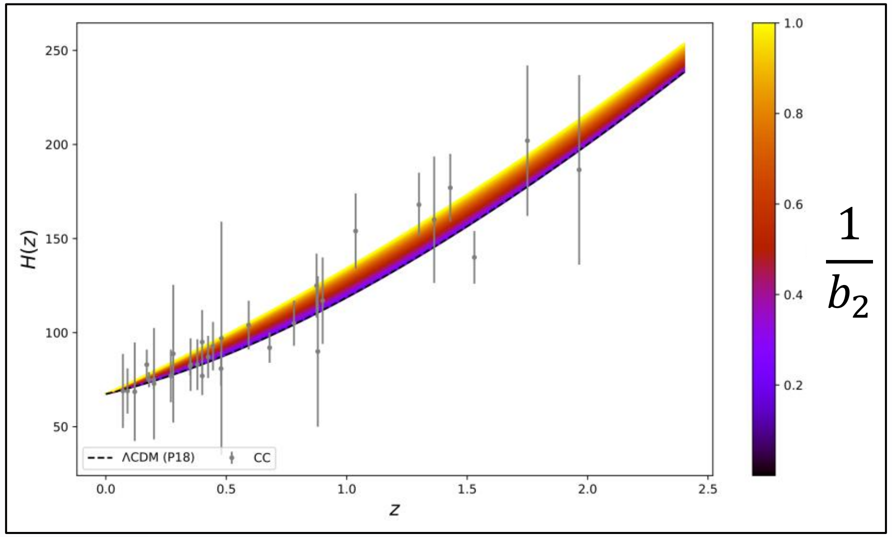
1. <u>Power-law</u>: $f_1(T) = \alpha_1 (T)^{b_1}$
2. <u>Linder</u>: $f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$
3. <u>Exponential</u>: $f_3(T) = \alpha_3 T_0 \left(1 - e^{-b_3 T/T_0}\right)$



$\frac{1}{b_3}$



b_1



$\frac{1}{b_2}$

$$\alpha_1 = \left(6H_0^2\right)^{1-p} \frac{1-\Omega_{M,0}}{2p-1}$$
$$\alpha_2 = \frac{1-\Omega_{M,0}}{1-(1-p)e^p}$$
$$\alpha_3 = \frac{1-\Omega_{M,0}}{1-(1-2p)e^p}$$

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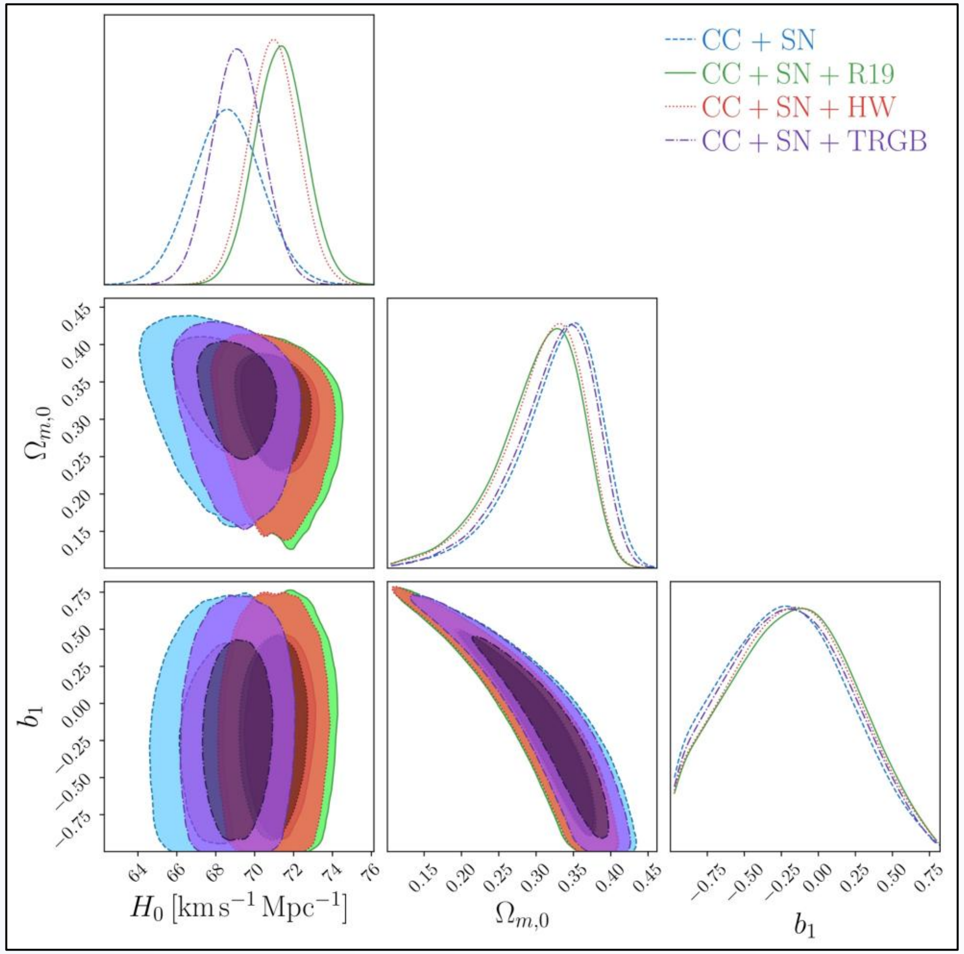
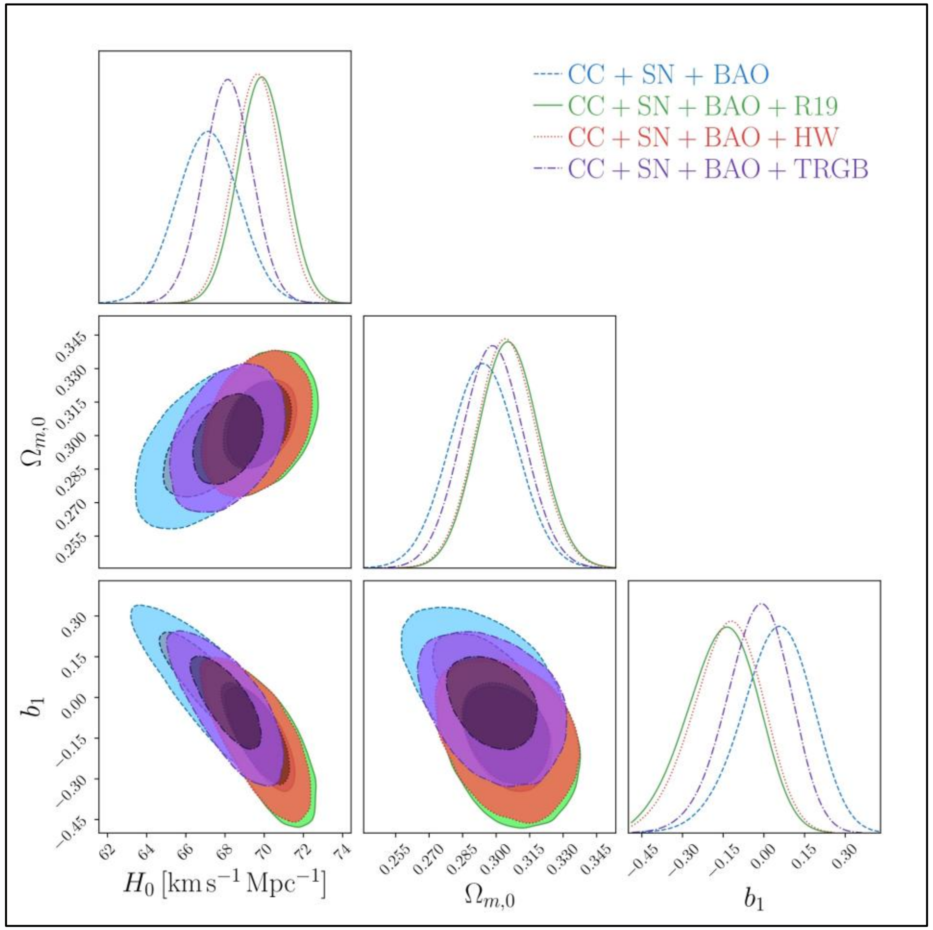
f_1CDM Posteriors

Power-law: $f_1(T) = \alpha_1(T)^{b_1}$





$\mathrm{f_1CDM}$ Eur.Phys.J.Plus 137 (2022) 5, 532

f_1CDM Constraints

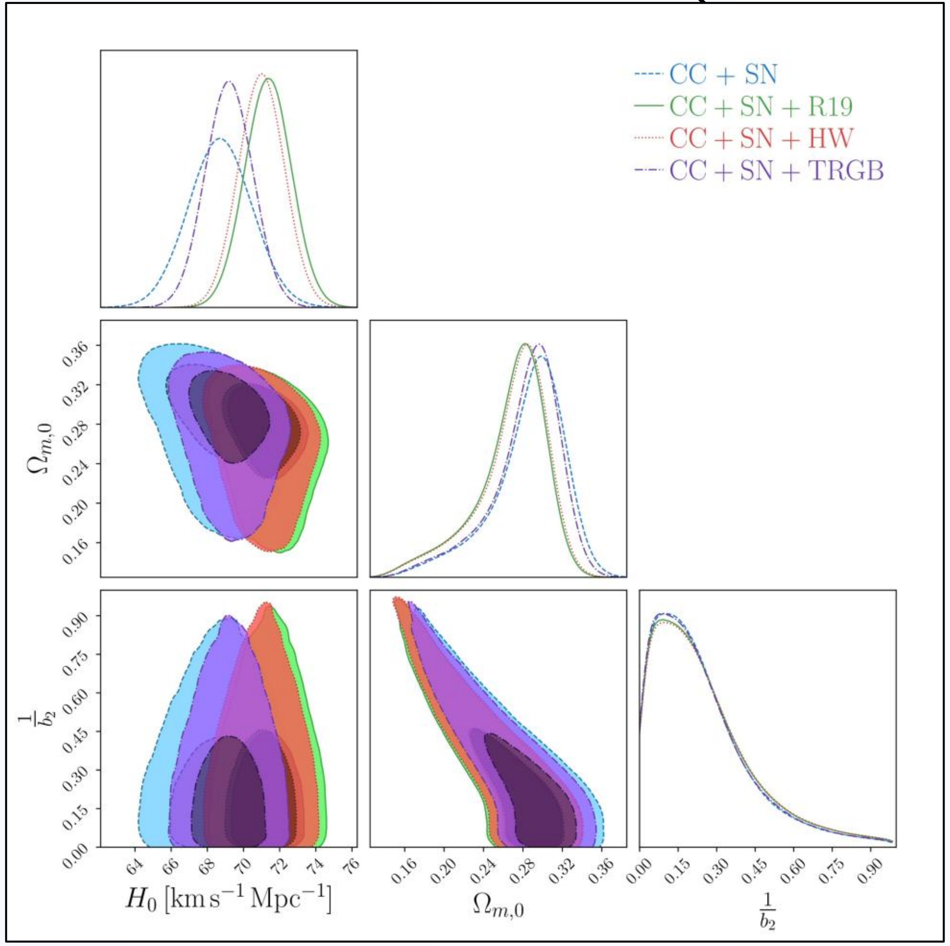
Power-law: $f_1(T) = \alpha_1(T)^{b_1}$

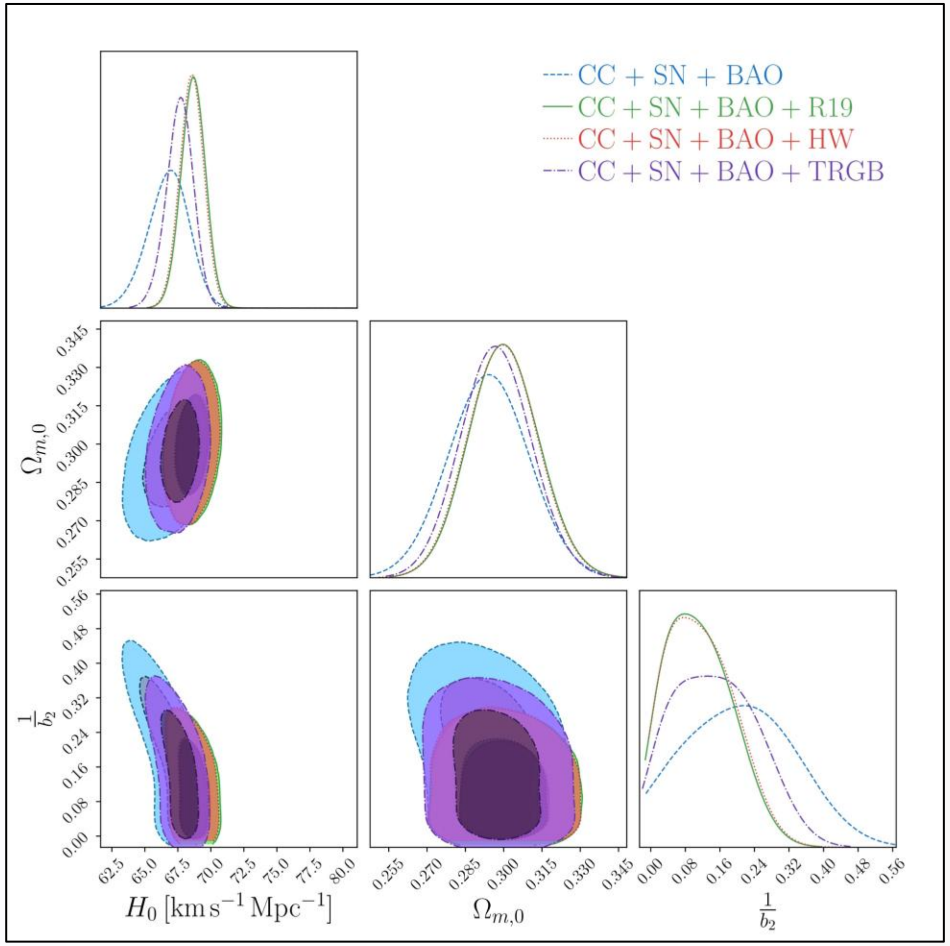
Data Sets	H_0[km/s/Mpc]	$\Omega_{m,0}$	b_1	M	$\chi^2_{\min}$	AIC	BIC	ΔAIC	ΔBIC
CC + SN	68.5 ± 1.8	$0.350^{+0.045}_{-0.064}$	$-0.22^{+0.41}_{-0.48}$	$-19.390^{+0.053}_{-0.055}$	1040.94	1048.94	1068.88	1.45	6.43
CC + SN + R19	$71.3^{+1.3}_{-1.4}$	$0.326^{+0.045}_{-0.065}$	$-0.13^{+0.40}_{-0.50}$	$-19.314^{+0.039}_{-0.038}$	1045.83	1053.83	1073.77	1.51	6.50
CC + SN + HW	71.0 ± 1.3	$0.329^{+0.045}_{-0.062}$	$-0.16^{+0.41}_{-0.48}$	$-19.324^{+0.038}_{-0.037}$	1044.50	1052.50	1072.44	1.51	6.50
CC + SN + TRGB	$69.1^{+1.4}_{-1.3}$	$0.344^{+0.045}_{-0.063}$	$-0.20^{+0.42}_{-0.47}$	-19.375 ± 0.040	1041.55	1049.55	1069.49	1.87	6.85
CC + SN + BAO	67.1 ± 1.6	0.294 ± 0.015	0.06 ± 0.13	-19.435 ± 0.047	1057.13	1065.13	1085.13	1.68	6.68
CC + SN + BAO + R19	69.9 ± 1.2	$0.305^{+0.014}_{-0.013}$	$-0.14^{+0.12}_{-0.13}$	$-19.359^{+0.035}_{-0.034}$	1066.87	1074.87	1094.87	0.56	5.56
CC + SN + BAO + HW	69.7 ± 1.2	$0.304^{+0.014}_{-0.012}$	$-0.12^{+0.12}_{-0.13}$	$-19.366^{+0.035}_{-0.033}$	1064.92	1072.92	1086.92	0.89	5.89
CC + SN + BAO + TRGB	68.1 ± 1.2	0.298 ± 0.014	$-0.01^{+0.11}_{-0.12}$	-19.407 ± 0.036	1058.56	1066.56	1086.56	2.00	7.00

f_1CDM Eur.Phys.J.Plus 137 (2022) 5, 532

f_2CDM Posteriors

Linder: $f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$





f_2CDM Eur.Phys.J.Plus 137 (2022) 5, 532

f_2CDM Constraints

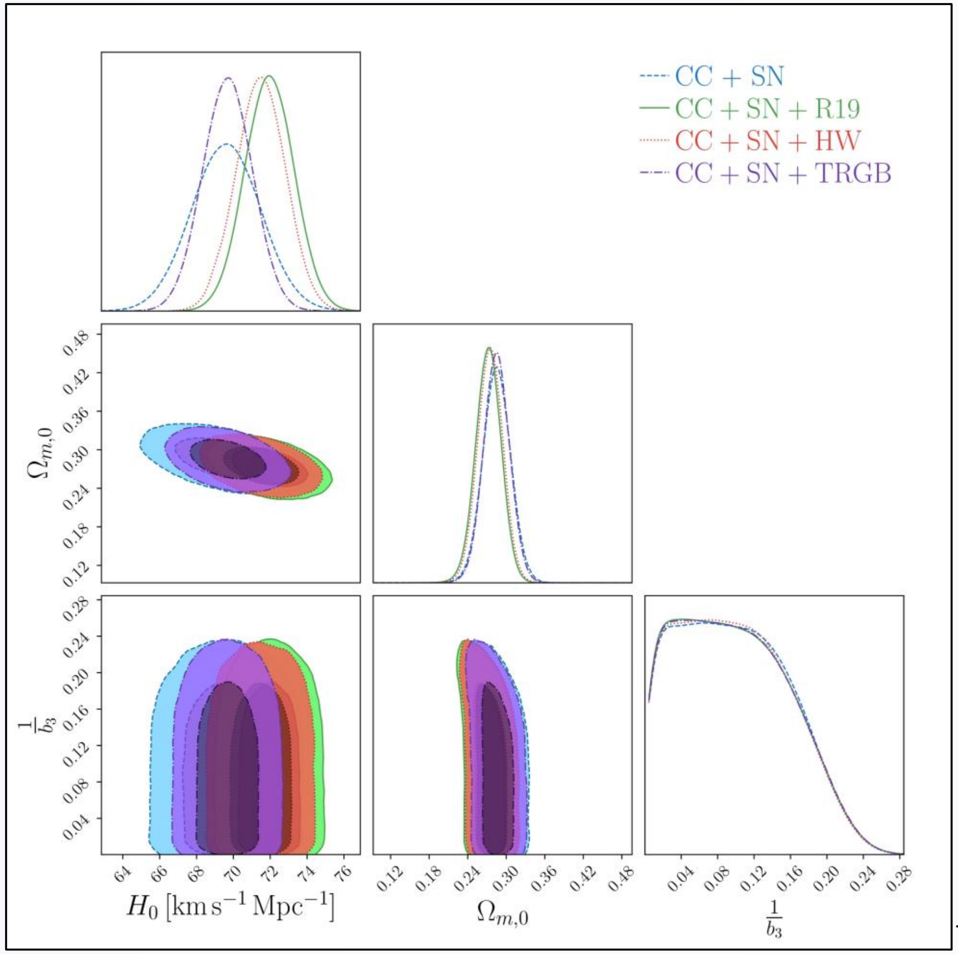
Linder: $f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$

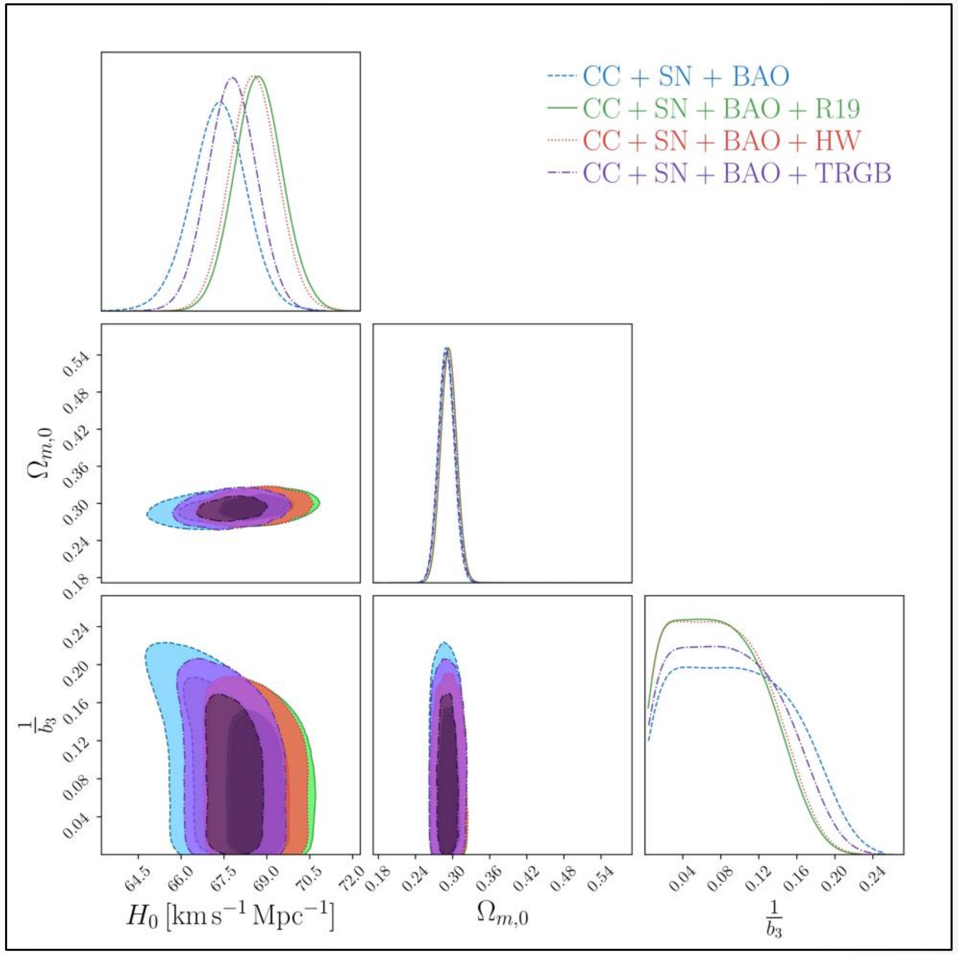
Data Sets	H_0[km/s/Mpc]	$\Omega_{m,0}$	$\frac{1}{b_2}$	M	χ^2_{min}	AIC	BIC	ΔAIC	ΔBIC
CC + SN	$68.7^{+1.8}_{-1.7}$	$0.298^{+0.031}_{-0.035}$	$0.101^{+0.227}_{-0.098}$	$-19.43^{+0.57}_{-0.47}$	1041.49	1049.49	1069.43	2.00	6.98
CC + SN + R19	71.4 ± 1.3	$0.283^{+0.027}_{-0.036}$	$0.088^{+0.252}_{-0.086}$	$-19.28^{+0.50}_{-0.51}$	1046.32	1054.32	1074.25	2.00	6.99
CC + SN + HW	$71.0^{+1.3}_{-1.2}$	$0.285^{+0.027}_{-0.036}$	$0.096^{+0.245}_{-0.093}$	$-19.37^{+0.45}_{-0.34}$	1044.99	1052.99	1072.93	2.00	6.99
CC + SN + TRGB	69.2 ± 1.3	$0.296^{+0.028}_{-0.035}$	$0.088^{+0.239}_{-0.085}$	$-19.36^{+0.36}_{-0.37}$	1041.69	1049.69	1069.62	2.00	6.99
CC + SN + BAO	$66.9^{+1.5}_{-1.6}$	0.294 ± 0.016	$0.22^{+0.12}_{-0.15}$	$-19.38^{+0.22}_{-0.35}$	1056.52	1064.62	1084.52	1.06	6.06
CC + SN + BAO + R19	$68.71^{+0.88}_{-0.96}$	0.300 ± 0.014	$0.079^{+0.098}_{-0.064}$	$-19.35^{+0.19}_{-0.24}$	1068.31	1076.31	1096.31	2.00	7.00
CC + SN + BAO + HW	$68.58^{+0.89}_{-0.92}$	$0.300^{+0.013}_{-0.014}$	$0.076^{+0.105}_{-0.060}$	$-19.389^{+0.045}_{-0.047}$	1066.03	1074.03	1094.03	2.00	7.00
CC + SN + BAO + TRGB	67.7 ± 1.0	0.297 ± 0.014	$0.128^{+0.111}_{-0.099}$	$-19.46^{+0.37}_{-0.26}$	1058.47	1066.47	1086.47	1.90	6.90

f_1CDM Eur.Phys.J.Plus 137 (2022) 5, 532

f_3CDM Posteriors

Exponential: $f_3(T) = \alpha_3 T_0\left(1 - e^{-b_3 T/T_0}\right)$





f_3CDM Eur.Phys.J.Plus 137 (2022) 5, 532

f_3CDM Constraints

Exponential: $f_3(T) = \alpha_3 T_0 \left(1 - e^{-b_3 T/T_0}\right)$

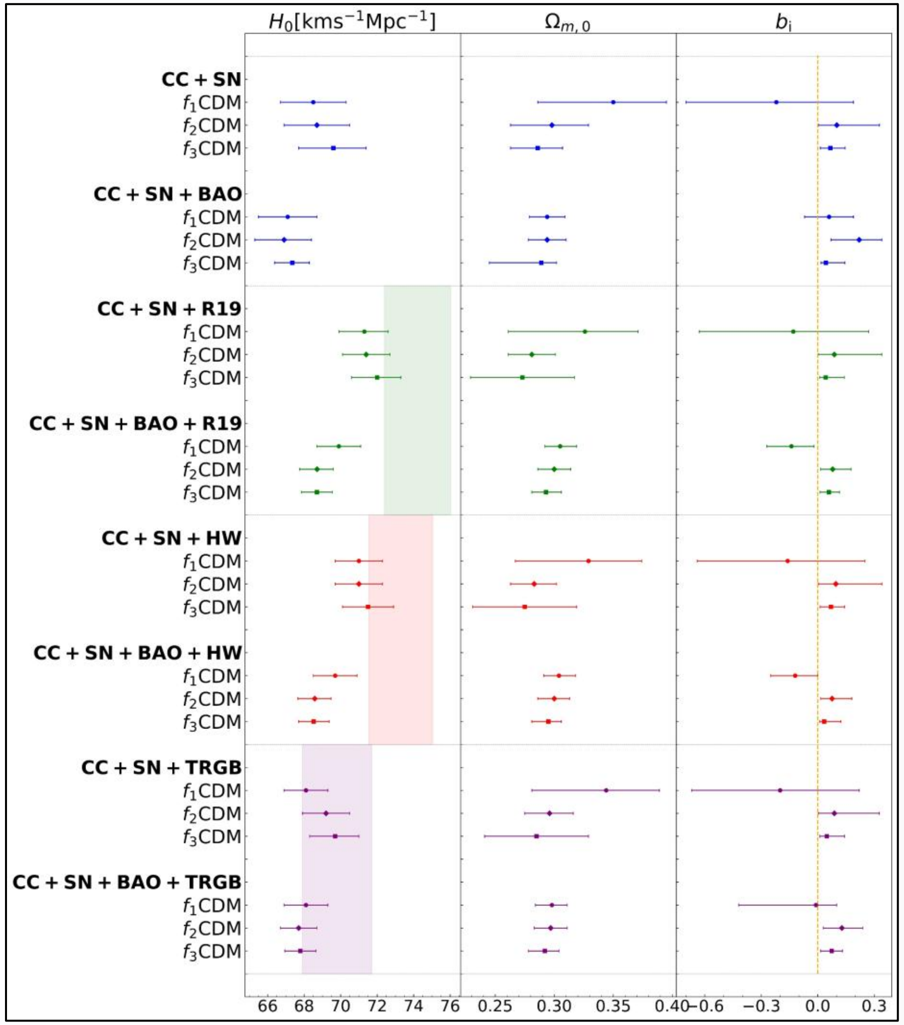
Data Sets	H_0[km/s/Mpc]	$\Omega_{m,0}$	$\frac{1}{b_3}$	M	$\chi^2_{\rm min}$	AIC	BIC	ΔAIC	ΔBIC
CC + SN	$69.6^{+1.8}_{-1.9}$	$0.286^{+0.021}_{-0.023}$	$0.067^{+0.078}_{-0.054}$	$-19.367^{+0.053}_{-0.056}$	1045.04	1053.04	1072.97	5.55	10.53
CC + SN + R19	$72.0^{+1.3}_{-1.4}$	0.273 ± 0.020	$0.042^{+0.099}_{-0.032}$	$-19.302^{+0.037}_{-0.039}$	1048.16	1056.16	1076.10	3.84	8.82
CC + SN + HW	71.5 ± 1.4	$0.275^{+0.019}_{-0.020}$	$0.070^{+0.072}_{-0.058}$	$-19.317^{+0.039}_{-0.038}$	1047.06	1055.07	1075.01	4.08	9.06
CC + SN + TRGB	$69.7^{+1.3}_{-1.4}$	$0.285^{+0.020}_{-0.021}$	$0.048^{+0.094}_{-0.037}$	-19.366 ± 0.040	1045.04	1053.04	1072.98	5.36	10.34
CC + SN + BAO	$67.35^{+0.94}_{-0.97}$	0.289 ± 0.013	$0.043^{+0.101}_{-0.026}$	$-19.441^{+0.032}_{-0.031}$	1060.55	1068.55	1088.55	5.09	10.09
CC + SN + BAO + R19	$68.70^{+0.84}_{-0.85}$	$0.293^{+0.013}_{-0.012}$	$0.059^{+0.056}_{-0.047}$	$-19.397^{+0.029}_{-0.028}$	1071.71	1079.71	1099.71	5.41	10.41
CC + SN + BAO + HW	$68.52^{+0.85}_{-0.82}$	$0.295^{+0.011}_{-0.014}$	$0.034^{+0.089}_{-0.024}$	$-19.401^{+0.028}_{-0.029}$	1069.03	1077.03	1097.03	4.99	9.10
CC + SN + BAO + TRGB	67.79 ± 0.85	$0.292^{+0.012}_{-0.014}$	$0.074^{+0.057}_{-0.059}$	$-19.425^{+0.027}_{-0.030}$	1061.78	1069.78	1089.78	5.21	10.21

f_1CDM Eur.Phys.J.Plus 137 (2022) 5, 532

$f(T)$ Cosmology Comparison

$f(T)$ models:

1. Power-law: $f_1(T) = \alpha_1 (T)^{b_1}$
2. Linder: $f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$
3. Exponential: $f_3(T) = \alpha_3 T_0 \left(1 - e^{-b_3 T/T_0}\right)$



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Scalar Perturbations

- $\boldsymbol{f(T)}$ **gravity** leaves imprints at the perturbative level

$$e^0{}_\mu = \delta^0_\mu(1+\psi)\,,e^i{}_\mu = \delta^i_\mu a(1-\phi) \;\Rightarrow\; ds^2 = (1+2\psi)dt^2 - a^2(1-2\phi)\delta_{ij}dx^i dx^j$$

- **Matter perturbation evolution** equation $\left(\delta_m = \frac{\delta\rho_m}{\rho_m}\right)$:

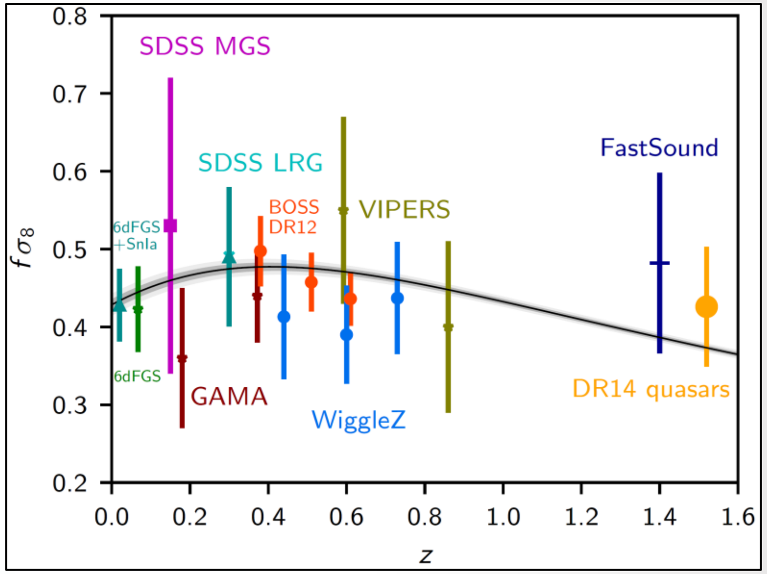
$$\ddot{\delta}_m + 2H\dot{\delta}_m + 4\pi G_{\text{eff}}\rho_m\delta_m = 0$$

where $G_{\text{eff}} = \frac{G_N}{1+f_T}$

- Using the **structure function and mass fluctuation metrics**

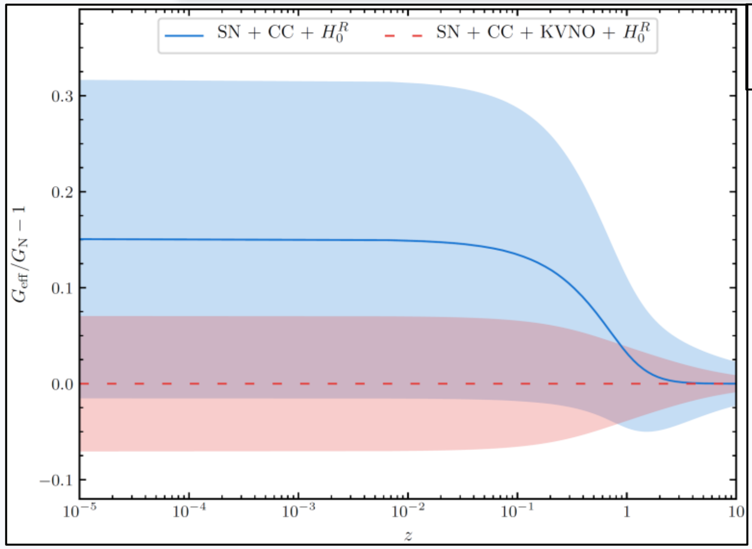
$$f(z) = \frac{d\ln\delta(z)}{d\ln a} \qquad \sigma_8(z) = \sigma_{8,0}\frac{\delta(z)}{\delta_0}$$

We can define the growth rath measure $f\sigma_8(z)$



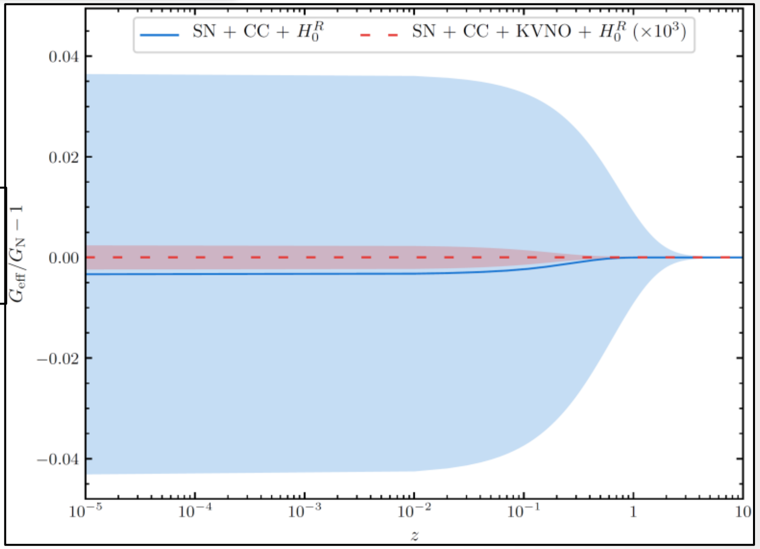
arXiv:2310.09159

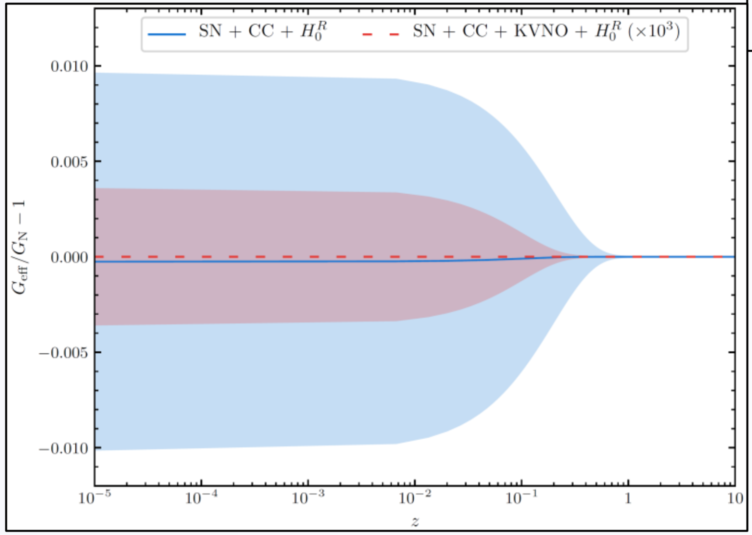
G_{eff} Constraints



$$f_1(T) = \alpha_1 (T)^{b_1}$$

$$f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$$



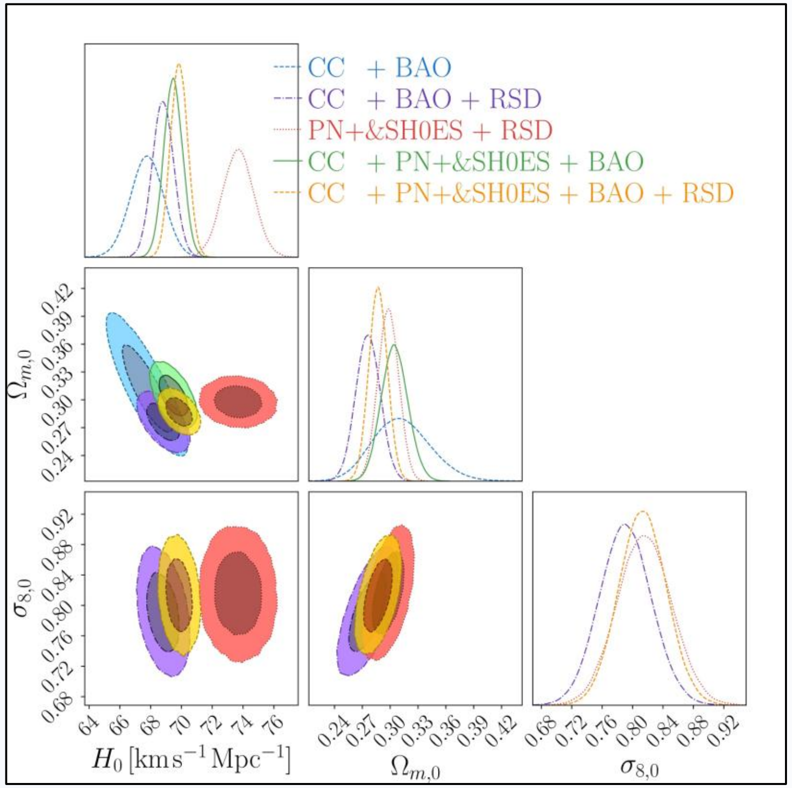


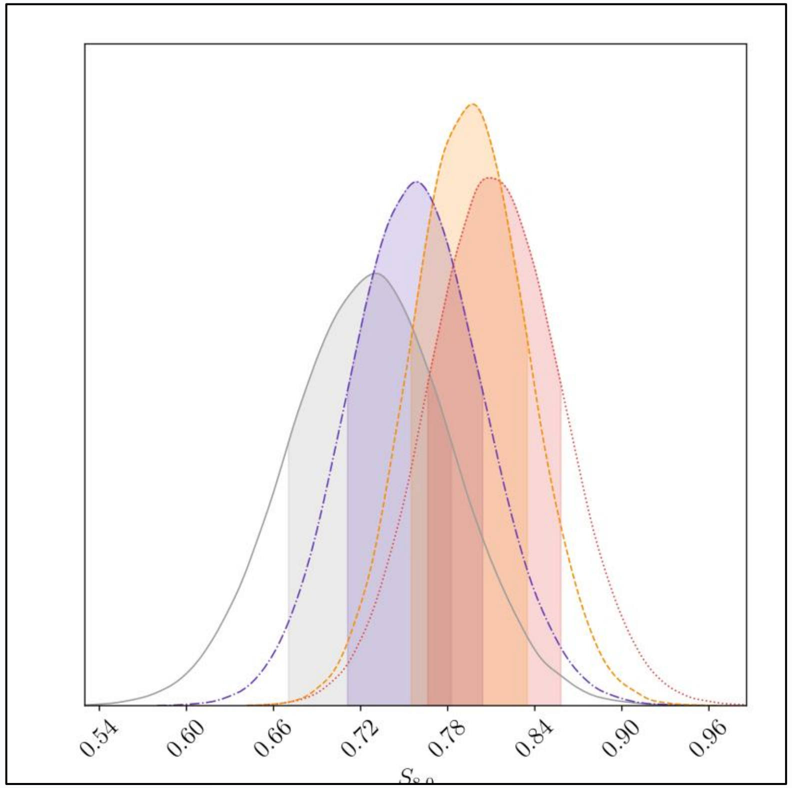
$$f_3(T) = \alpha_3 T_0 \left(1 - e^{-b_3 T/T_0}\right)$$

$\Delta\alpha/\alpha$ data

- Keck Observatory (K) and Very Large Telescope [VLT] (V): Measuring quasar absorption lines
- 21 New Measurements (N): Collected in arXiv:1709.02923
- Oklo nuclear reactor (O)

ΛCDM Growth





Model	$S_{8,0}$
— RSD	$0.729^{+0.053}_{-0.059}$
-·-CC + BAO + RSD	$0.758^{+0.046}_{-0.047}$
····PN^{+} & SH0ES + RSD	$0.809^{+0.050}_{-0.042}$
- - -CC + PN^{+} & SH0ES + BAO + RSD	$0.797^{+0.038}_{-0.042}$

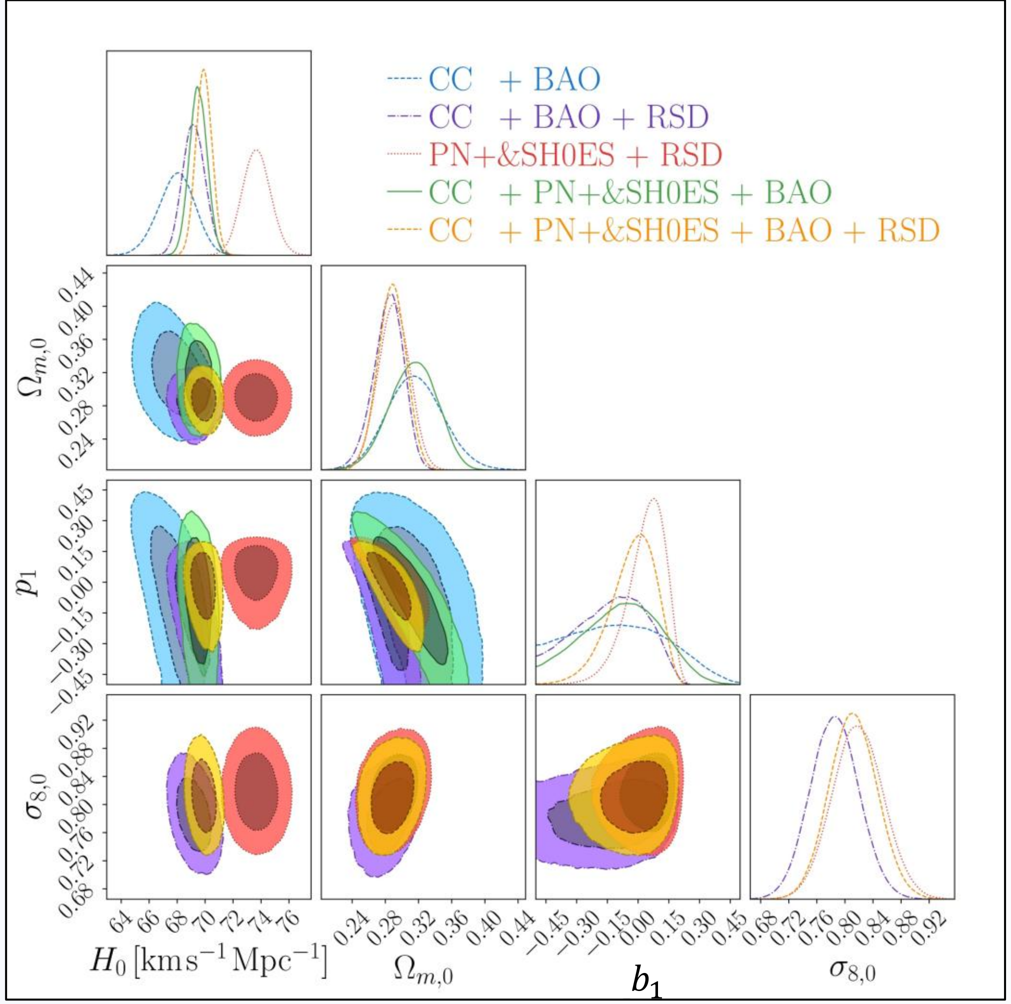
$$S_8 = \sigma_{8,0}\sqrt{\Omega_{m,0}/0.3}$$

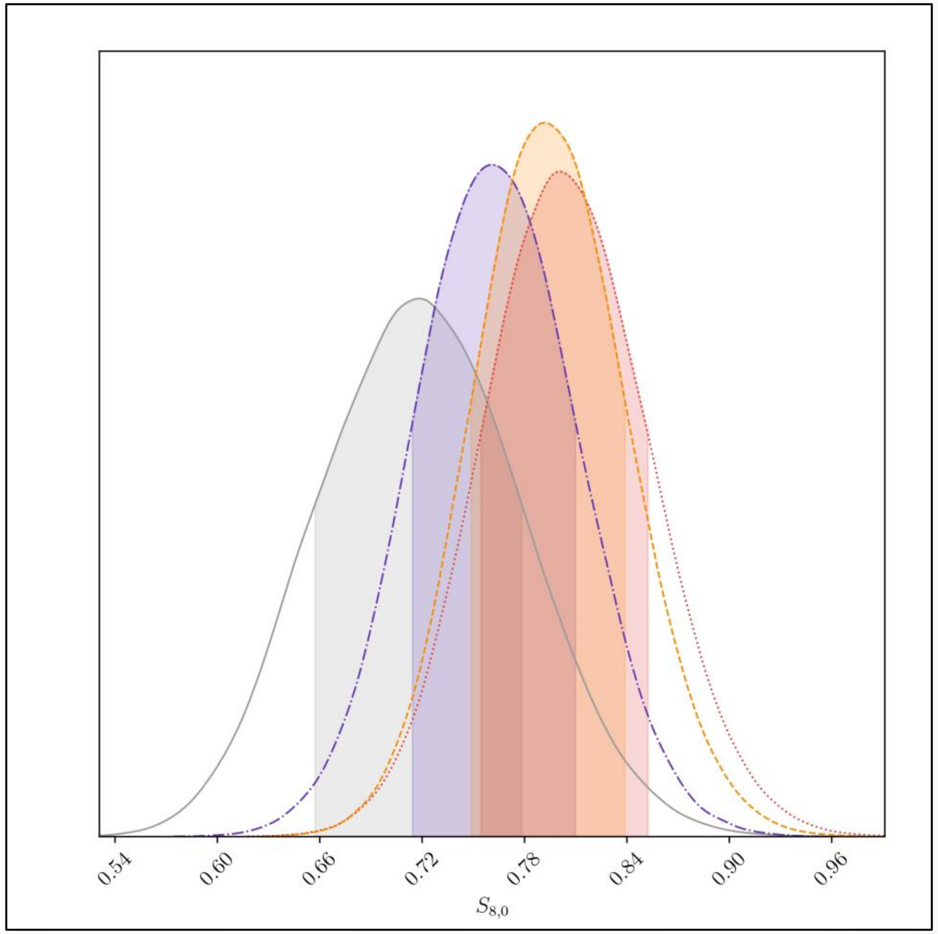
Redshift Space Distortion (RSD) data:
Spatial distortion of distribution of galaxies (eBOSS, WiggleZ, GAMA, VIPERS, 2dFGRS)

arXiv:2310.09159

f_1CDM Growth Constraints

$f_1(T) = \alpha_1(T)^{b_1}$



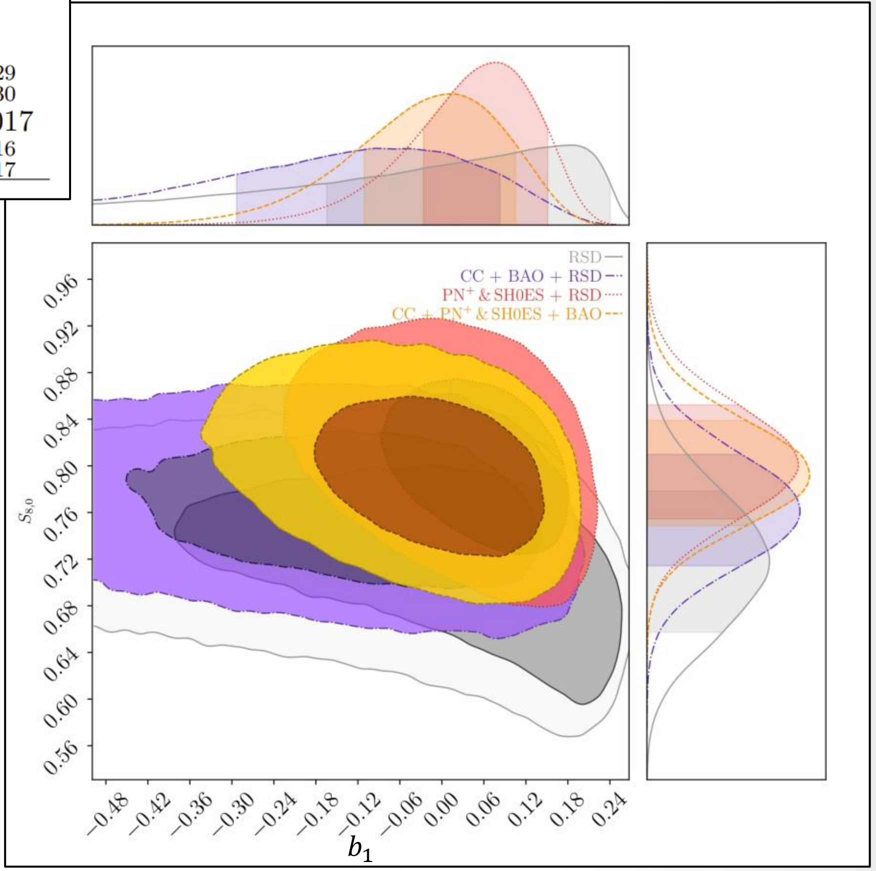


arXiv:2310.09159

f_1CDM Growth Constraints

Data sets	H_0 [km s^{-1} Mpc^{-1}]	$\Omega_{m,0}$	b_1	$\sigma_{8,0}$	M
CC + BAO	$68.1^{+1.2}_{-1.4}$	$0.314^{+0.034}_{-0.033}$	$-0.09^{+0.24}_{-0.30}$	–	–
CC + BAO + RSD	69.17 ± 0.81	$0.287^{+0.016}_{-0.020}$	$-0.09^{+0.17}_{-0.20}$	0.785 ± 0.035	–
PN$^+$ & SH0ES + RSD	73.7 ± 1.0	$0.290^{+0.019}_{-0.018}$	$0.076^{+0.075}_{-0.102}$	$0.817^{+0.037}_{-0.035}$	$-19.252^{+0.029}_{-0.030}$
CC + PN$^+$ & SH0ES + BAO	$69.45^{+0.69}_{-0.58}$	$0.316^{+0.028}_{-0.029}$	$-0.06^{+0.19}_{-0.22}$	–	-19.375 ± 0.017
CC + PN$^+$ & SH0ES + BAO + RSD	69.90 ± 0.58	$0.289^{+0.016}_{-0.018}$	$0.014^{+0.091}_{-0.125}$	$0.810^{+0.036}_{-0.033}$	$-19.367^{+0.016}_{-0.017}$

Data sets	$S_{8,0}$
— RSD	$0.718^{+0.061}_{-0.060}$
-·-·-CC + BAO + RSD	$0.761^{+0.049}_{-0.046}$
·····PN$^+$ & SH0ES + RSD	$0.801^{+0.052}_{-0.046}$
- - -CC + PN$^+$ & SH0ES + BAO + RSD	$0.792^{+0.047}_{-0.043}$

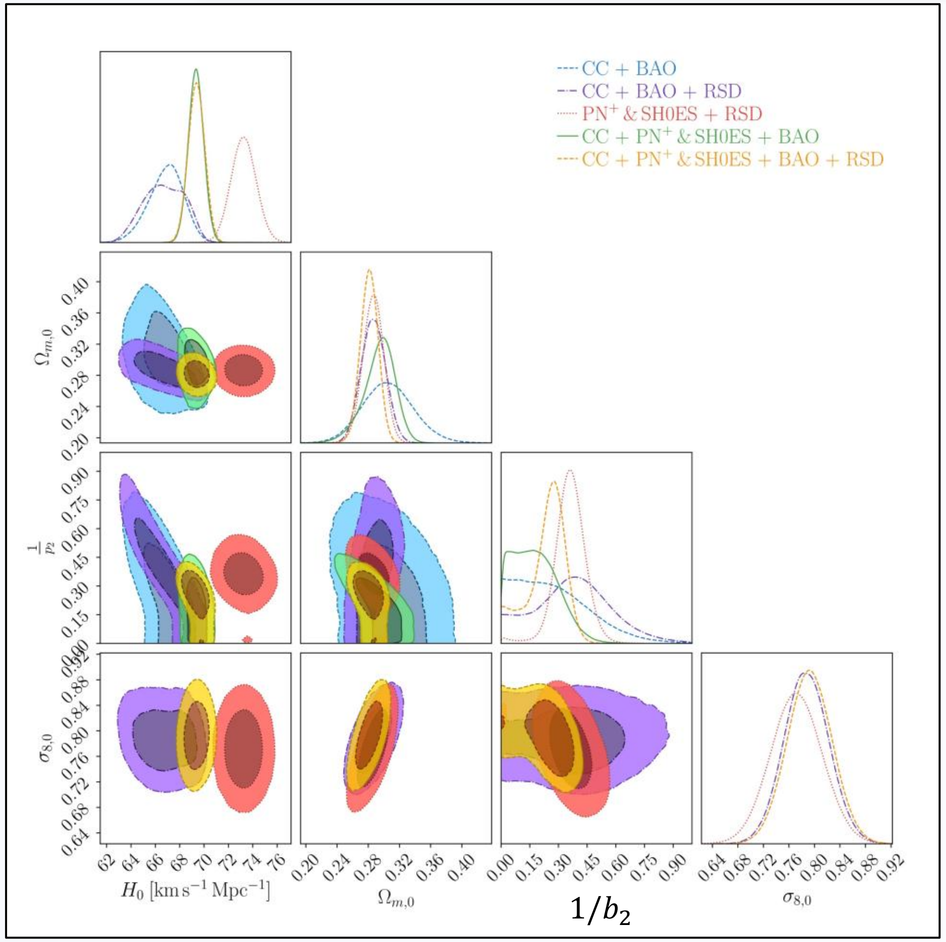


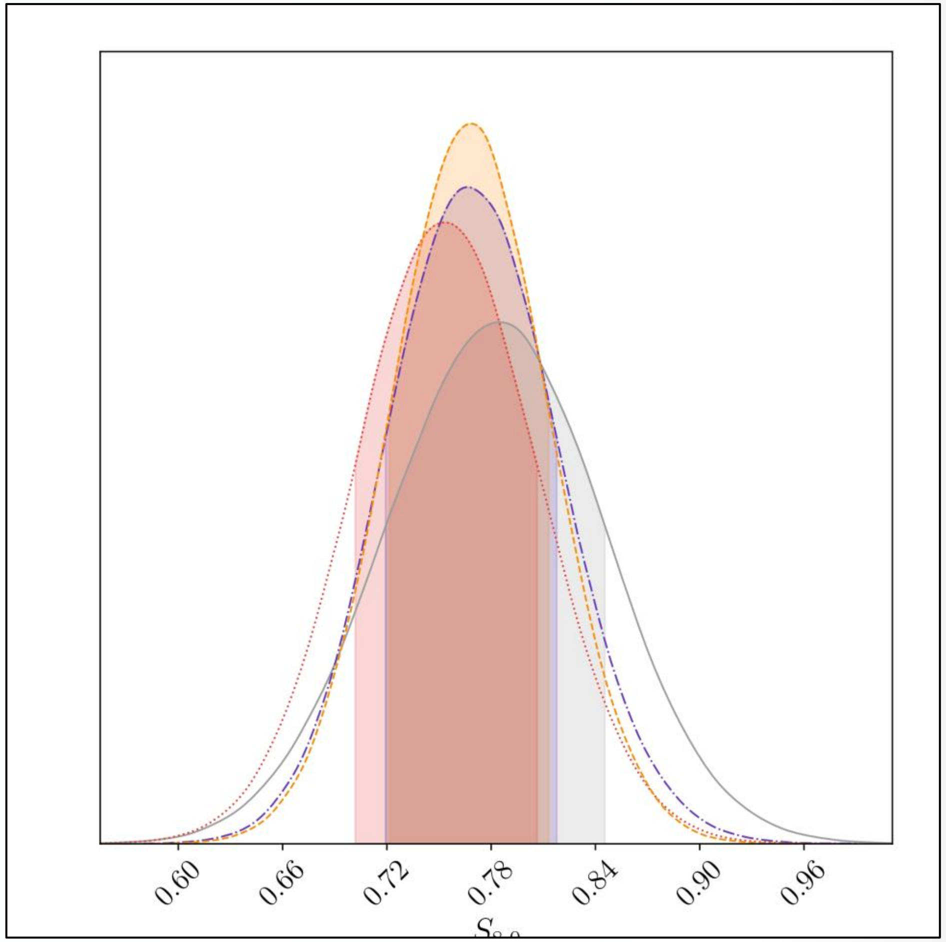
$$f_3(T) = \alpha_3 T_0 \left(1 - e^{-b_3 T/T_0}\right)$$

arXiv:2310.09159

f_2CDM Growth Posteriors

$$f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$$

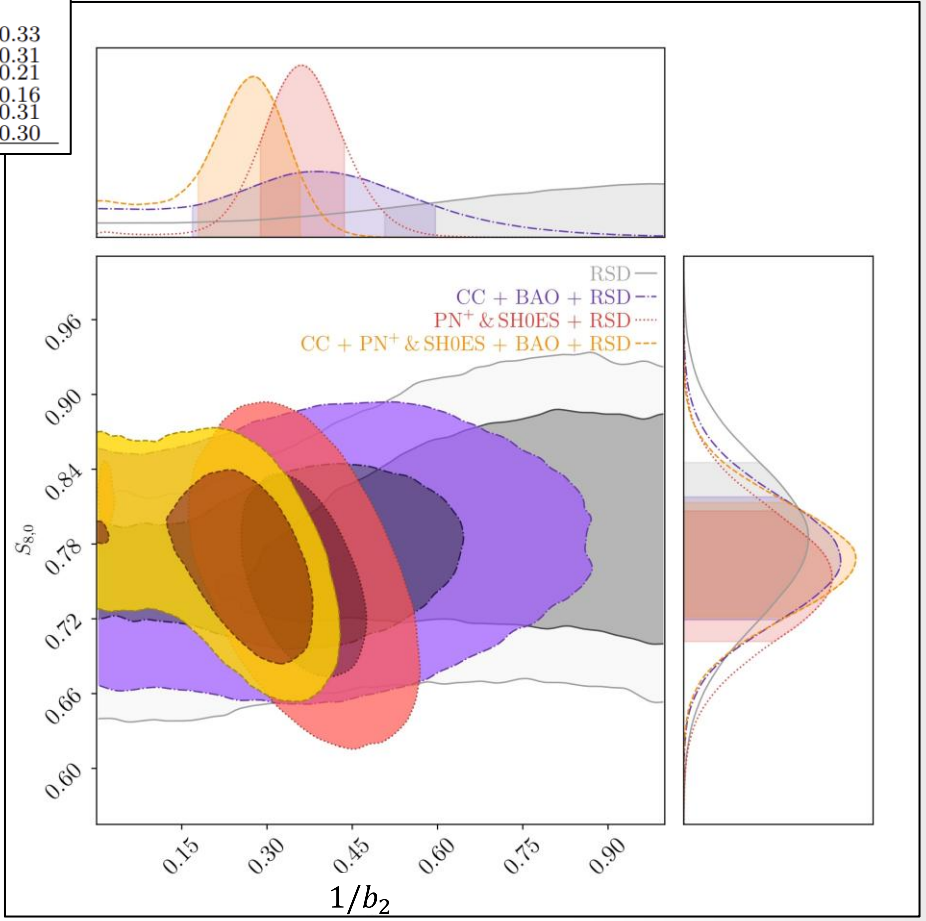




arXiv:2310.09159

f_2CDM Growth Constraints

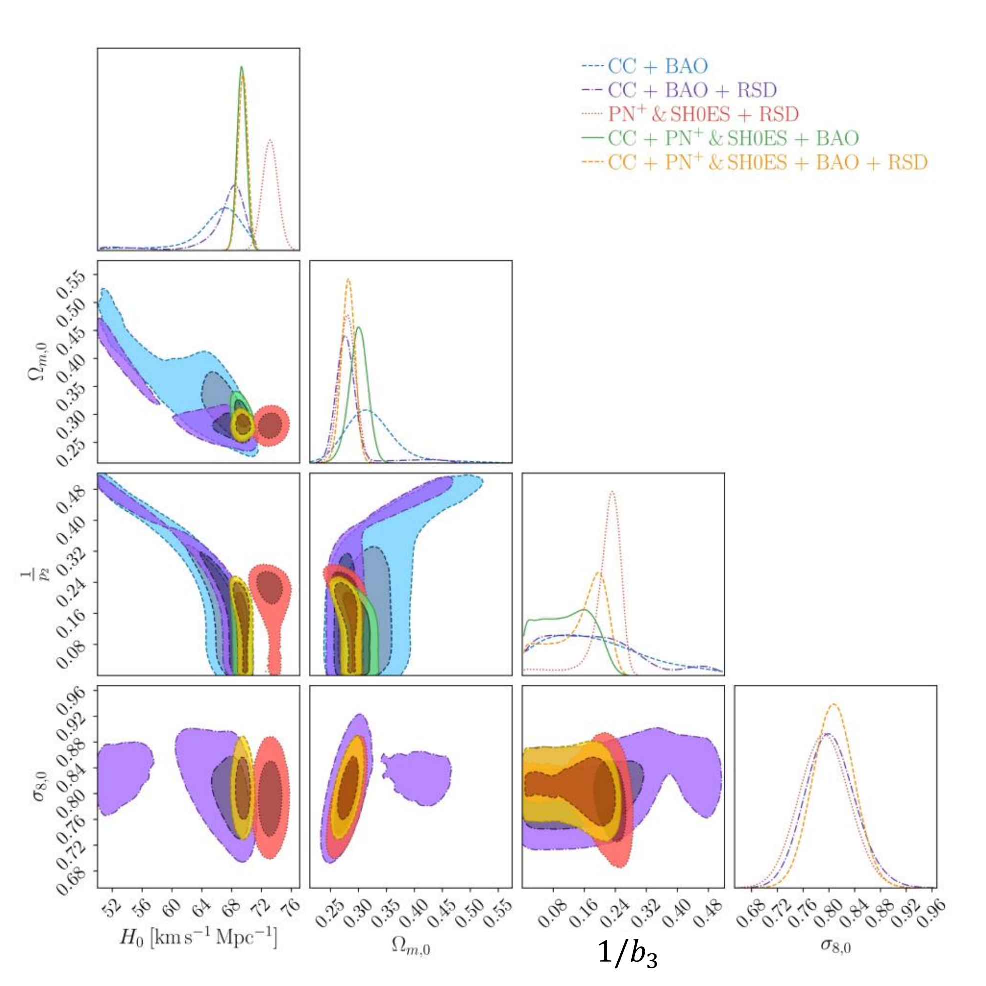
Data Sets	H_0 [km s^{-1} Mpc^{-1}]	$\Omega_{m,0}$	$1/b_2$	$\sigma_{8,0}$	M
CC + BAO	$67.2^{+1.2}_{-1.6}$	$0.302^{+0.035}_{-0.030}$	$0.00^{+0.37}_{-0.00}$	–	–
CC + BAO + RSD	$66.5^{+2.2}_{-1.3}$	$0.286^{+0.016}_{-0.015}$	$0.39^{+0.21}_{-0.22}$	$0.784^{+0.038}_{-0.032}$	–
PN$^+$ & SH0ES + RSD	73.2 ± 1.0	0.287 ± 0.013	$0.359^{+0.077}_{-0.071}$	$0.770^{+0.042}_{-0.039}$	$-19.26^{+0.33}_{-0.31}$
CC + PN$^+$ & SH0ES + BAO	$69.35^{+0.61}_{-0.63}$	$0.299^{+0.017}_{-0.021}$	$0.167^{+0.080}_{-0.154}$	–	$-19.40^{+0.21}_{-0.16}$
CC + PN$^+$ & SH0ES + BAO + RSD	$69.38^{+0.67}_{-0.68}$	0.282 ± 0.011	$0.275^{+0.083}_{-0.096}$	0.793 ± 0.035	$-19.37^{+0.31}_{-0.30}$



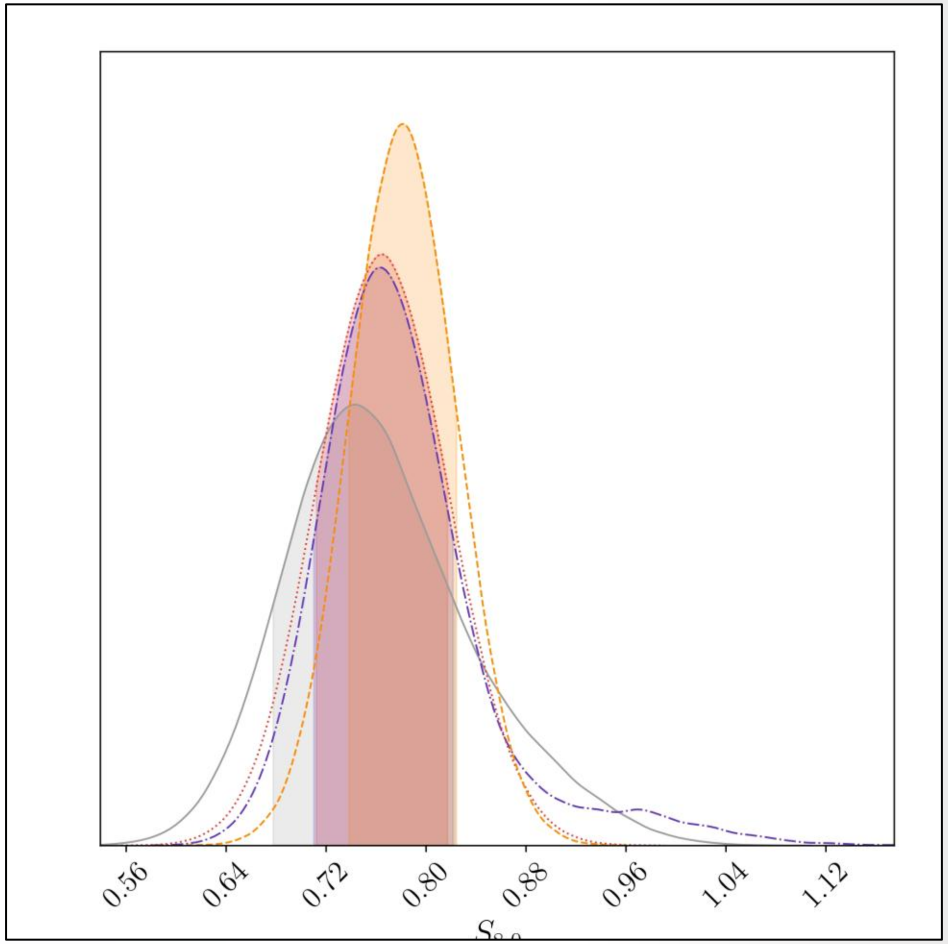
Data sets	$S_{8,0}$
— RSD	$0.784^{+0.061}_{-0.065}$
-·-·-CC + BAO + RSD	$0.765^{+0.052}_{-0.046}$
·····PN$^+$ & SH0ES + RSD	$0.753^{+0.054}_{-0.051}$
- - -CC + PN$^+$ & SH0ES + BAO + RSD	$0.768^{+0.045}_{-0.046}$

arXiv:2310.09159

f_3CDM Growth Constraints



$$f_3(T) = \alpha_3 T_0\left(1 - e^{-b_3 T/T_0}\right)$$

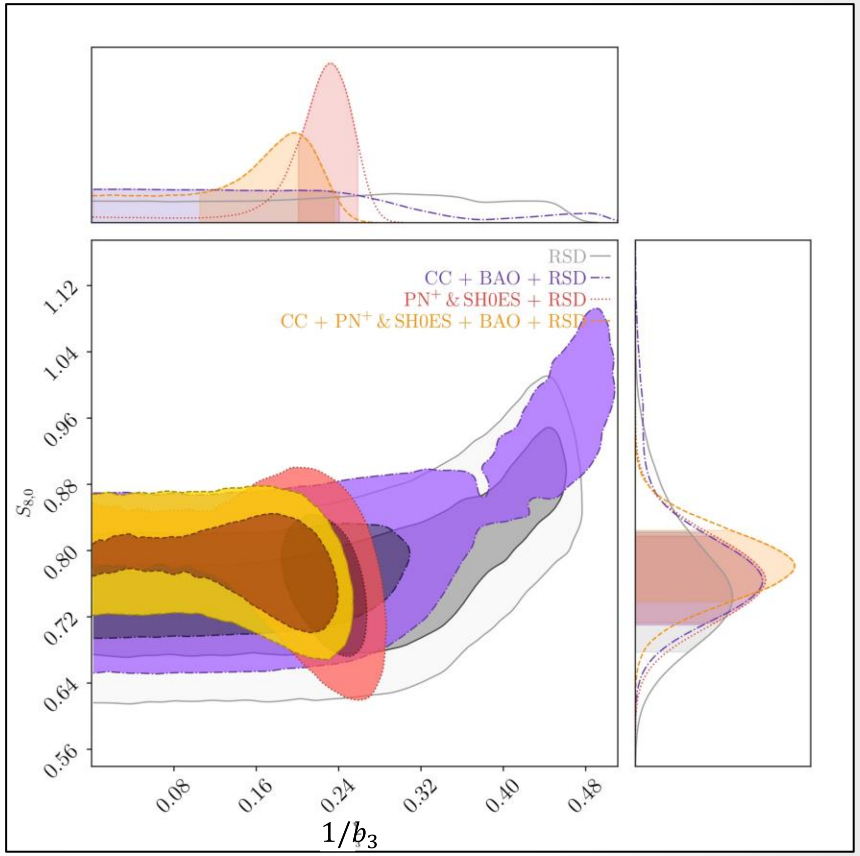


arXiv:2310.09159

f_3CDM Growth Constraints

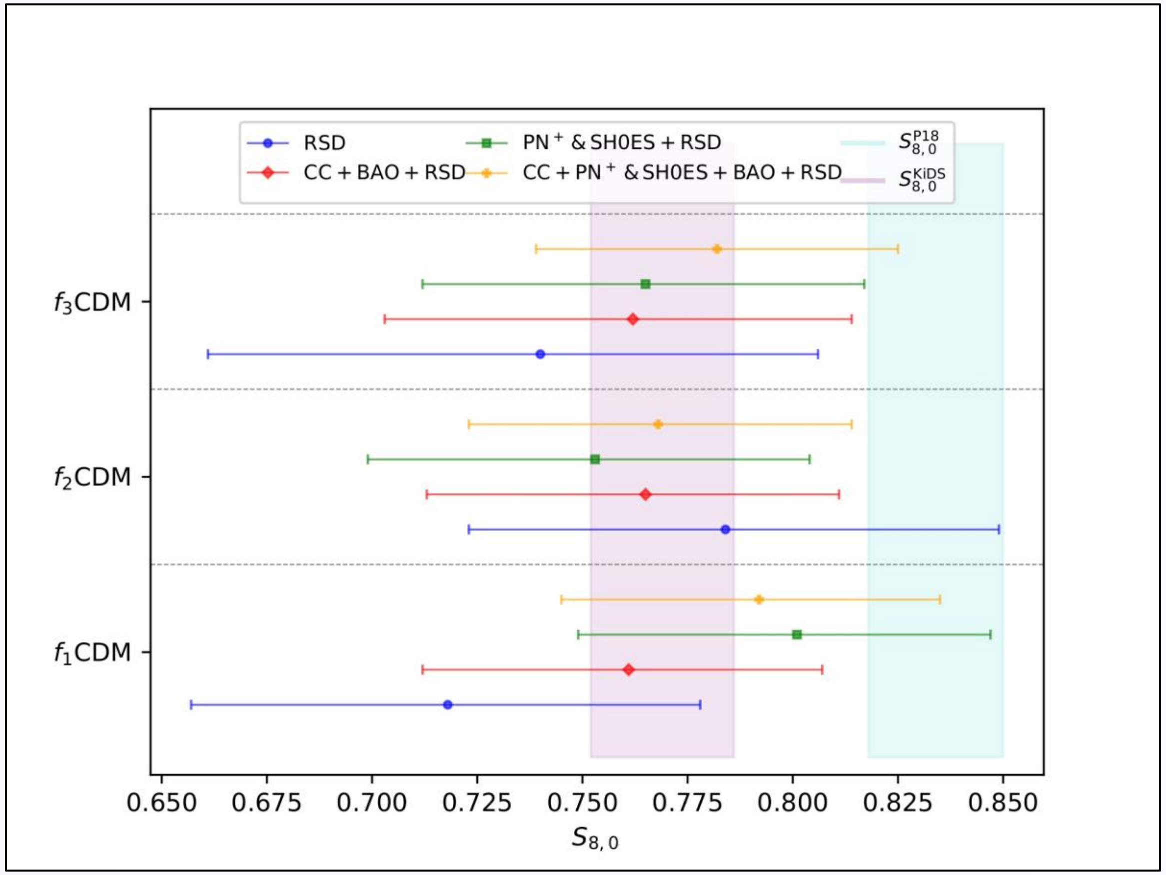
Data Sets	H_0 [km s^{-1} Mpc^{-1}]	$\Omega_{m,0}$	$1/b_3$	$\sigma_{8,0}$	M
CC + BAO	$67.5^{+1.7}_{-2.3}$	$0.311^{+0.039}_{-0.034}$	$0.058^{+0.182}_{-0.056}$	–	–
CC + BAO + RSD	$68.6^{+1.3}_{-1.9}$	$0.276^{+0.016}_{-0.015}$	$0.026^{+0.214}_{-0.025}$	$0.798^{+0.040}_{-0.036}$	–
PN$^+$ & SH0ES + RSD	$73.2^{+1.0}_{-1.1}$	$0.280^{+0.014}_{-0.015}$	$0.232^{+0.027}_{-0.031}$	$0.793^{+0.038}_{-0.039}$	-19.25 ± 0.11
CC + PN$^+$ & SH0ES + BAO	$69.34^{+0.65}_{-0.64}$	$0.300^{+0.017}_{-0.016}$	$0.160^{+0.029}_{-0.126}$	–	$-19.34^{+0.24}_{-0.31}$
CC + PN$^+$ & SH0ES + BAO + RSD	$69.54^{+0.64}_{-0.66}$	0.282 ± 0.012	$0.197^{+0.038}_{-0.092}$	0.807 ± 0.032	$-19.38^{+0.20}_{-0.19}$

Data Sets	$S_{8,0}$
— RSD	$0.744^{+0.079}_{-0.066}$
-·-·-CC + BAO + RSD	$0.762^{+0.059}_{-0.052}$
·····PN$^+$ & SH0ES + RSD	$0.765^{+0.053}_{-0.052}$
- - -CC + PN$^+$ & SH0ES + BAO + RSD	0.782 ± 0.043



arXiv:2310.09159

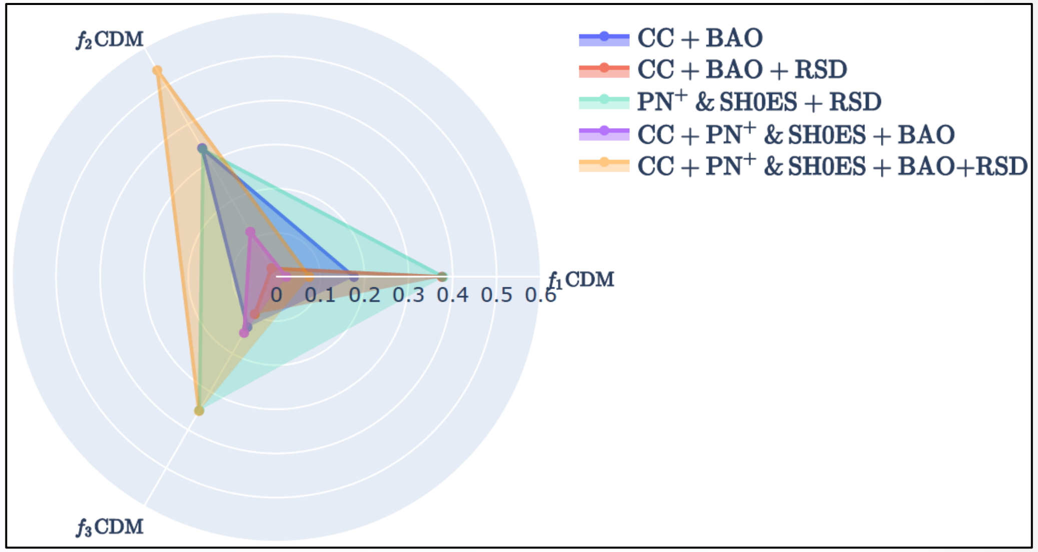
f_3CDM Growth Constraints



arXiv:2310.09159

$f(T)$ models:

1. Power-law: $f_1(T) = \alpha_1(T)^{b_1}$
2. Linder: $f_2(T) = \alpha_2 T_0 \left(1 - e^{-b_2\sqrt{T/T_0}}\right)$
3. Exponential: $f_3(T) = \alpha_3 T_0 \left(1 - e^{-b_3 T/T_0}\right)$



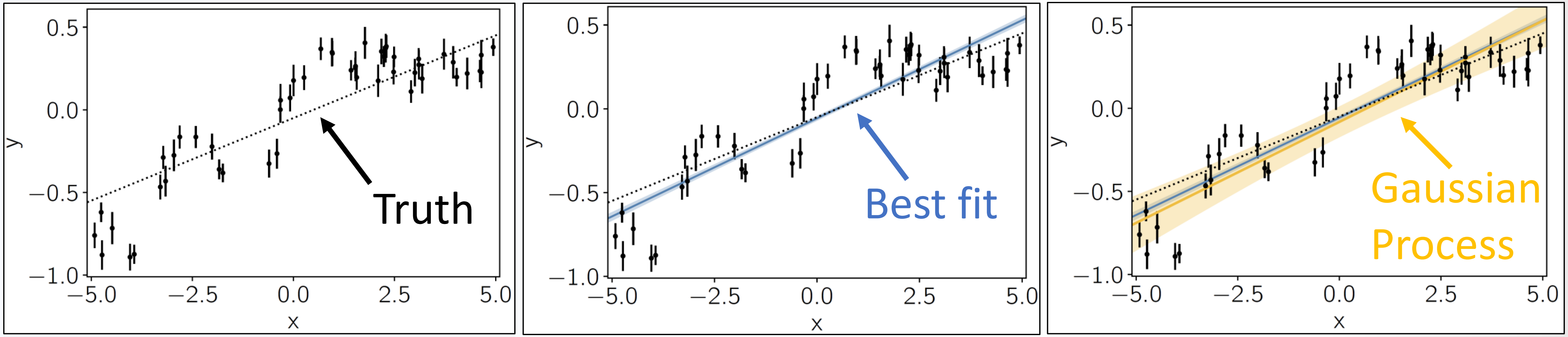
Is there a model independent way of performing background analyses?

What about Gaussian Processes?

Definition: A GP is a stochastic (random) process where any finite subset is a **multivariant Gaussian distribution** with mean $\mu(x)$ and covariance $k(x, x')$

Setting the each $\mu(x)$ to zero, the **covariance function** can be used to **learn the behavior** that produced the data points

$$y(x_i) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{Exp}\left[-\frac{1}{2}\left(\frac{x_i - \mu}{\sigma}\right)^2\right]$$



Gaussian Processes Regression

- The covariance function contains **non-physical hyperparameters** θ which define the distribution $k(\theta, x, x')$
- Iterating over these values using **Bayesian inference** (or others) can produce better hyperparameters
- The result is a (physics) **model independent reconstruction** of the behavior of some parameter
- This is superior to regular fitting because it is nonparametric and so **assumes no physical model** whatsoever

The Covariance Functions

- **Squared Exponential (Gaussian)**

$$k(x,x') = \sigma_f^2 \,\mathrm{Exp}\left[-\frac{1}{2}\left(\frac{x-x'}{l_f}\right)^2\right]$$

- **Cauchy**

$$k(x,x') = \frac{\sigma_f^2 l_f}{(x-x')^2 + l_f^2}$$

- **Matérn**

$$k(x,x') = \sigma_f^2 \left(1 + \frac{\sqrt{3}|x-x'|}{l_f}\right) \mathrm{Exp}\left[-\frac{\sqrt{3}|x-x'|}{l_f}\right]$$

- **Rational quadratic**

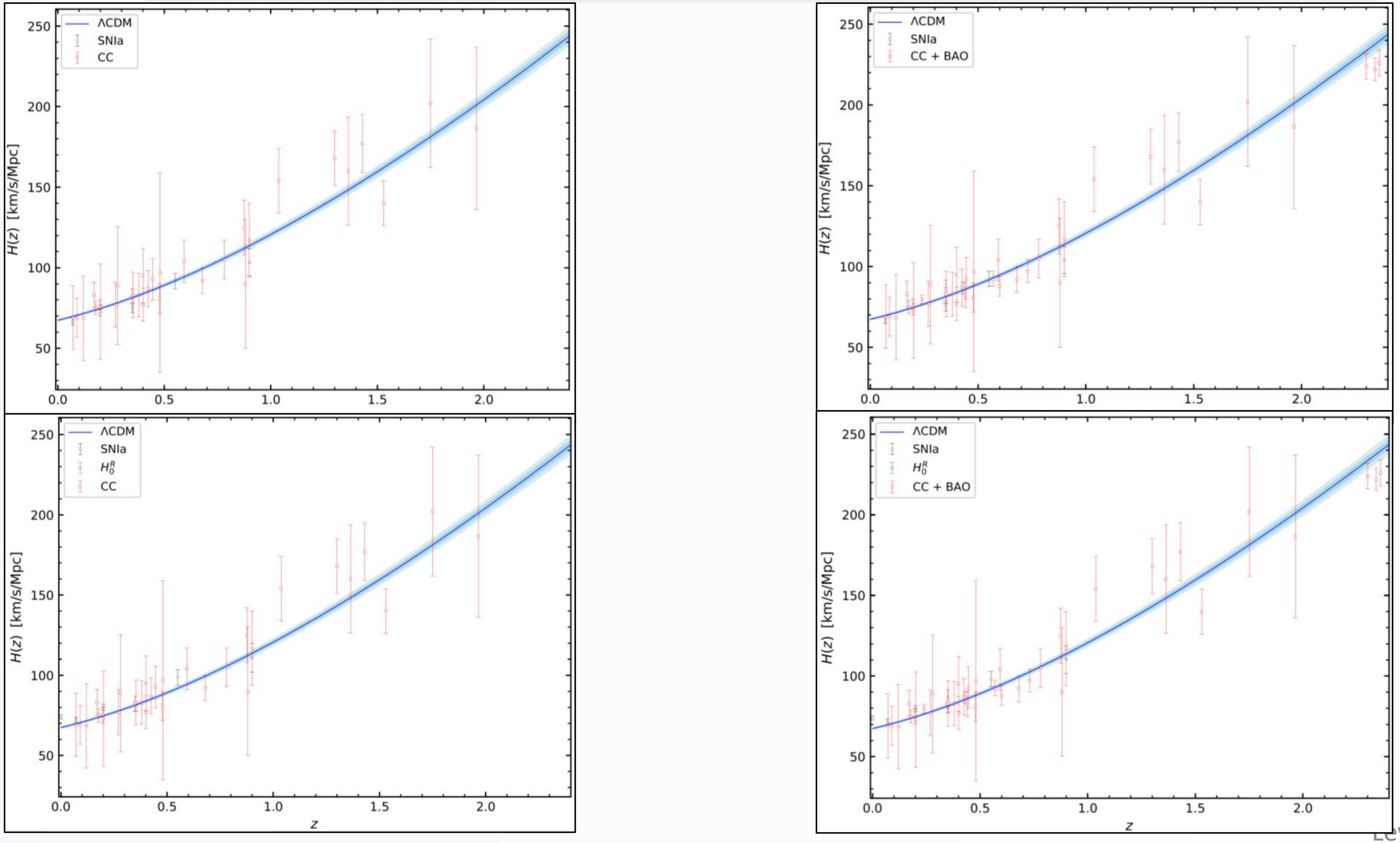
$$k(x,x') = \sigma_f^2 \left[1 + \frac{(x-x')^2}{2\alpha l_f^2}\right]^{-\alpha}$$

H_0 Priors

- Planck Collaboration (18): **ΛCDM Model dependent** → $H_0 = 67.4 \pm 0.5\ \mathrm{km\ s^{-1} Mpc^{-1}}$
- SH0ES Survey [H_0^R]: Riess et al. (2019) calibrated with **Cepheid variables** → $H_0 = 74.03 \pm 1.42\ \mathrm{km\ s^{-1} Mpc^{-1}}$
- Tip of the Red Giant Branch [TRGB]: Freedman et al. (2019) reports $H_0 = 69.8 \pm 1.9\ \mathrm{km\ s^{-1} Mpc^{-1}}$
- H0licow [HW]: Based on strong lensing → $H_0 = 73.3 \pm 1.8\ \mathrm{km\ s^{-1} Mpc^{-1}}$

Distance (in σ units) between the H_0 arguments: $d\left(H_{0,i}, H_{0,j}\right) = \frac{H_{0,i} - H_{0,j}}{\sqrt{\sigma_i^2 + \sigma_j^2}}$

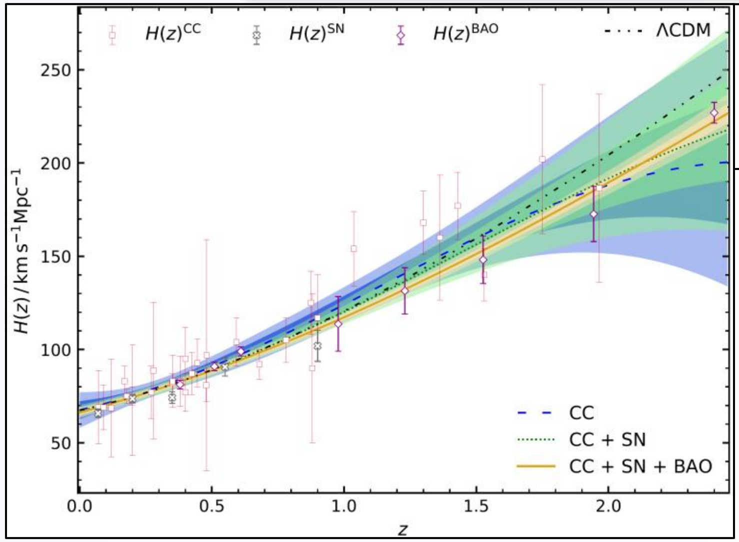
Hubble Data ($H(z)$)



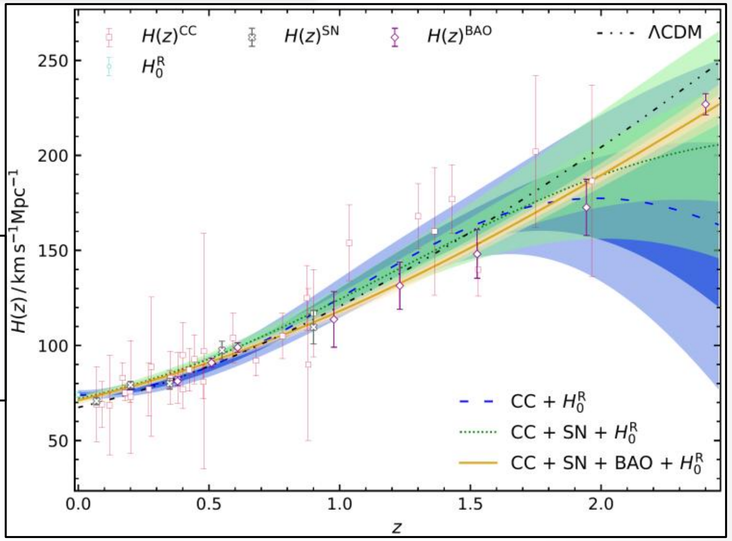
ΛCDM using Planck parameters

Square Exponential H_0 GP

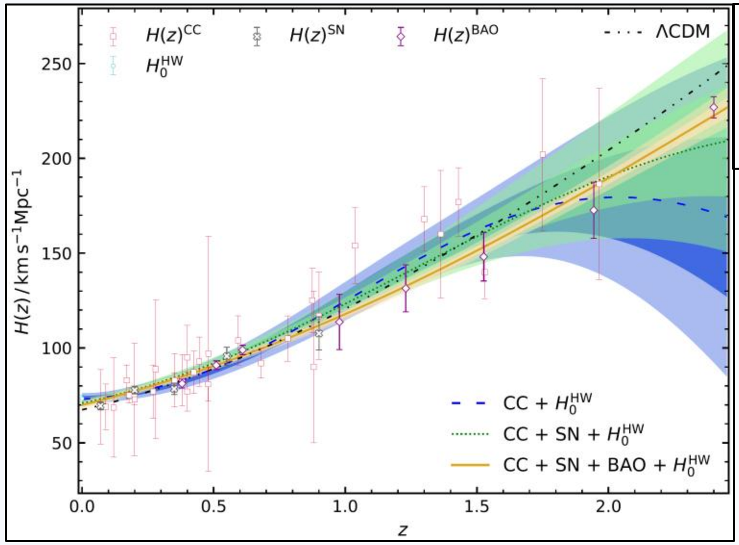
GaPP code from Seikel et al. (2012)



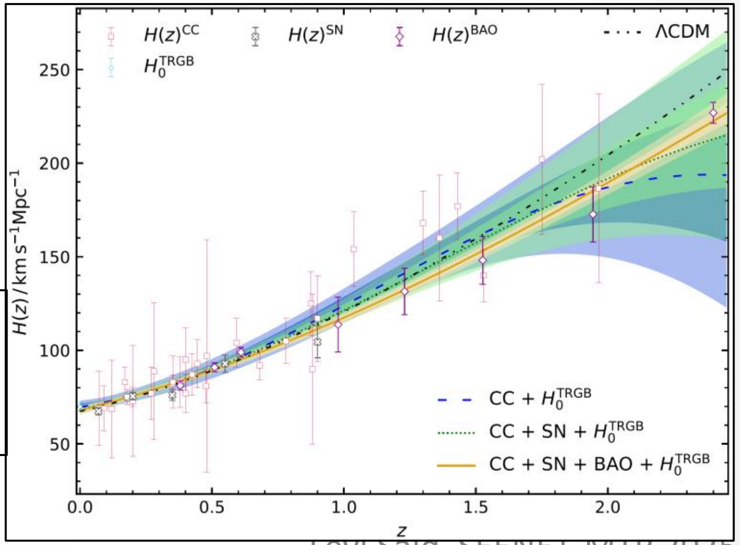
$H_0 = 67.539 \pm 4.772 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 66.001 \pm 1.653 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 66.197 \pm 1.464 \mathrm{km\,s^{-1}Mpc^{-1}}$



$H_0 = 73.782 \pm 1.374 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 72.022 \pm 1.076 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 71.180 \pm 1.025 \mathrm{km\,s^{-1}Mpc^{-1}}$



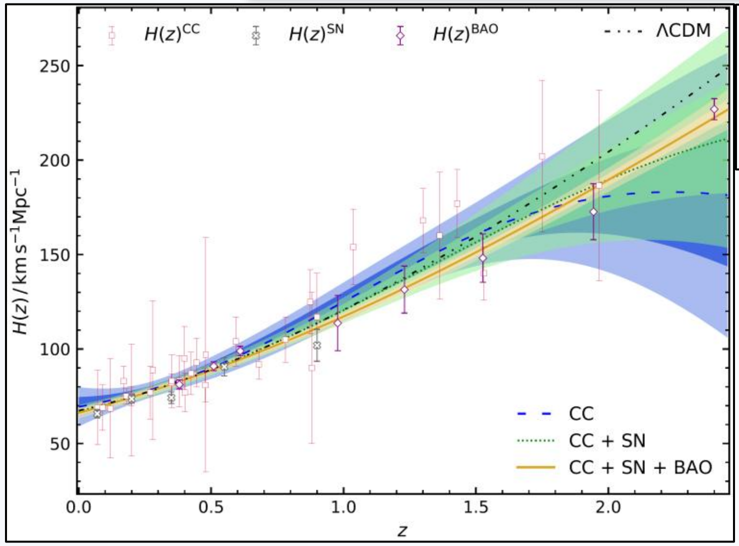
$H_0 = 69.604 \pm 1.756 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 68.468 \pm 1.221 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 67.811 \pm 1.147 \mathrm{km\,s^{-1}Mpc^{-1}}$



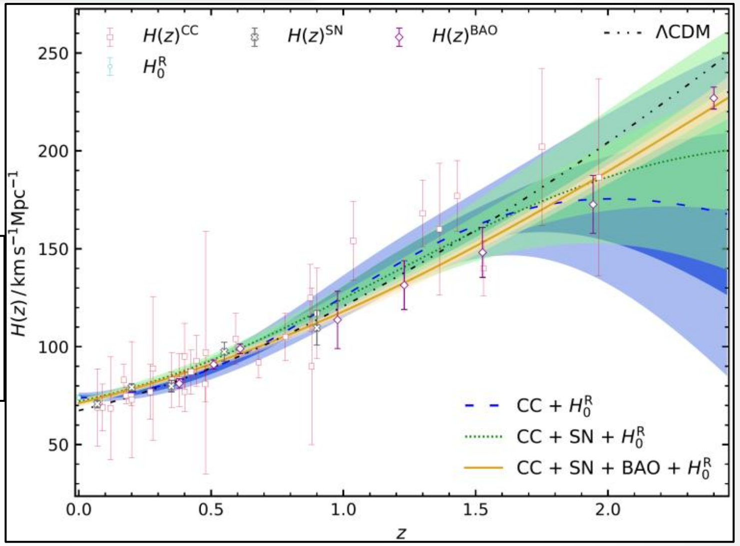
$H_0 = 72.966 \pm 1.664 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 70.850 \pm 1.199 \mathrm{km\,s^{-1}Mpc^{-1}}$
$H_0 = 69.911 \pm 1.128 \mathrm{km\,s^{-1}Mpc^{-1}}$

arXiv:2009.14582

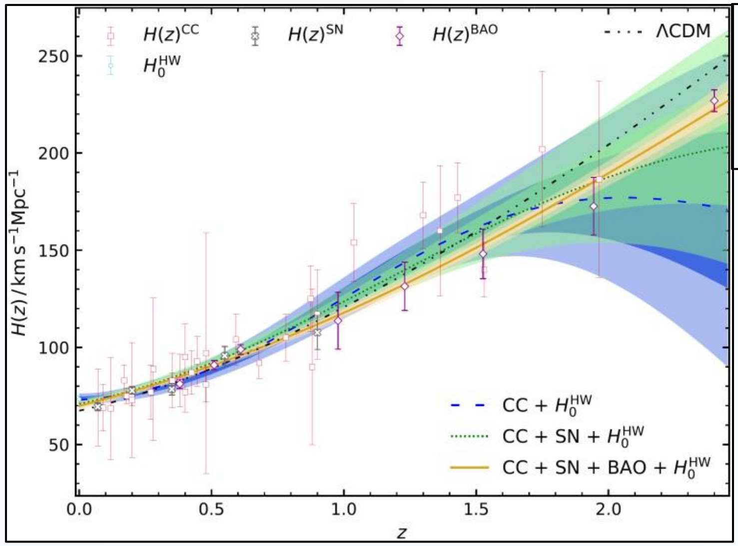
Cauchy H_0 GP



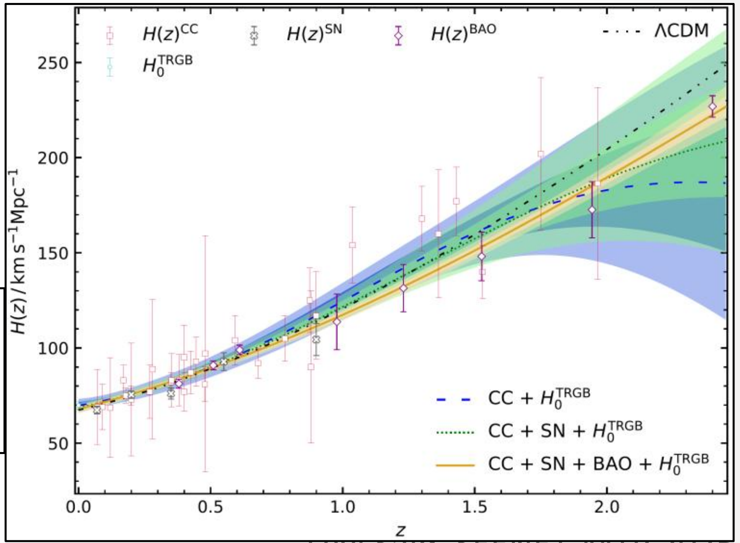
$H_0 = 69.396 \pm 5.186 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 67.082 \pm 1.682 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 66.179 \pm 1.472 \text{km s}^{-1}\text{Mpc}^{-1}$



$H_0 = 73.802 \pm 1.376 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 72.056 \pm 1.083 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 71.166 \pm 1.028 \text{km s}^{-1}\text{Mpc}^{-1}$



$H_0 = 69.695 \pm 1.760 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 68.508 \pm 1.233 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 67.796 \pm 1.151 \text{km s}^{-1}\text{Mpc}^{-1}$



$H_0 = 73.003 \pm 1.667 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 70.892 \pm 1.209 \text{km s}^{-1}\text{Mpc}^{-1}$
$H_0 = 69.895 \pm 1.132 \text{km s}^{-1}\text{Mpc}^{-1}$

Square Exponential Covariance for H_0

Data set(s)	H_0	$d(H_0, H_0^{\mathrm{R}})$	$d(H_0, H_0^{\mathrm{TRGB}})$	$d(H_0, H_0^{\mathrm{HW}})$	$d(H_0, H_0^{\mathrm{Planck18}})$
CC	67.539 ± 4.772	-1.304	-0.441	-1.133	0.029
CC+SN	67.001 ± 1.653	-3.225	-1.118	-2.617	-0.231
CC+SN+BAO	$\mathbf{66.197 \pm 1.464}$	**-3.841**	**-1.513**	**-3.113**	**-0.778**
CC+H_0^{R}	$\mathbf{73.782 \pm 1.374}$	**-0.126**	**1.711**	**0.217**	**4.364**
CC+SN+H_0^{R}	72.022 ± 1.076	-1.127	1.027	-0.622	3.897
CC+SN+BAO+H_0^{R}	71.180 ± 1.025	-1.628	0.645	-1.046	3.316
CC+H_0^{TRGB}	69.604 ± 1.756	-1.960	-0.076	-1.491	1.208
CC+SN+H_0^{TRGB}	68.468 ± 1.221	-2.970	-0.594	-2.264	0.810
CC+SN+BAO+H_0^{TRGB}	67.811 ± 1.147	-3.407	-0.903	-2.623	0.328
CC+H_0^{HW}	72.966 ± 1.664	-0.486	1.262	-0.138	3.204
CC+SN+H_0^{HW}	70.850 ± 1.199	-1.711	0.471	-1.155	2.656
CC+SN+BAO+H_0^{HW}	69.911 ± 1.128	-2.272	0.051	-1.628	2.036

Cauchy Covariance for H_0

Data set(s)	H_0	$d\left(H_0, H_0^{\mathrm{R}}\right)$	$d\left(H_0, H_0^{\mathrm{TRGB}}\right)$	$d\left(H_0, H_0^{\mathrm{HW}}\right)$	$d\left(H_0, H_0^{\mathrm{Planck18}}\right)$
CC	69.396 ± 5.186	-0.862	-0.073	-0.713	0.383
CC+SN	67.082 ± 1.682	-3.157	-1.078	-2.562	-0.181
CC+SN+BAO	$\mathbf{66.179 \pm 1.472}$	**-3.839**	**-1.512**	**-3.114**	**-0.786**
CC+H_0^{R}	$\mathbf{73.802 \pm 1.376}$	**-0.115**	**1.719**	**0.226**	**4.374**
CC+SN+H_0^{R}	72.056 ± 1.083	-1.106	1.040	-0.605	3.905
CC+SN+BAO+H_0^{R}	71.166 ± 1.028	-1.634	0.638	-1.052	3.294
CC+H_0^{TRGB}	69.695 ± 1.760	-1.917	-0.041	-1.452	1.254
CC+SN+H_0^{TRGB}	68.508 ± 1.233	-2.937	-0.575	-2.239	0.833
CC+SN+BAO+H_0^{TRGB}	67.796 ± 1.151	-3.410	-0.909	-2.628	0.316
CC+H_0^{HW}	73.003 ± 1.667	-0.469	1.276	-0.122	3.221
CC+SN+H_0^{HW}	70.892 ± 1.209	-1.683	0.489	-1.132	2.670
CC+SN+BAO+H_0^{HW}	69.895 ± 1.132	-2.277	0.043	-1.634	2.016

Can we test $\Lambda\mathrm{CDM}$ with the GP reconstructions?

- The $\Lambda\mathrm{CDM}$ **Friedmann equation** for a **dark fluid** $w(z)$ is

$$E^2(z) := \left(\frac{H(z)}{H_0}\right)^2 = \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}\mathrm{Exp}\left[3\int_0^z \frac{1+w(z')}{1+z'}\mathrm{d}z'\right]$$

- Solving for $\Omega_{m,0}$ ($w(z) = -1$) we can define

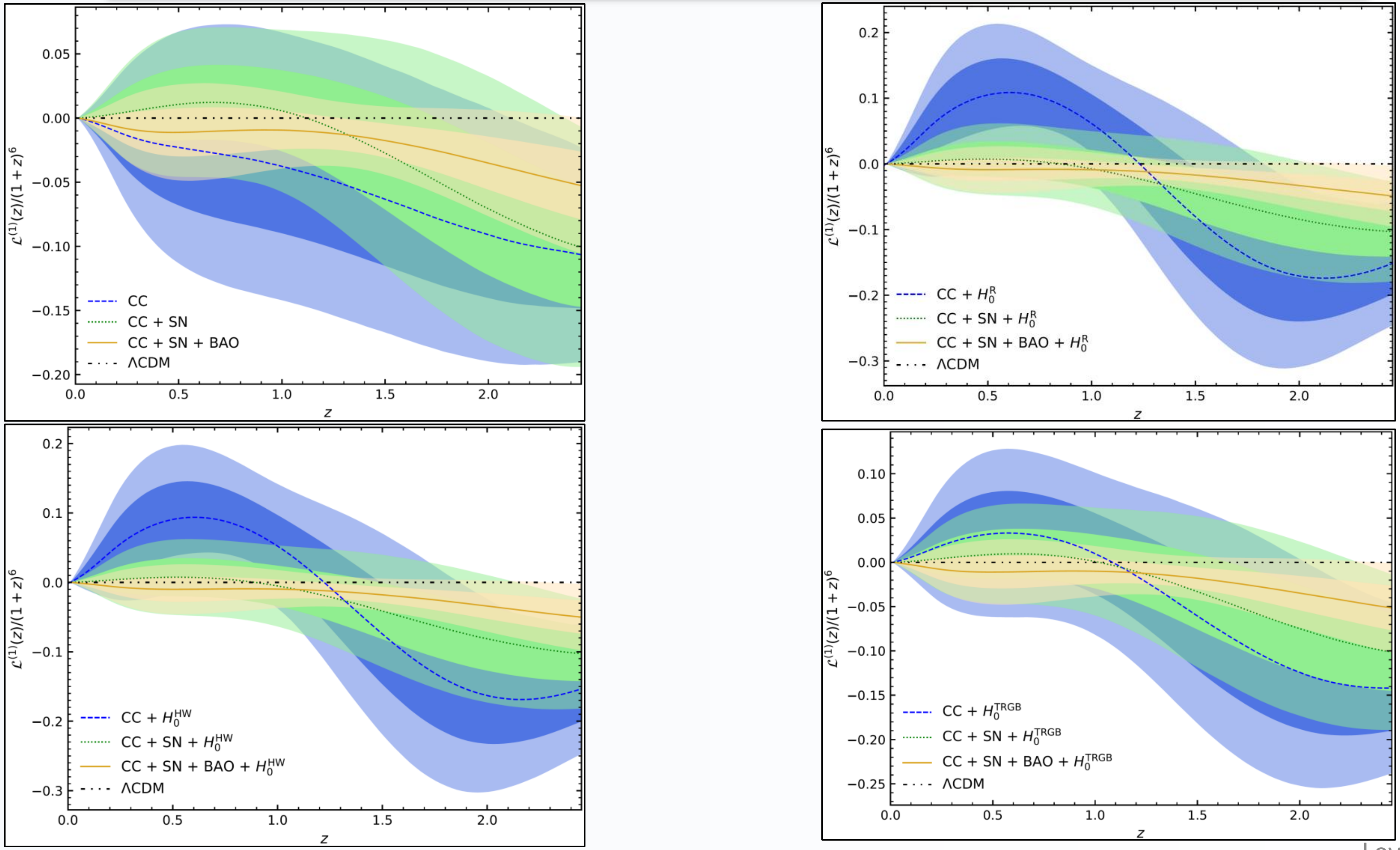
$$\mathcal{O}_m^{(1)}(z) := \frac{E^2(z)-1}{z(3+3z+z^2)}$$

- Therefore we can define the **diagnostic test**

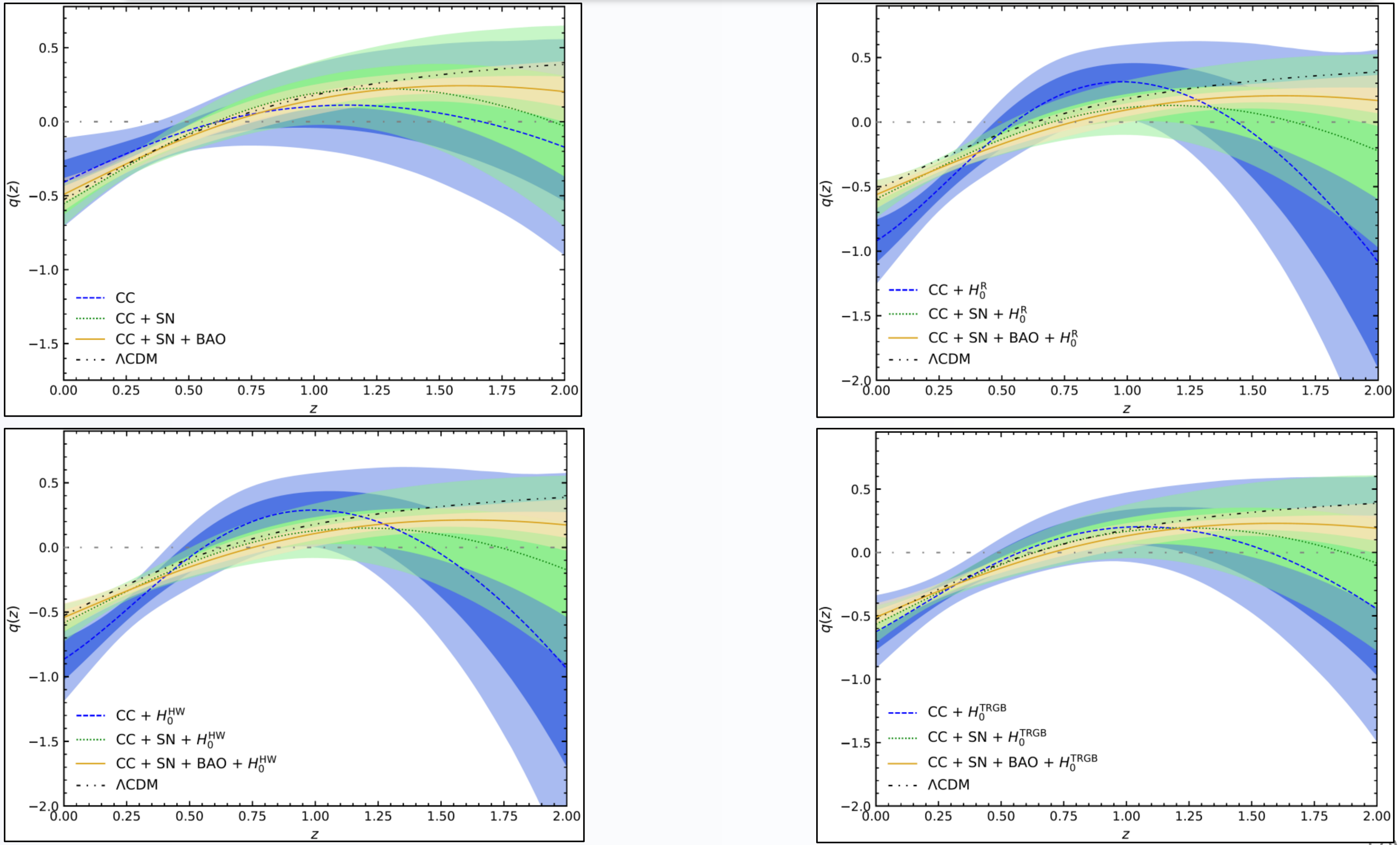
$$\mathcal{L}^{(1)}(z) := \frac{\mathrm{d}\mathcal{O}_m^{(1)}}{\mathrm{d}z} = 3\big(1-E^2(z)\big)(1+z)^2 + 2z\big(3+3z+z^2\big)E(z)E'(z)$$

- **For $\boldsymbol{\Lambda\mathrm{CDM}}$**: $\mathcal{O}_m^{(1)}(z) = \Omega_{m,0}$ and $\mathcal{L}^{(1)}(z) = 0$

Diagnostic Tests of ΛCDM



Square Exponential $q(z)$ GP

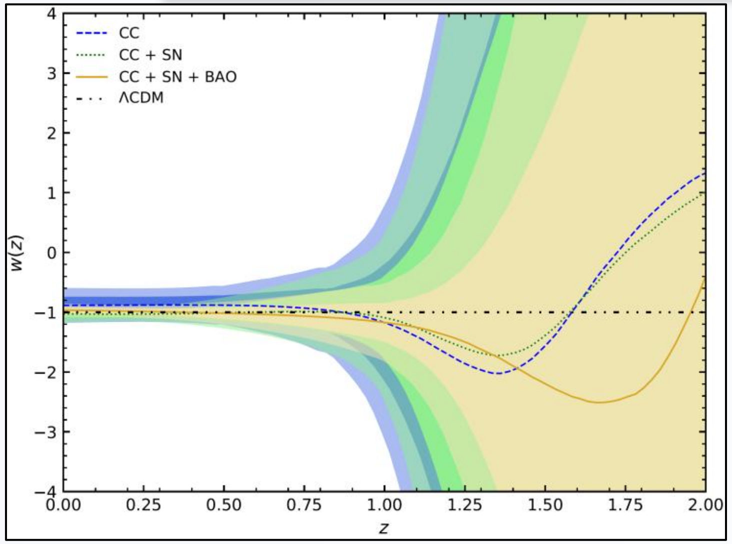


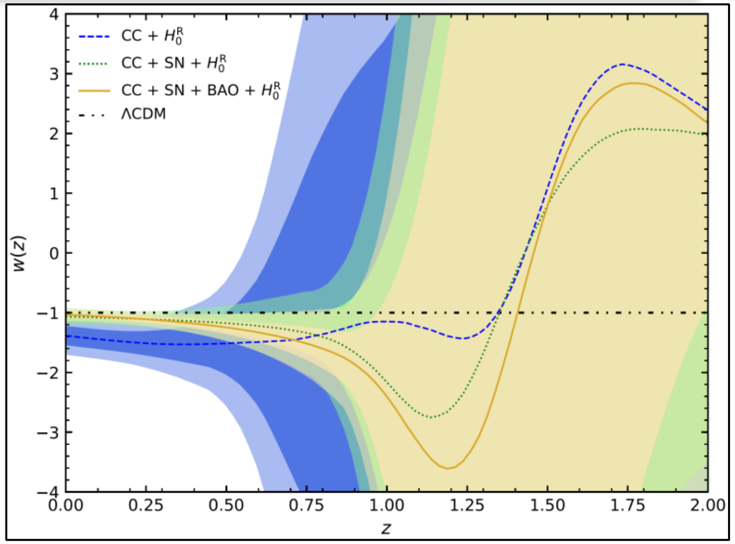
What does this say about the transition redshift

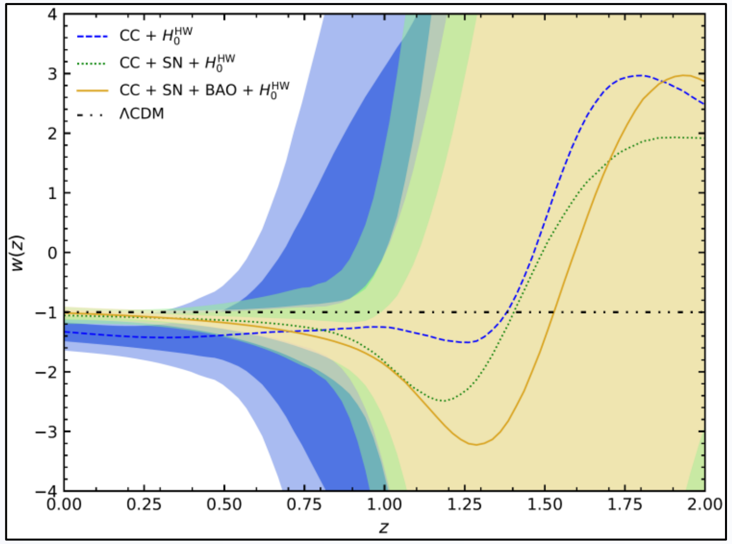
z_t

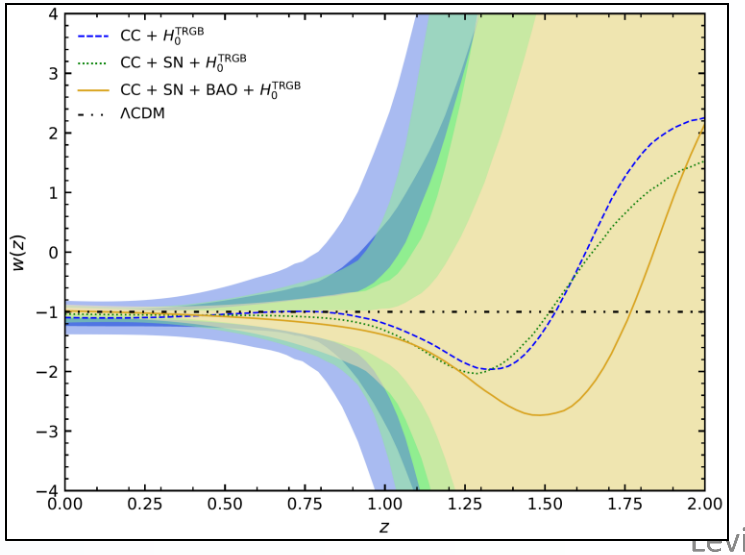
Data set(s)	Square exponential	Cauchy	Matérn	Rational quadratic
CC	$0.616^{+na}_{-0.136}$	$0.572^{+0.267}_{-0.118}$	$0.591^{+na}_{-0.129}$	$0.553^{+0.211}_{-0.110}$
CC+SN	$\mathbf{0.607^{+0.132}_{-0.084}}$	$\mathbf{0.600^{+0.132}_{-0.087}}$	$\mathbf{0.605^{+0.131}_{-0.089}}$	$\mathbf{0.598^{+0.133}_{-0.088}}$
CC+SN+BAO	$0.667^{+0.095}_{-0.075}$	$0.658^{+0.105}_{-0.081}$	$0.659^{+0.108}_{-0.082}$	$0.664^{+0.101}_{-0.079}$
CC+H_0^{R}	$0.551^{+0.112}_{-0.079}$	$0.540^{+0.114}_{-0.082}$	$0.546^{+0.115}_{-0.082}$	$0.528^{+0.114}_{-0.082}$
CC+SN+H_0^{R}	$0.702^{+0.246}_{-0.112}$	$0.687^{+0.268}_{-0.114}$	$0.694^{+0.263}_{-0.116}$	$0.679^{+na}_{-0.114}$
CC+SN+BAO+H_0^{R}	$\mathbf{0.783^{+0.107}_{-0.088}}$	$\mathbf{0.770^{+0.119}_{-0.095}}$	$\mathbf{0.772^{+0.123}_{-0.097}}$	$\mathbf{0.783^{+0.118}_{-0.095}}$
CC+H_0^{TRGB}	$0.573^{+0.165}_{-0.096}$	$0.552^{+0.161}_{-0.099}$	$0.564^{+0.167}_{-0.102}$	$0.537^{+0.159}_{-0.098}$
CC+SN+H_0^{TRGB}	$0.633^{+0.149}_{-0.091}$	$0.624^{+0.148}_{-0.094}$	$0.629^{+0.148}_{-0.094}$	$0.622^{+0.148}_{-0.094}$
CC+SN+BAO+H_0^{TRGB}	$0.703^{+0.099}_{-0.079}$	$0.693^{+0.109}_{-0.085}$	$0.695^{+0.112}_{-0.087}$	$0.699^{+0.106}_{-0.084}$
CC+H_0^{HW}	$0.556^{+0.120}_{-0.083}$	$0.544^{+0.122}_{-0.085}$	$0.551^{+0.124}_{-0.087}$	$0.531^{+0.121}_{-0.085}$
CC+SN+H_0^{HW}	$0.679^{+0.196}_{-0.104}$	$0.666^{+0.197}_{-0.107}$	$0.671^{+0.200}_{-0.108}$	$0.661^{+0.201}_{-0.107}$
CC+SN+BAO+H_0^{HW}	$0.752^{+0.106}_{-0.085}$	$0.740^{+0.116}_{-0.091}$	$0.743^{+0.119}_{0.094}$	$0.745^{+0.113}_{-0.091}$

Square Exponential $w(z)$ GP









Propagating $f(T(z))$ with GP data

- The Friedmann equation contains f_T which **need to by finite difference** methods
- Using a **central differencing** approach (error $\sim \mathcal{O}(\Delta z^2)$), we can assume

$$f'(z_i) \simeq \frac{f(z_{i+1}) - f(z_{i-1})}{z_{i+1} - z_{i-1}}$$

$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3}\boldsymbol{f_T}$$

- Therefore, we can remove the $f_T(T) = f'(z)/T'(z)$
- This then gives a **propagation equation**

$$f(z_{i+1}) = f(z_{i-1}) + 2(z_{i+1} - z_{i-1})\frac{H'(z_i)}{H(z_i)}\left(3H(z_i)^2 + \frac{f(z_i)}{2} - 3H_0^2\Omega_{m_0}(1+z_i)^3\right)$$

- Using **forward differencing**, we can produce a second boundary condition

Boundary Conditions

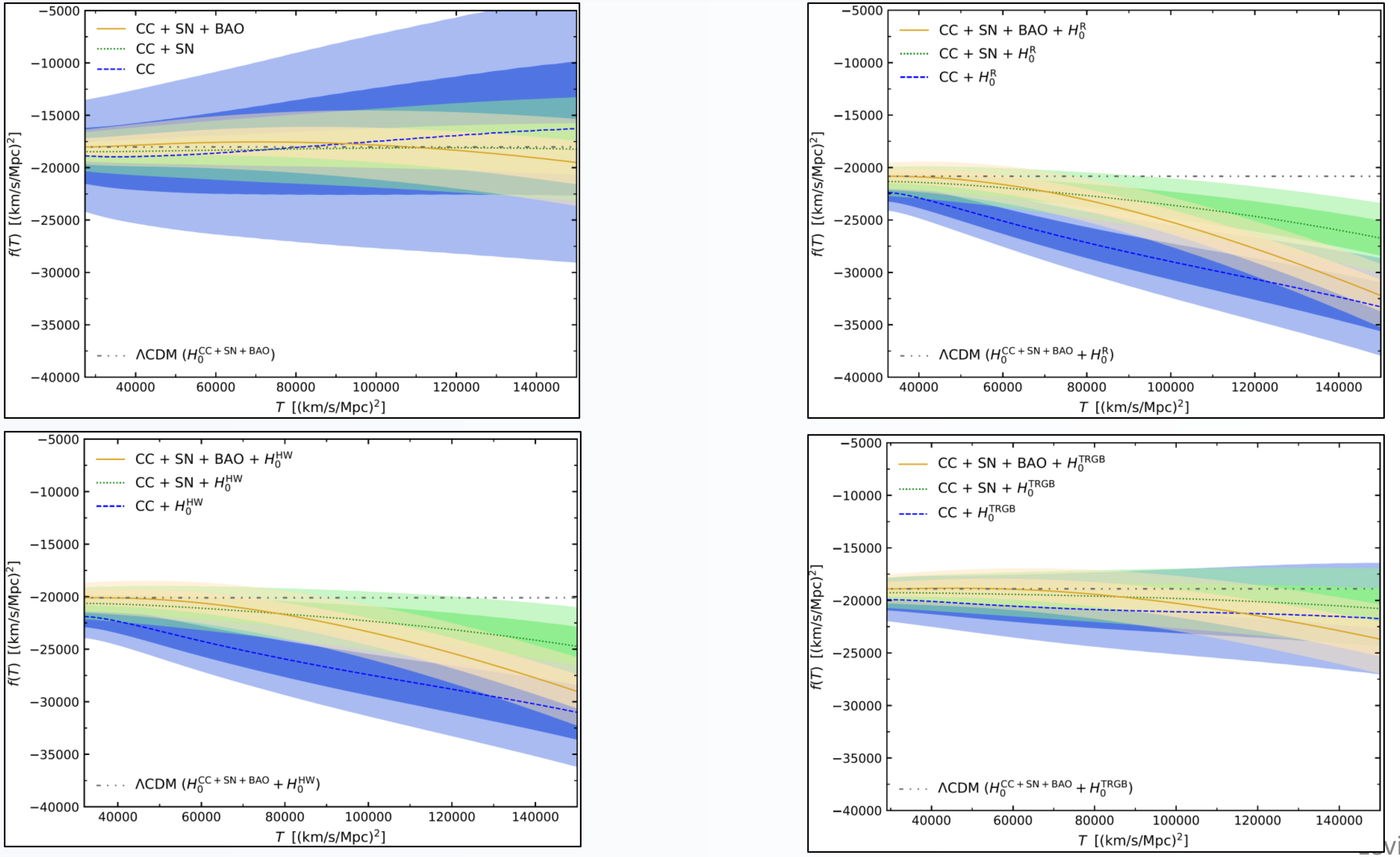
ΛCDM (or $f(T) = \Lambda$) at works at late cosmological times

This implies that

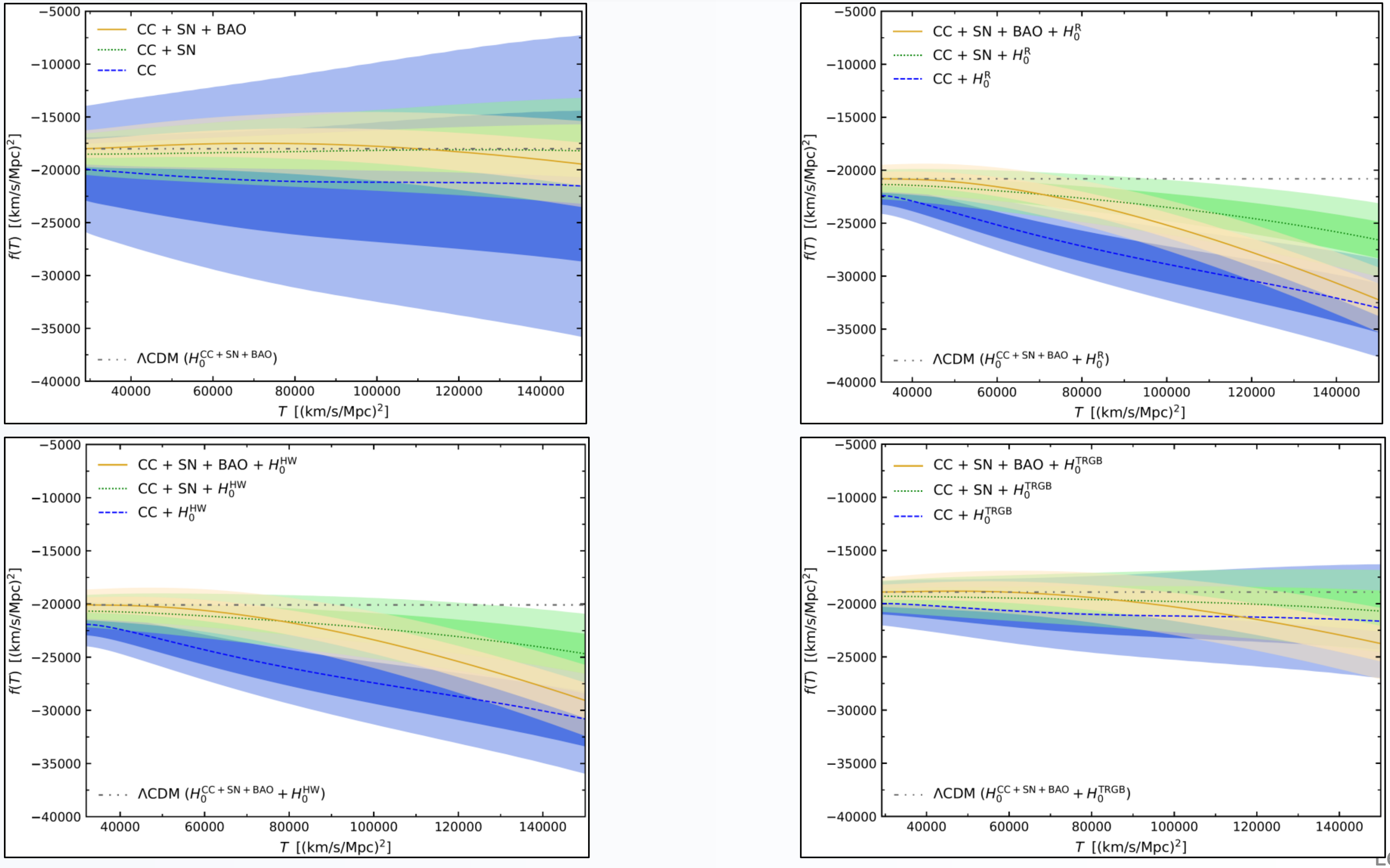
$$f_T(z \simeq 0) \simeq 0$$

$$\Rightarrow f(z \simeq 0) = 6H_0^2\left(\Omega_{m_0} - 1\right)$$

Square Exponential $f(T)$ GP



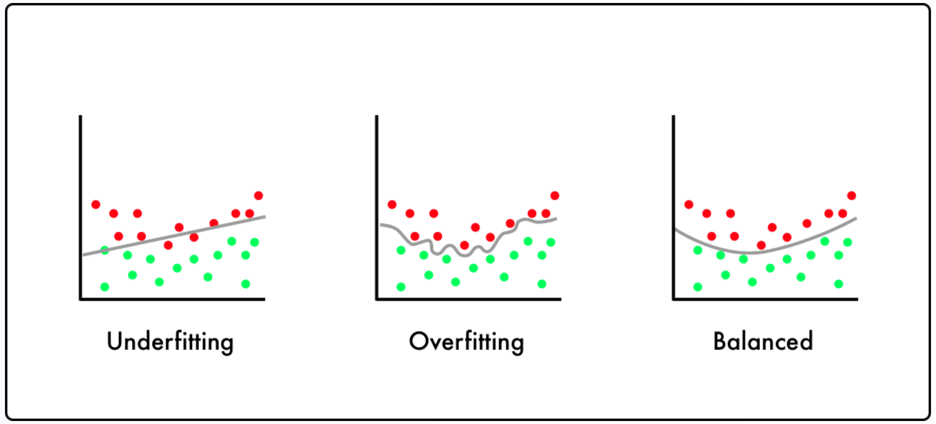
Cauchy $f(T)$ GP



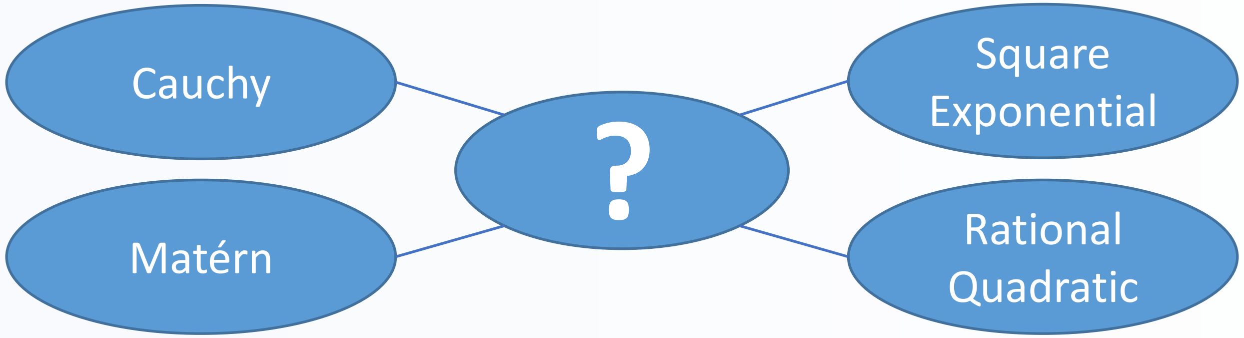
CC + SN + BAO
CC + SN
CC
ΛCDM ($H_0^{CC+SN+BAO}$)
f(T) [(km/s/Mpc)²]
T [(km/s/Mpc)²]
CC + SN + BAO + H_0^R
CC + SN + H_0^R
CC + H_0^R
ΛCDM ($H_0^{CC+SN+BAO}$ + H_0^R)
CC + SN + BAO + H_0^{HW}
CC + SN + H_0^{HW}
CC + H_0^{HW}
ΛCDM ($H_0^{CC+SN+BAO}$ + H_0^{HW})
CC + SN + BAO + H_0^{TRGB}
CC + SN + H_0^{TRGB}
CC + H_0^{TRGB}
ΛCDM ($H_0^{CC+SN+BAO}$ + H_0^{TRGB})

Open Problems with GP Reconstructions

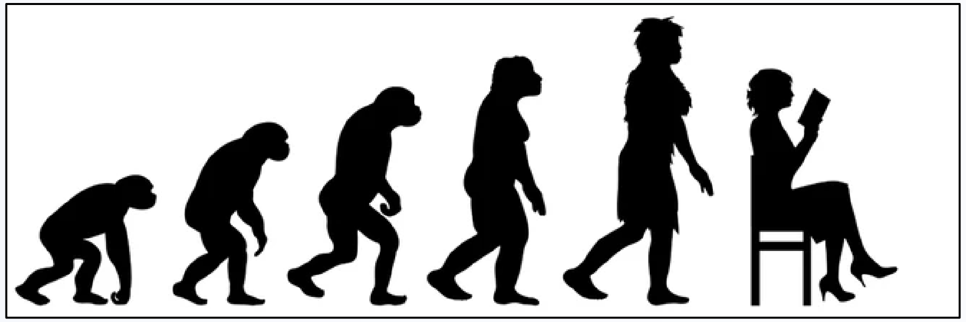
- **<u>Overfitting at origin</u>:** GP is very prone to overfitting for small data sets, which is especially pronounced at the origin, i.e. Hubble constant

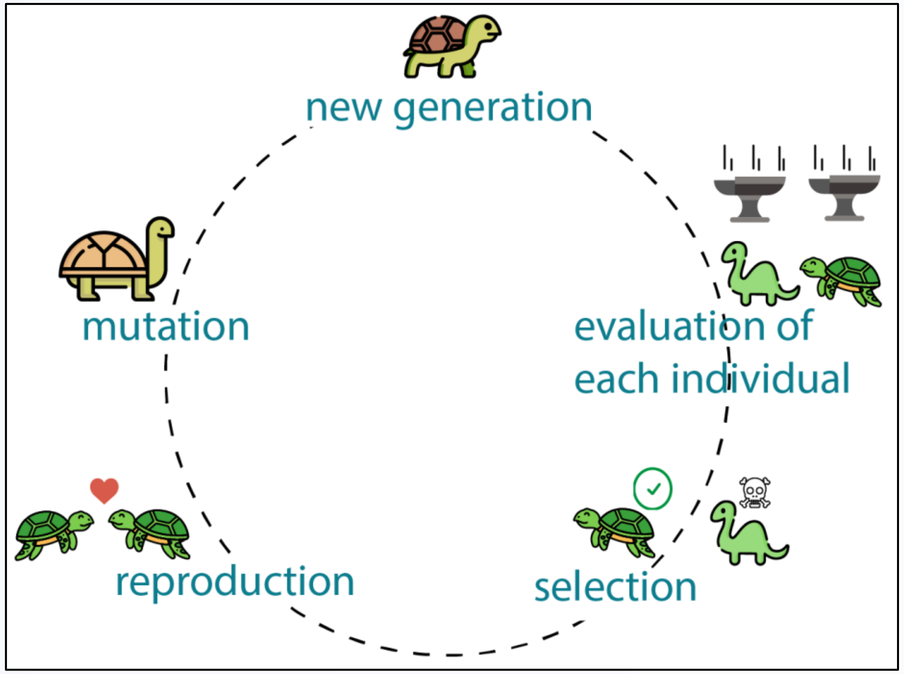


- **<u>Kernel Selection Problem</u>:** There is no natural kernel for cosmology



Genetic Algorithms (GAs)





Fitness function: Score to characterize the performance of each generation (BIC inspired)

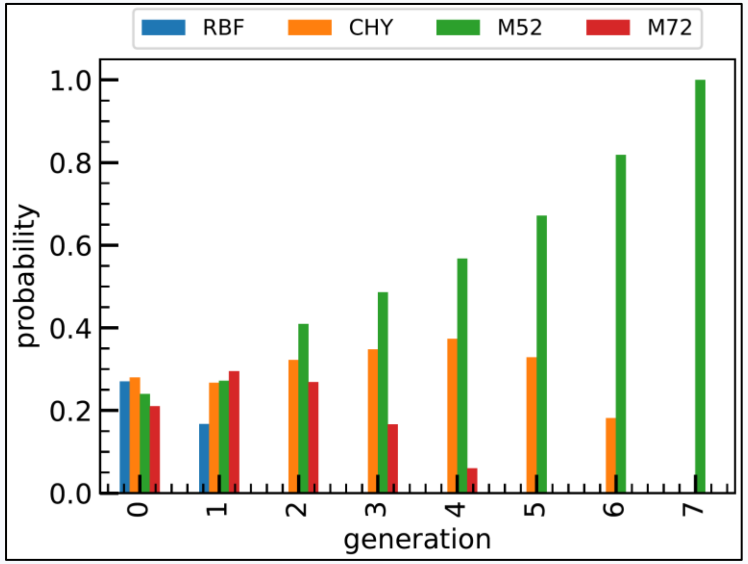
$$\mathcal{F} = \ln \mathcal{L} - \frac{k_{\mathrm{eff}} \ln N}{2}$$

Selection: Population that will survive

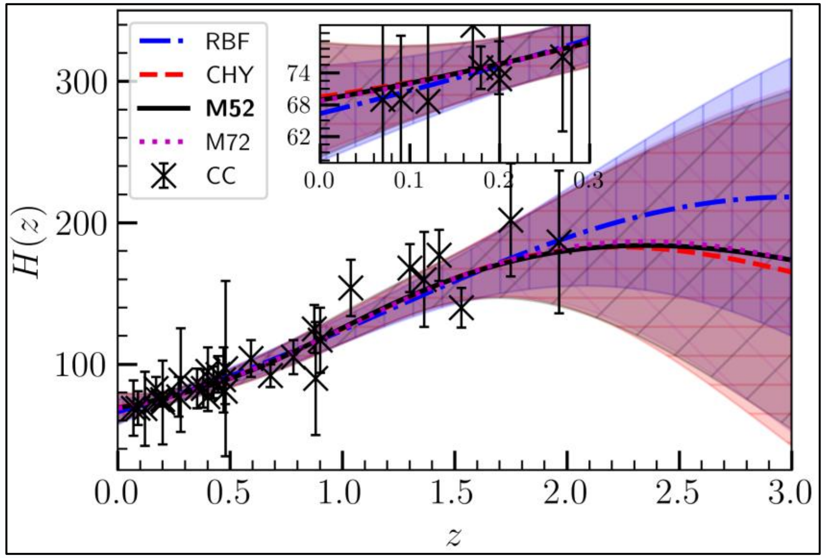
Crossover: Inheritance of kernels

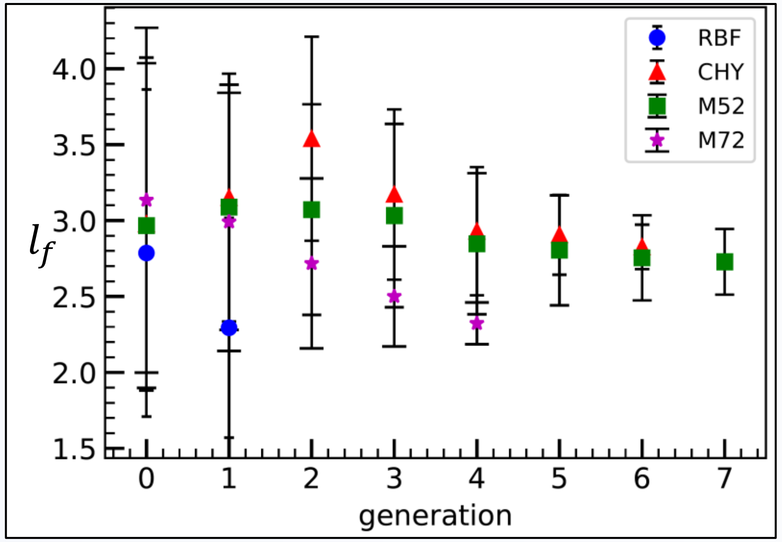
Mutation: Changes in addition to crossover

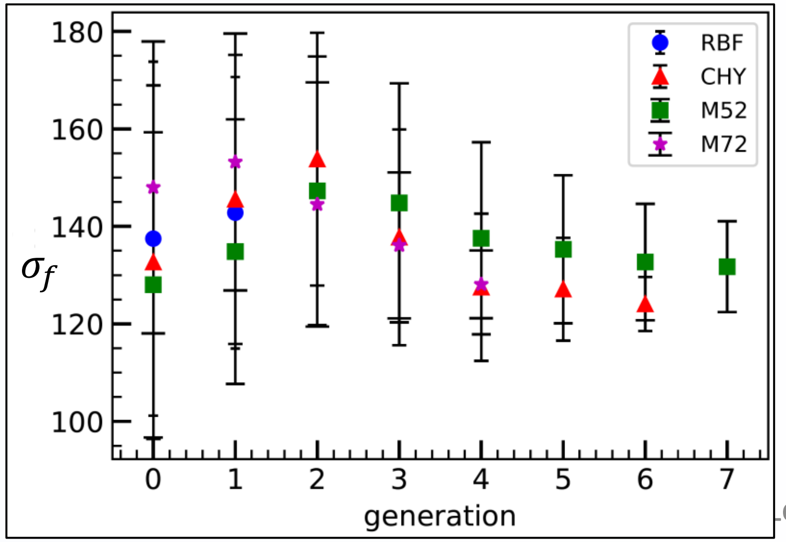
Genetic Algorithms (GAs)



CC dataset





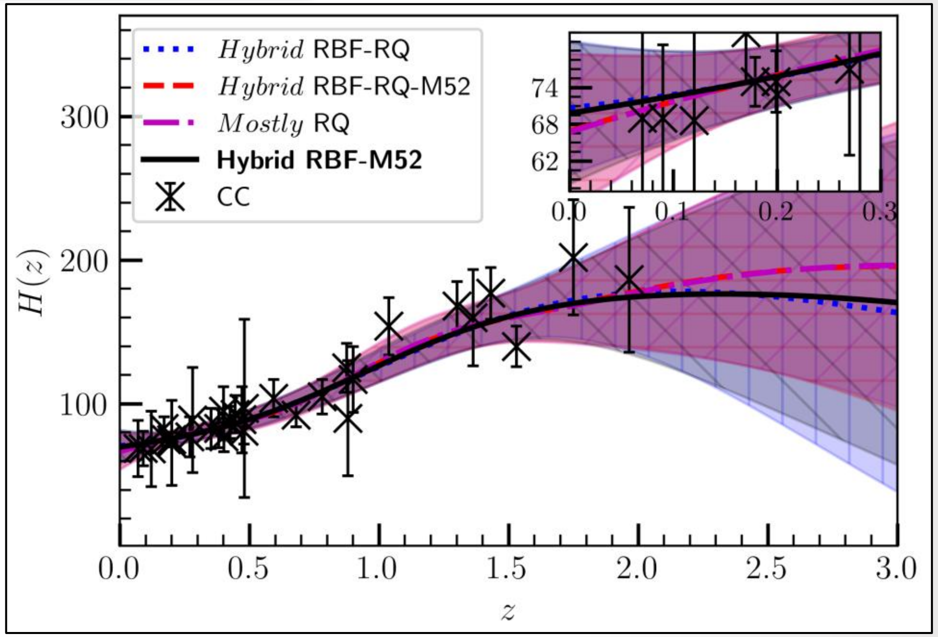


arXiv:2106.08688

Trials for GAs

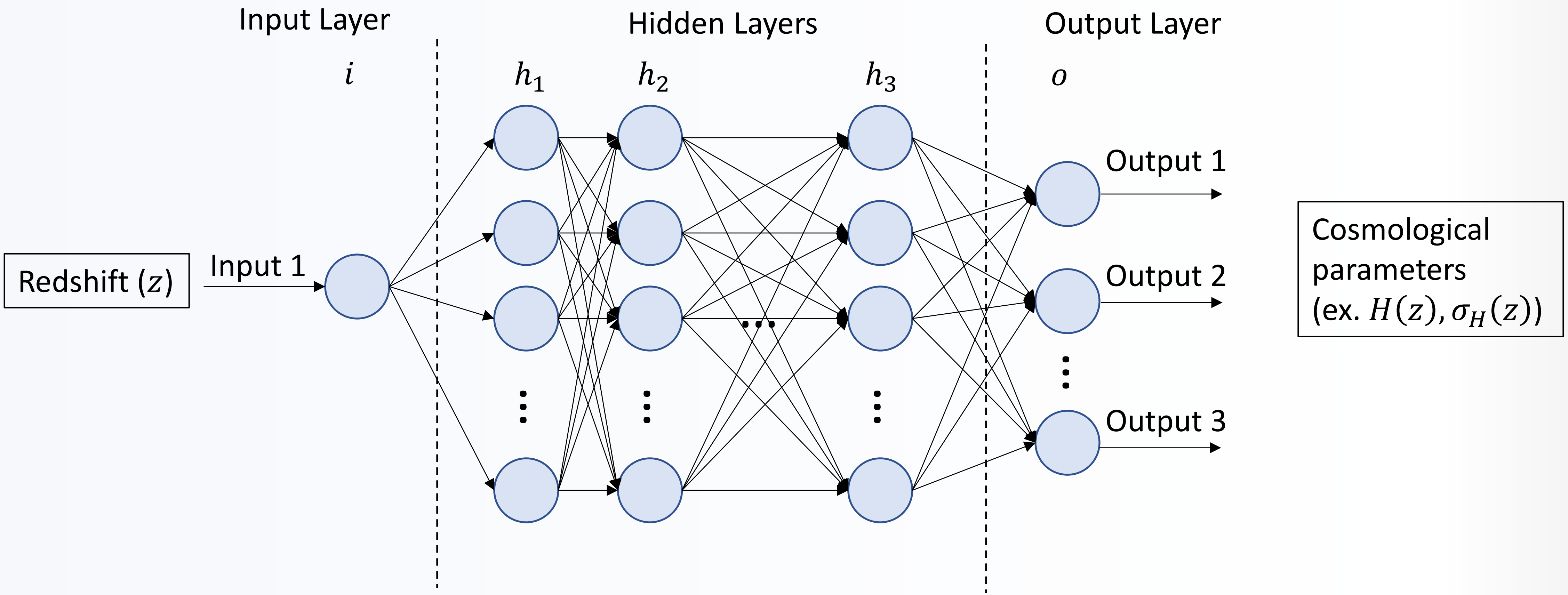
Trial	Population size	Selection rate	Mutation rate	No. of generations	Best fitness
1	10^4	0.5	0.15	10^1	-143.5
2	10^4	0.3	0.30	10^1	-148.5
3	10^3	0.1	0.10	10^2	-143.4
4	$\mathbf{10^3}$	$\mathbf{0.3}$	$\mathbf{0.50}$	$\mathbf{10^2}$	$\mathbf{-141.8}$

Kernel	H_0	$\ln \mathcal{L}$	χ	fitness	Penalty
Hybrid RBF-RQ	70.6 ± 5.5	-131.49	13.1	-143.5	12.0
Hybrid RBF-RQ-M52	66.9 ± 6.3	-131.38	12.0	-148.5	17.2
Mostly RQ	66.7 ± 6.4	-131.36	11.7	-143.4	12.0
***Hybrid* RBF-M52**	$\mathbf{69.8 \pm 5.8}$	$\mathbf{-131.48}$	$\mathbf{12.7}$	$\mathbf{-141.8}$	$\mathbf{10.3}$



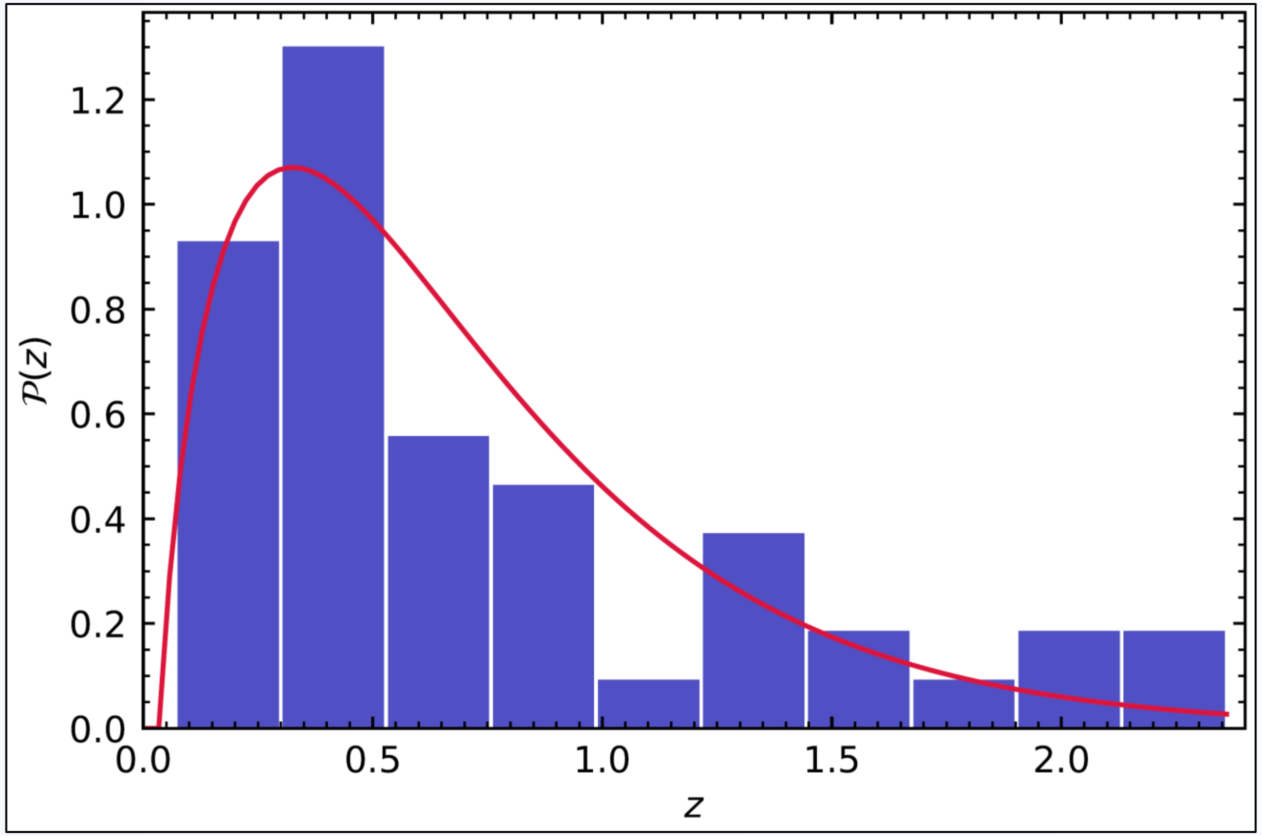
$$\text{Penalty} = \frac{k_{\text{eff}} \ln N}{2}$$

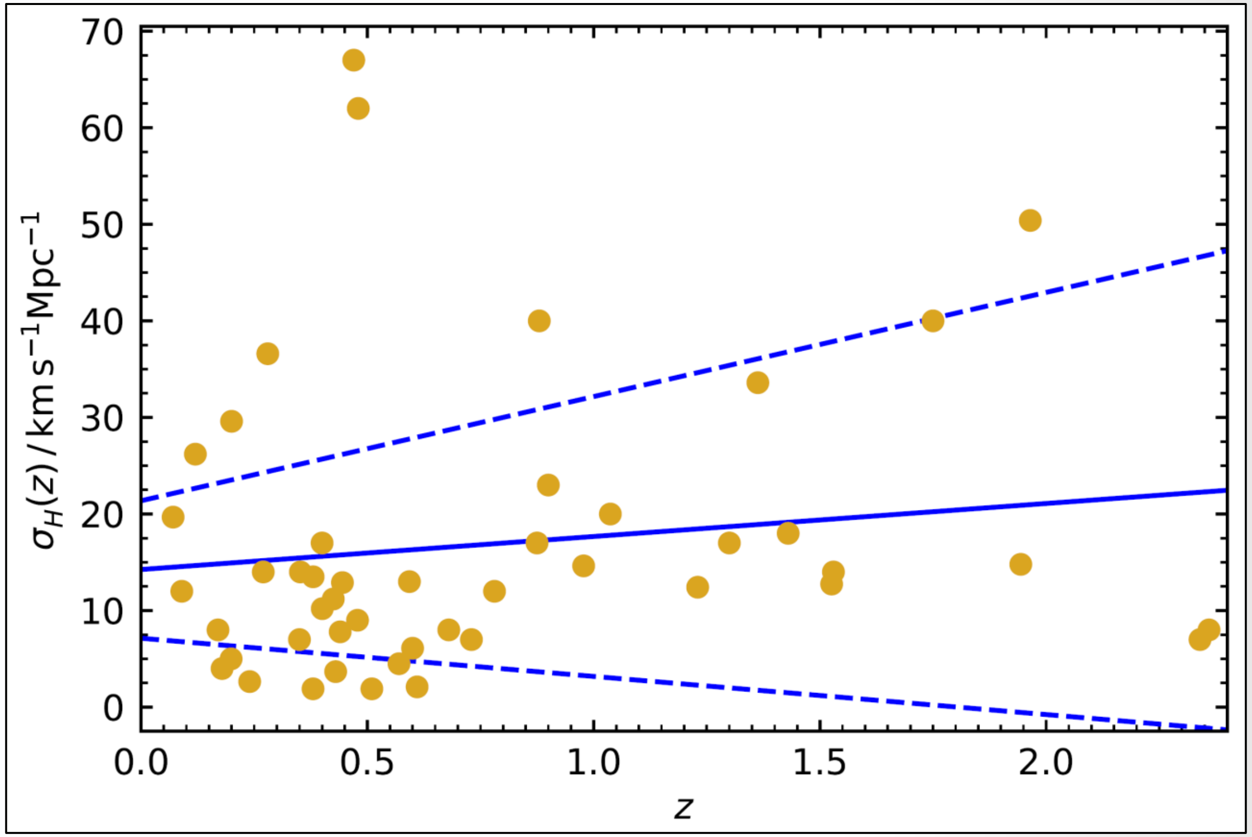
Artificial Neural Networks (ANNs)



Training Data for the ANN

CC+BAO dataset





This observes the gamma distribution:

$$\mathcal{P}(z,\alpha,\lambda)=\frac{\lambda^{\alpha}}{\Gamma(\alpha)}z^{\alpha-1}e^{-\lambda z}$$

Mean: $\sigma_H = 14.25 + 3.42z$

Upper error: $\sigma_H = 21.37 + 10.79z$

Lower error: $\sigma_H = 7.14 - 3.95z$

Designing the ANN

- **Risk** – Optimizes the **number of hidden layers and neurons** in an ANN

$$\text{risk} = \sum_{i=1}^{N} (\text{Bias}_i^2 + \text{Variance}_\text{i}) = \sum_{i=1}^{N} \left(\left[H_{Obs}(z_i) - H_{pred}(z_i) \right]^2 + \sigma_H^2(z_i) \right)$$

- **Loss** – Balances the **number of iterations** a system needs to predict the observational data

    1. L1 (Least absolute deviation)

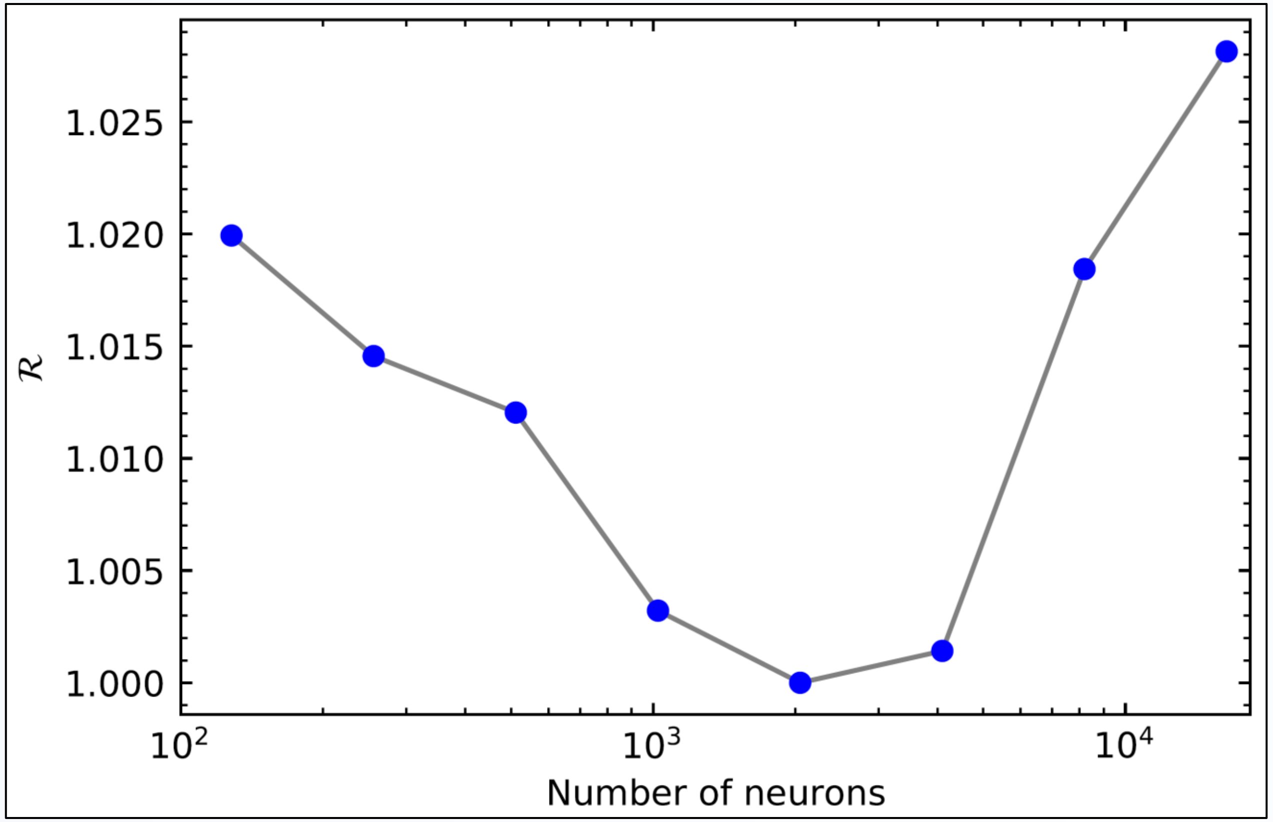
$$\text{L1} = \sum_{i=1}^{N} \left| H_{Obs}(z_i) - H_{pred}(z_i) \right|$$

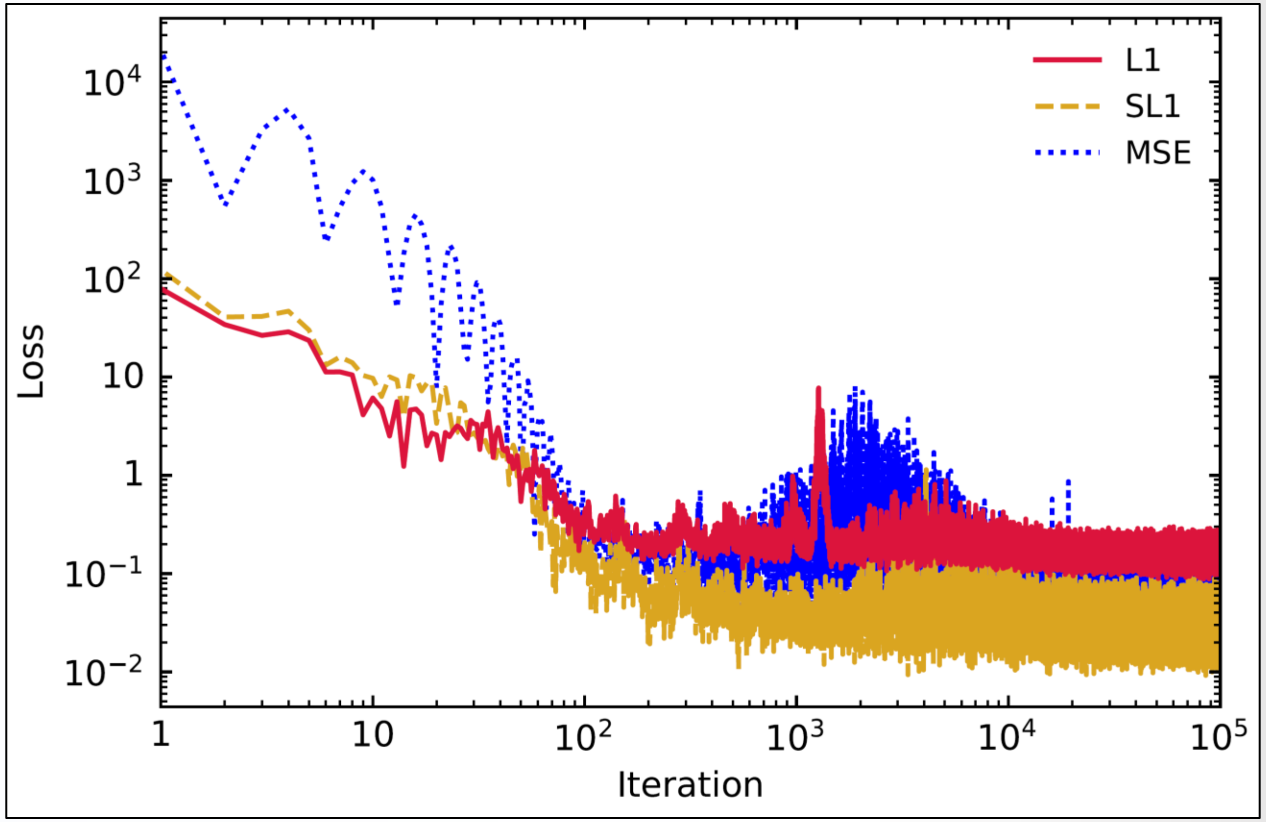
    2. Smoothed L1 (**SL1**)

    3. Mean Square Error (**MSE**)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} \left(H_{Obs}(z_i) - H_{pred}(z_i) \right)^2$$

Building the ANN

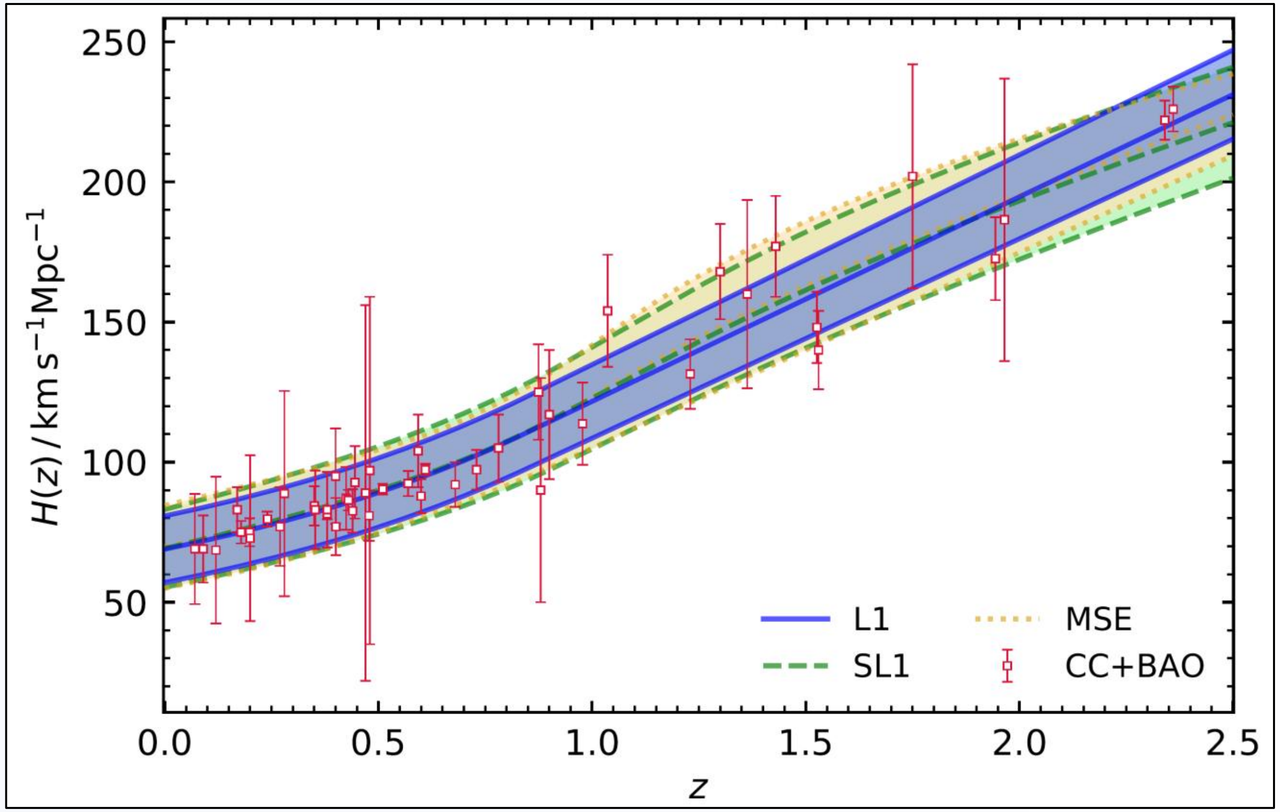


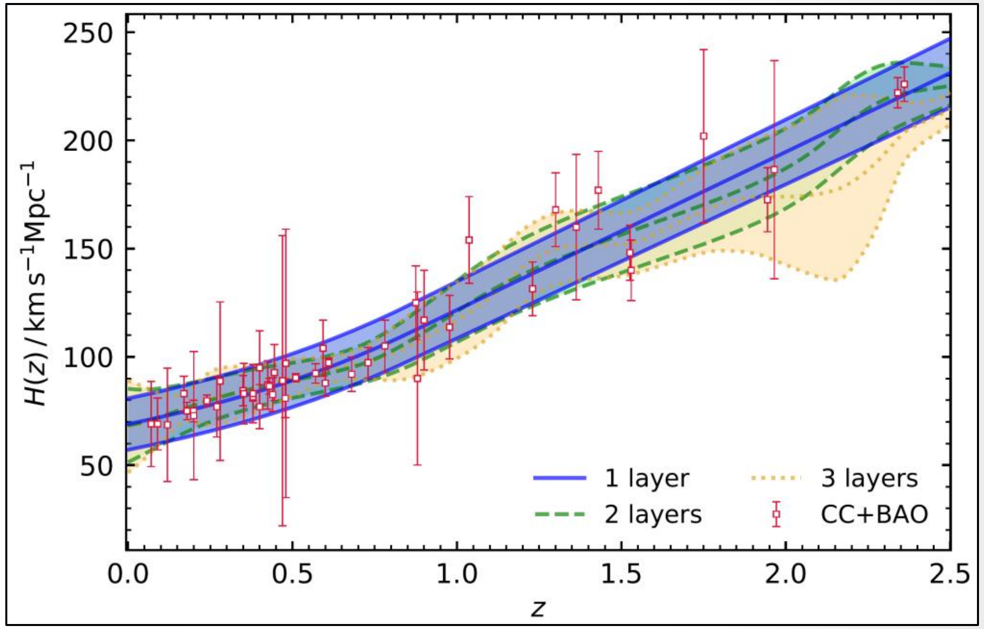


Risk function for **one layer** ($number\ of\ neurons = 2^n$
$n \in \{7, .. 14\}$)

arXiv:2111.11462

Using the ANN





One layer is preferred

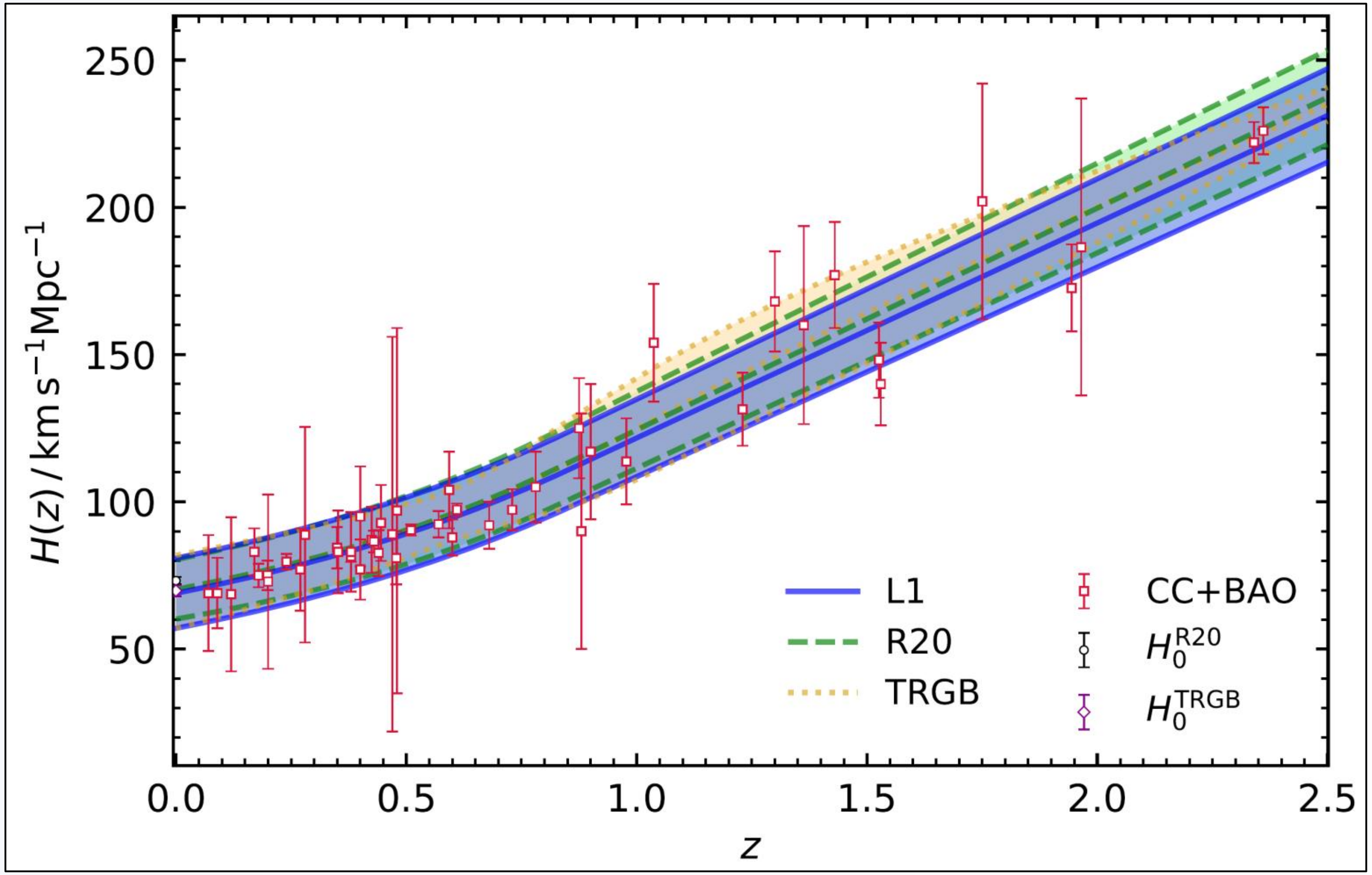
MSE: $H_0 = 69.76 \pm 14.82 \text{ km s}^{-1}\text{Mpc}^{-1}$

L1: $H_0 = 68.93 \pm 11.90 \text{ km s}^{-1}\text{Mpc}^{-1}$

SL1: $H_0 = 69.18 \pm 13.92 \text{ km s}^{-1}\text{Mpc}^{-1}$

arXiv:2111.11462

What about priors?



Priors:

$H_0^{\mathrm{R20}} = 73.2 \pm 1.3\ \mathrm{km\ s^{-1}Mpc^{-1}}$

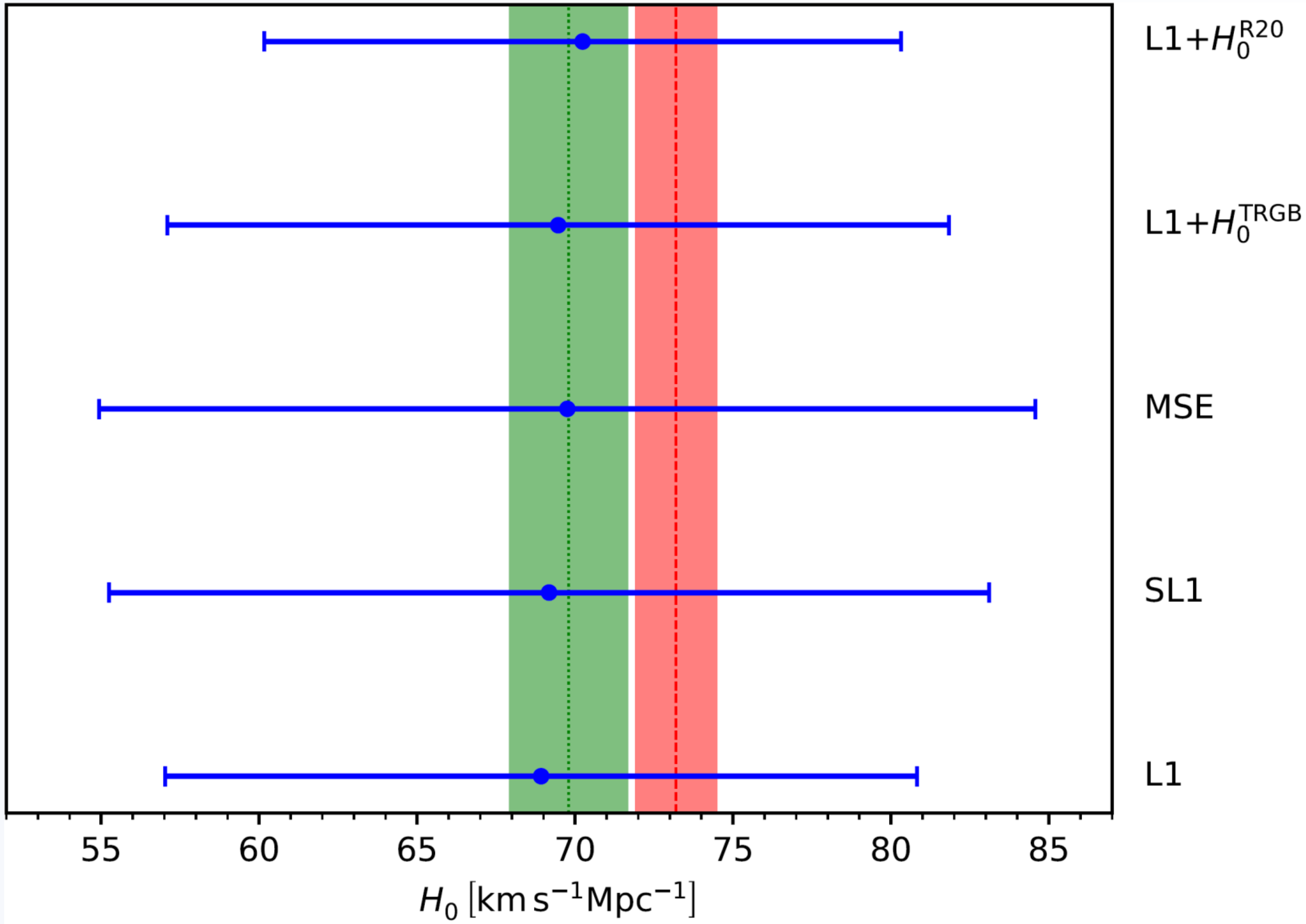
$H_0^{\mathrm{TRGB}} = 69.8 \pm 1.9\ \mathrm{km\ s^{-1}Mpc^{-1}}$

$\boldsymbol{H_0^{\mathrm{R20}}}$: $H_0 = 70.24 \pm 10.08\ \mathrm{km\ s^{-1}Mpc^{-1}}$

$\boldsymbol{H_0^{\mathrm{TRGB}}}$: $H_0 = 69.47 \pm 12.37\ \mathrm{km\ s^{-1}Mpc^{-1}}$

arXiv:2111.11462

Whisker Plot of Results



Priors:

$H_0^{\text{R20}} = 73.2 \pm 1.3 \text{ km s}^{-1}\text{Mpc}^{-1}$

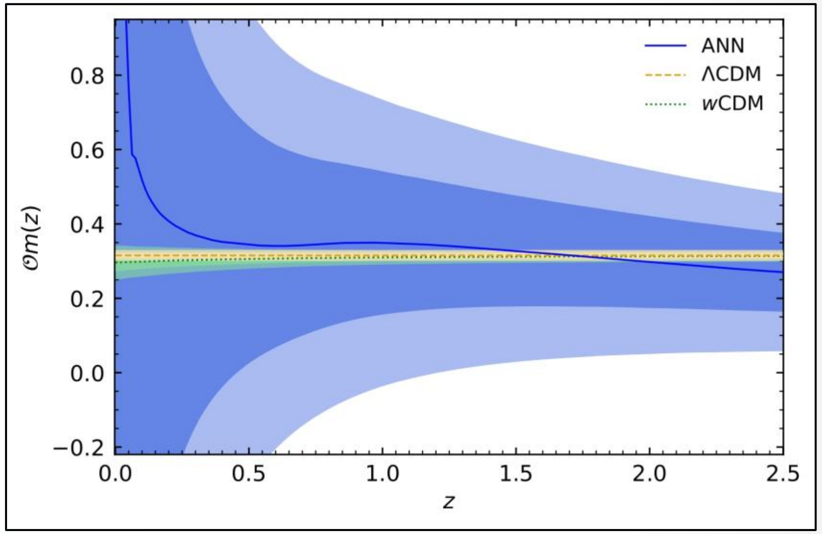
$H_0^{\text{TRGB}} = 69.8 \pm 1.9 \text{ km s}^{-1}\text{Mpc}^{-1}$

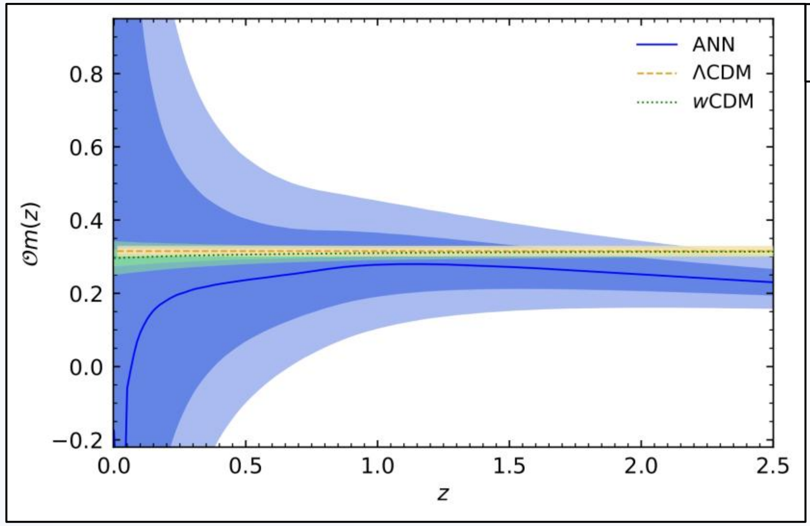
arXiv:2111.11462

Diagnostic Testing on $\Omega_\mathbf{m}$

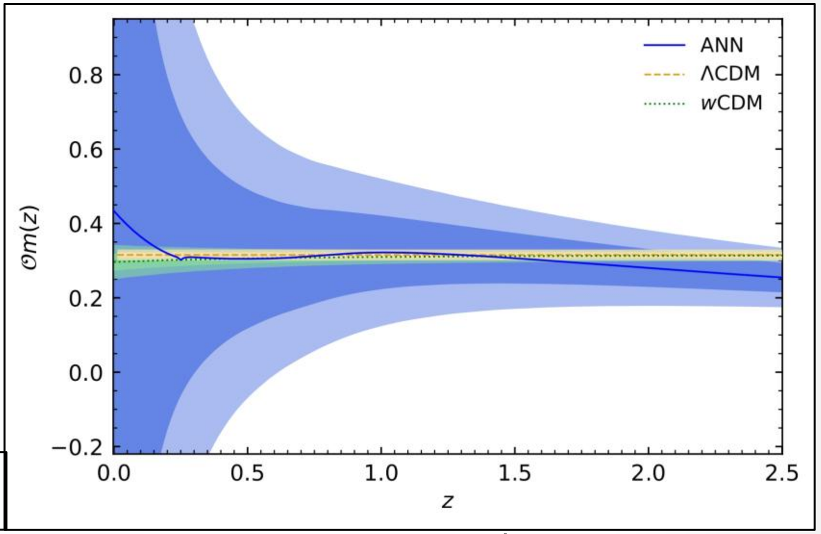
Null Testing:

$$\mathcal{O}m(z) = \frac{(H(z)/H_0)^2 - 1}{(1+z)^3 - 1} \xrightarrow[\Lambda\text{CDM}]{} \Omega_{m,0}$$





SH0ES prior



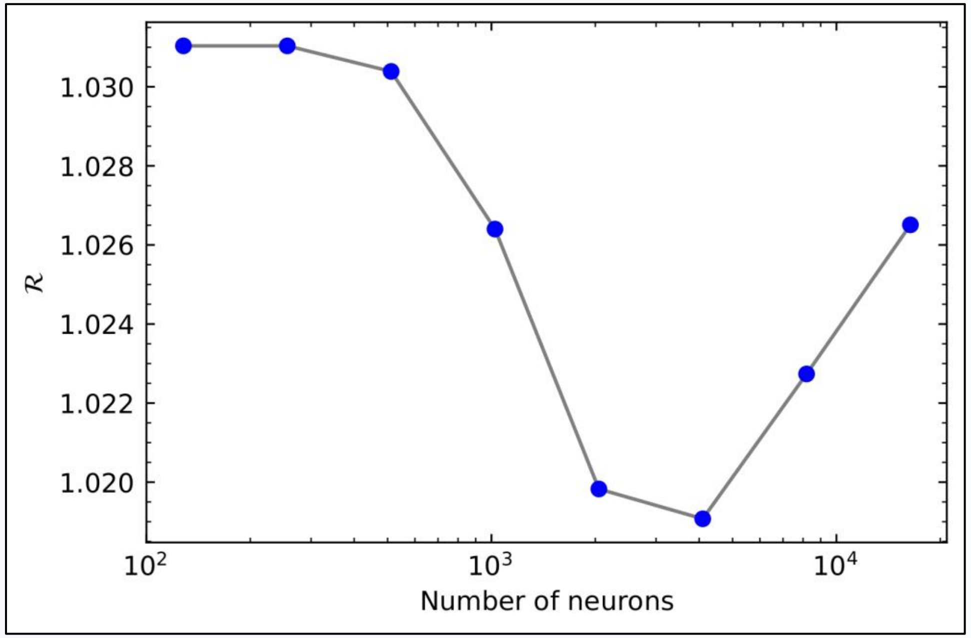
TRGB prior

arXiv:2111.11462

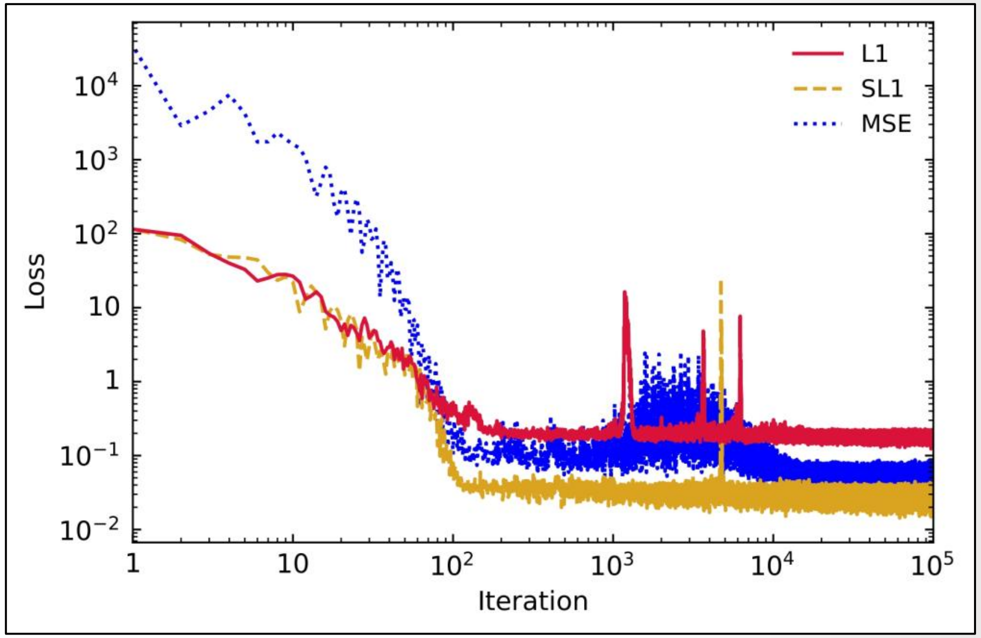
ANN Design for $f\sigma_8$ Reconstruction

Matter perturbation evolution:

$$\delta_m''(a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)\delta_m'(a) - \frac{3}{2}\frac{\Omega_m(a)}{a^2}\rho_m\delta_m(a) = 0$$



arXiv:2111.11462



Risk function for **one layer** ($number\ of\ neurons = 2^n$
$n \in \{7, \ldots 14\}$)

Reconstructing $f\sigma_8$ using ANNs

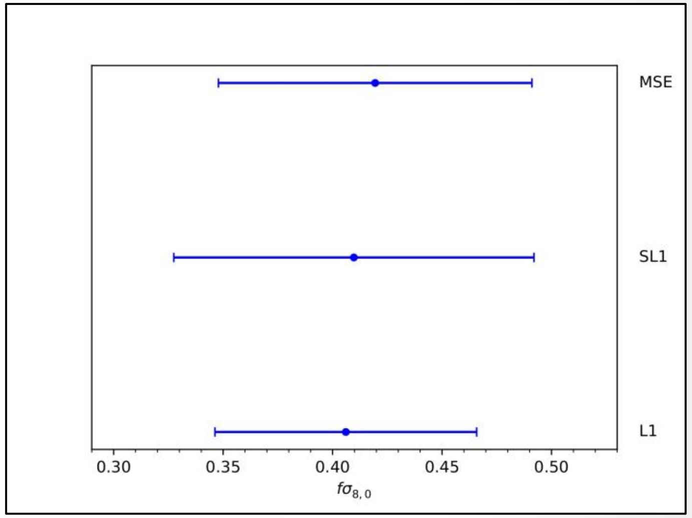
Matter perturbation evolution:

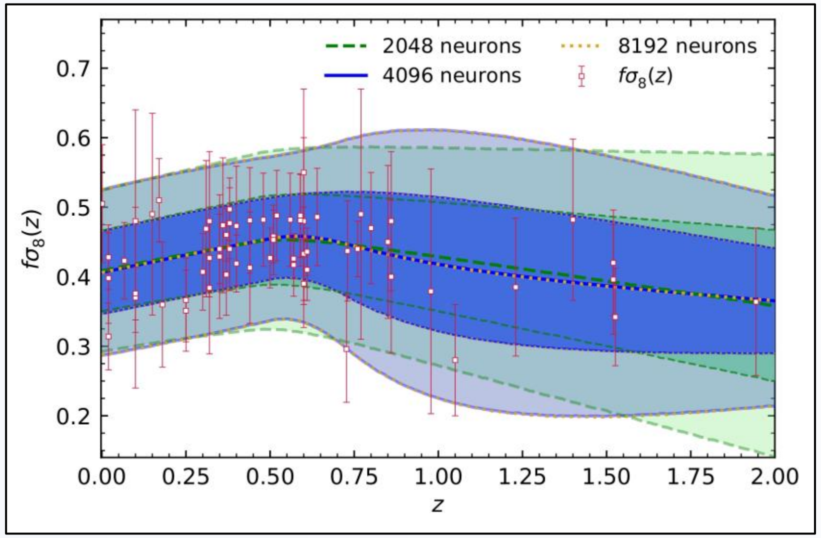
$$\delta_m''(a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)\delta_m'(a) - \frac{3}{2}\frac{\Omega_m(a)}{a^2}\rho_m\delta_m(a) = 0$$

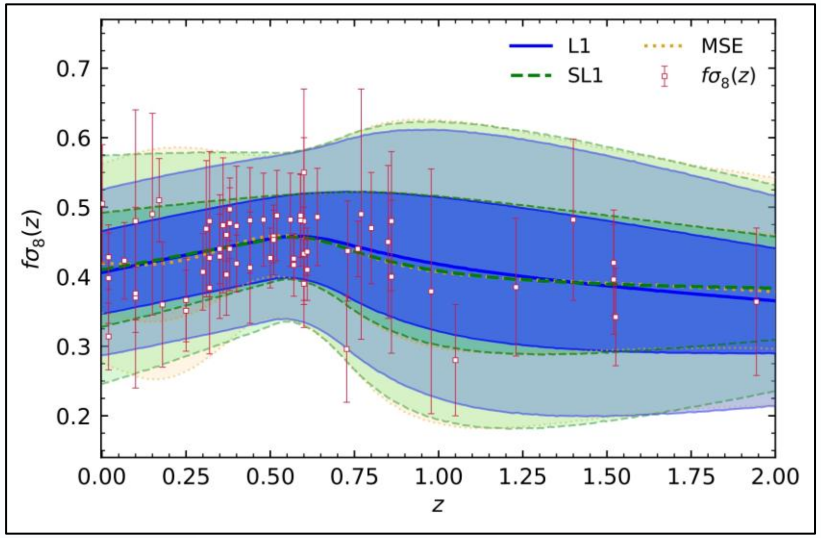
Structure and matter fluctuation equations:

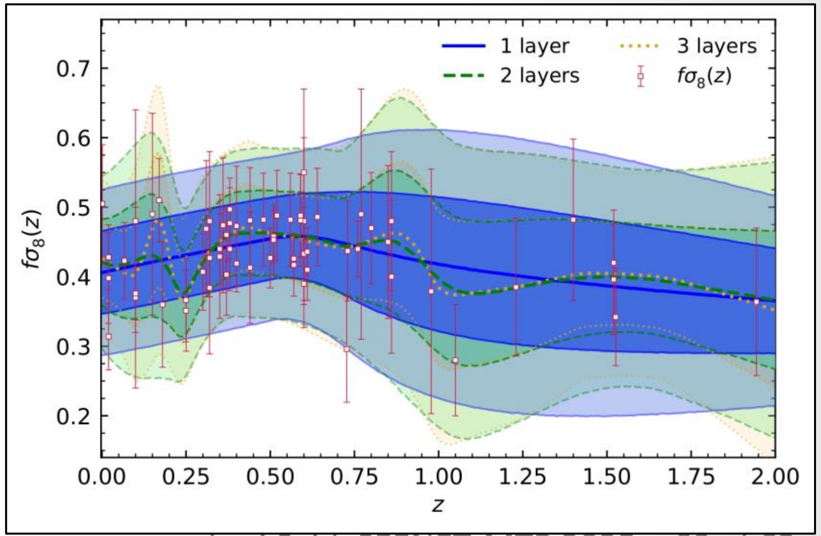
$$f(z) = \frac{d\ln\delta(z)}{d\ln a} \qquad \sigma_8(z) = \sigma_{8,0}\frac{\delta(z)}{\delta_0}$$

arXiv:2111.11462









Conclusion and Prospects

- **Cosmological observations** indicate that we may need to re-visit our **gravitational foundations**
- **TG offers an interesting alternative** to traditional ways to modify gravity
- TG satisfies a number of preliminary **observational tests**, and offers a more consistent picture of modified gravity
- TG can fix problems in **traditional modified gravity**

Thank You