Numerical Cosmology 1:

Cosmological Tensions

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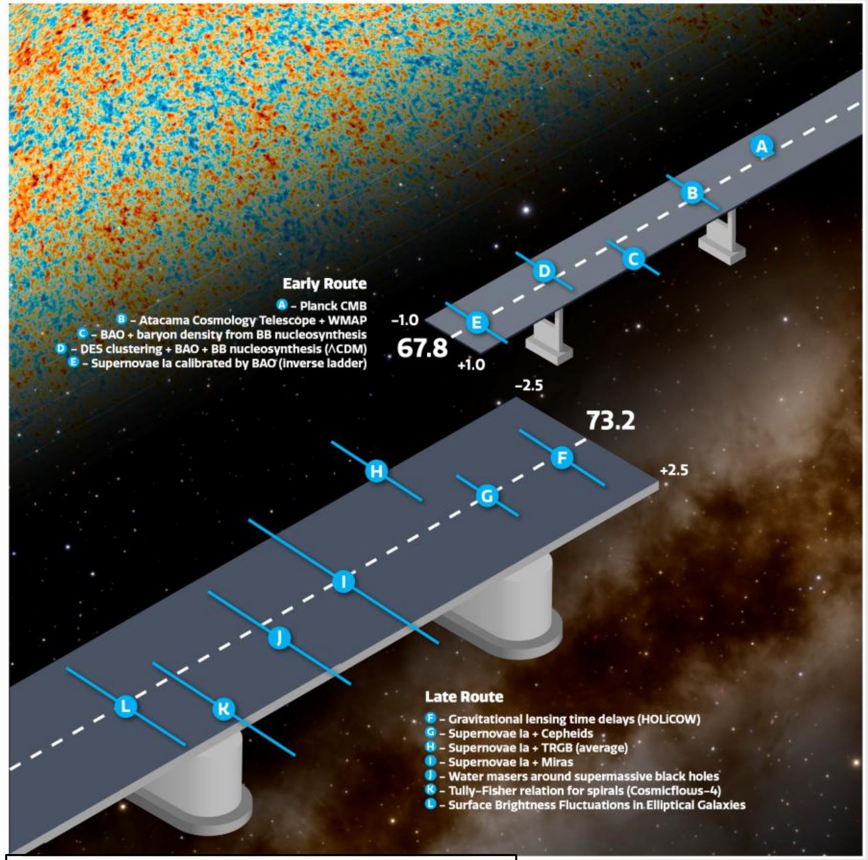
Main take away message

Why care about the Hubble constant?

Adam Riess (2019): “H_0 is the ultimate end-to-end test for ΛCDM”

- The H_0 tension is more than just a **tension between CMB and the SH0ES** measurement
- Its also a tension between the **inverse distance ladder and high-z measurements**
- We are very far from a solution!



Riess, A. Nat. Rev. Phys. 2 (2020) 10

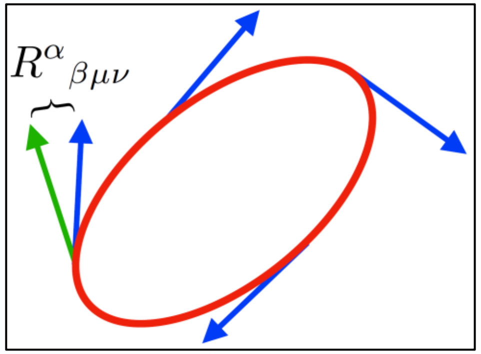
Why do we need modifications to standard cosmology?

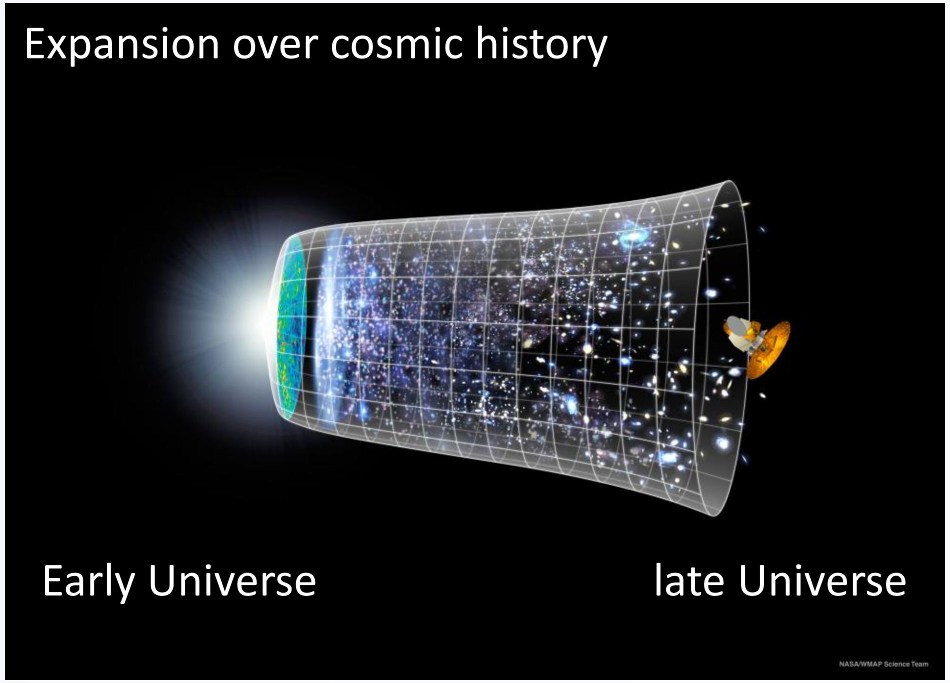
General Relativity and Concordance Cosmology

Einstein-Hilbert action for GR:

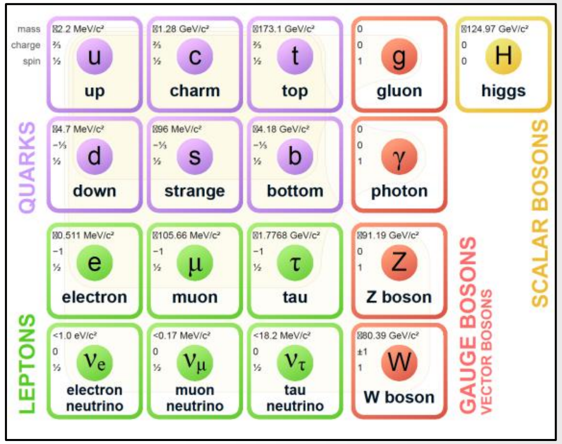
$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}[\mathcal{R}] + \int d^4x\sqrt{-g}\mathcal{L}_m(g_{\mu\nu},\psi)$$

Einstein 1915: **General Relativity (GR)**
Energy-momentum source of curvature
Levi-Civita connection: Zero Torsion, Metricity

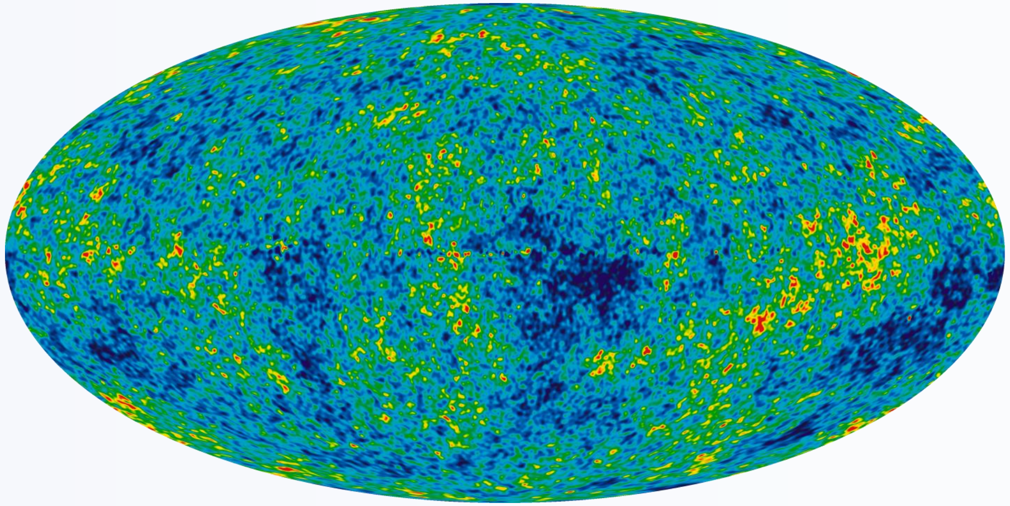


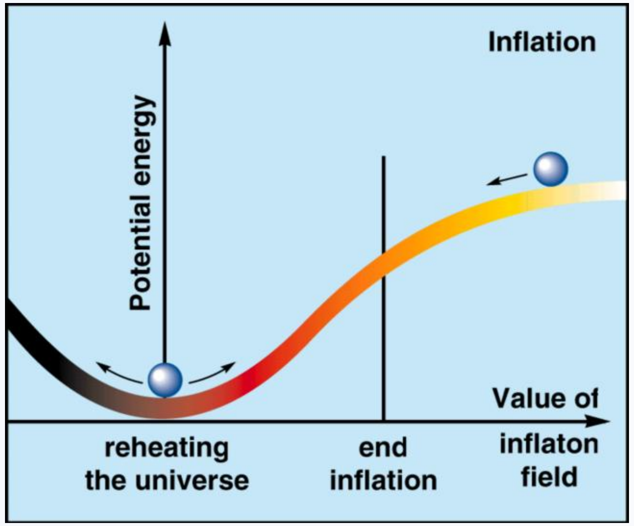


Standard model of particle physics:
$SU(3) \times SU(2) \times U(1)$



Early Universe Concordance Cosmology





Cosmic inflation

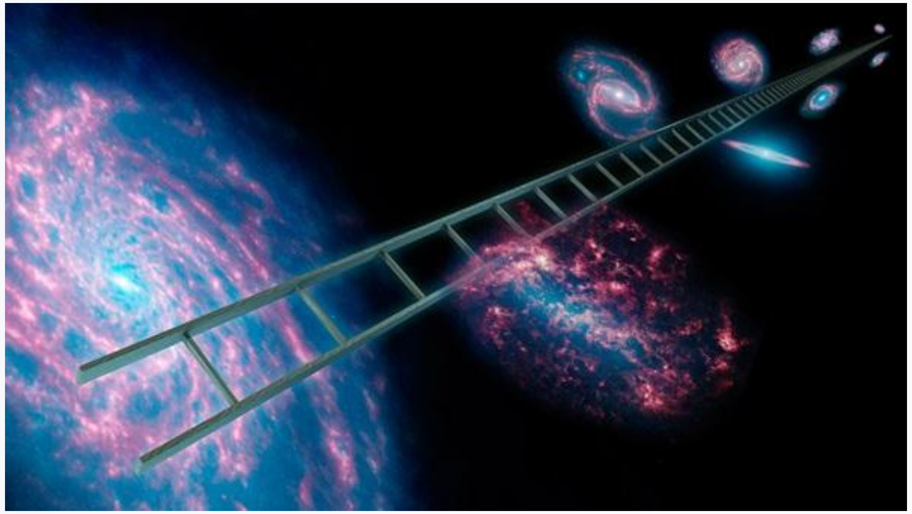
Pros: Horizon and flatness problems

Cons: Fine-tuning

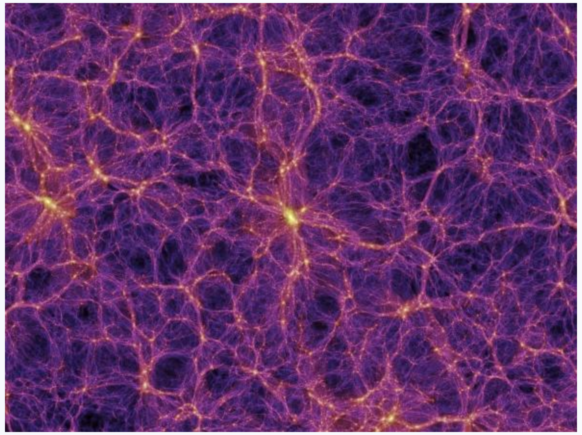
Anomalies and problems:

- The Lithium problem
- Hints of a closed Universe
- Large angular scale anomalies in the CMB
- Anomalously strong ISW effect
- Cosmic dipoles (cosmological principle)
- Lyman-α forest BAO anomalies
- Cosmic birefringence
- Discordance in dark matter abundance at smaller scales

Late Universe Concordance Cosmology



Anomalies and problems:
- Cold dark matter problems (core-cusp, missing satellites, satellite plane alignment)
- Dark energy in fundamental physics
- Oscillations of best-fit parameters across the sky
- Baryonic Tully-Fisher Relation

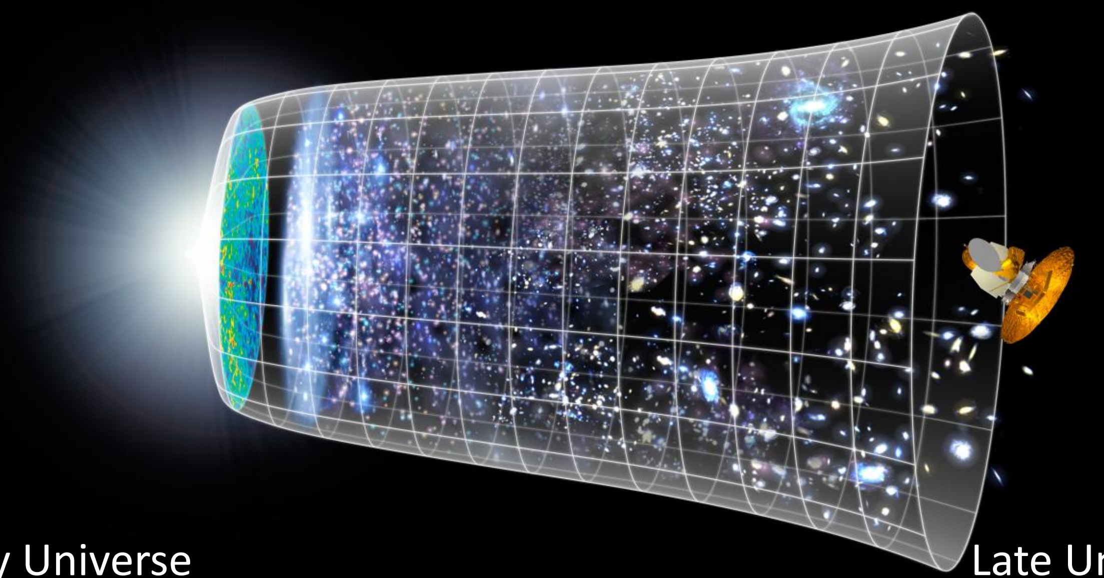


Requirements:
Dark matter
Dark energy

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}[\mathcal{R} - 2\Lambda] + \int d^4x\sqrt{-g}\mathcal{L}_m(g_{\mu\nu}, \psi)$$

The Hubble Tension

Cosmic Tension > 5σ



Early Universe estimate

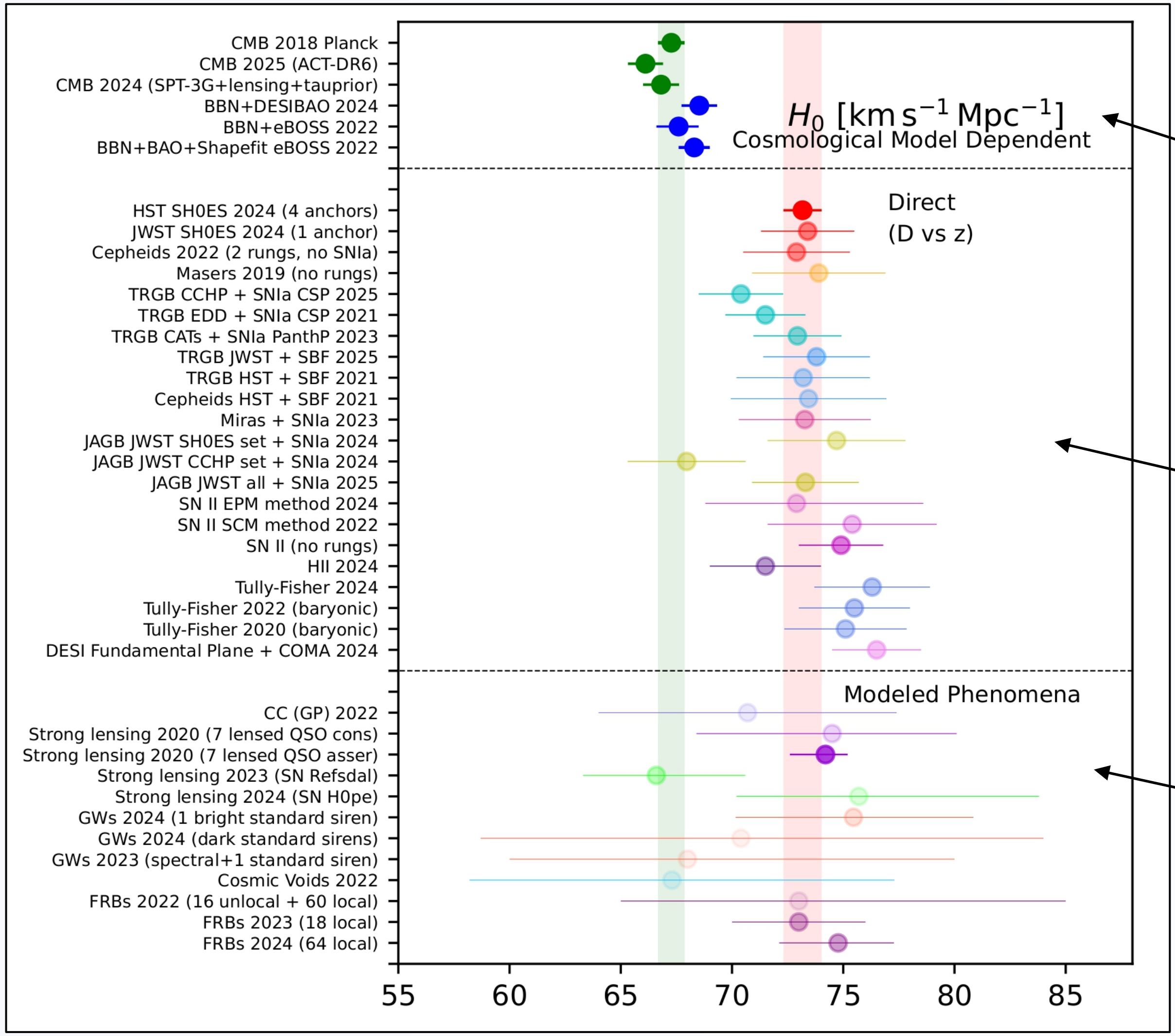
Late Universe estimate

$H_0^{\mathrm{P18}} = 67.27 \pm 0.60\ \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0^{\mathrm{S22}} = 73.04 \pm 1.04\ \mathrm{km\ s^{-1} Mpc^{-1}}$

NASA/WMAP Science Team

Cosmic Tensions



Indirect measures predict H_0 using ΛCDM

$$r_s = \int_{z_{\rm LS}}^{\infty} \frac{c_s(z', \rho_b)}{H(z')} \mathrm{d}z'$$

Direct measures estimate H_0 using astrophysics

$$d_L(z) = (1+z) \int_0^z \frac{\mathrm{d}z'}{H(z')}$$

Indirect measures predict H_0 using statistical or astrophysical modeling

CosmoVerse White Paper. arXiv:2504.01669

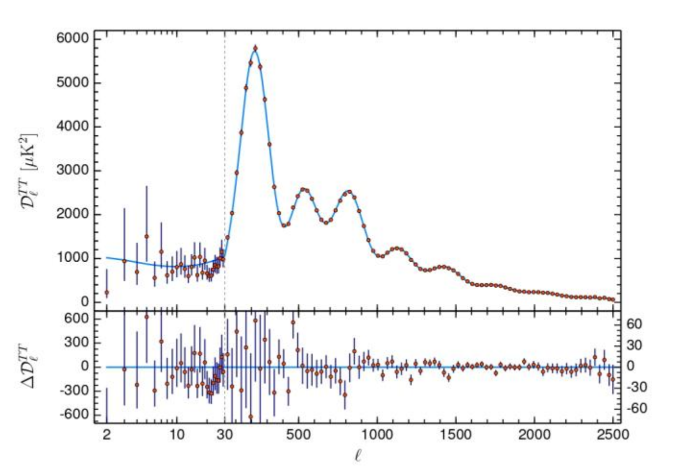
Cosmic Tensions: CMB

Parameter	Plik best fit	Plik [1]	CamSpec [2]	([2] − [1])/σ_1	Combined
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	−0.5	0.02233 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	−0.3	0.1198 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031	1.04087 ± 0.00031	−0.2	1.04089 ± 0.00031
τ	0.0543	0.0544 ± 0.0073	$0.0536^{+0.0069}_{-0.0077}$	−0.1	0.0540 ± 0.0074
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014	3.041 ± 0.015	−0.3	3.043 ± 0.014
n_s	0.96605	0.9649 ± 0.0042	0.9656 ± 0.0042	+0.2	0.9652 ± 0.0042
$\Omega_m h^2$	0.14314	0.1430 ± 0.0011	0.1426 ± 0.0011	−0.3	0.1428 ± 0.0011
H_0 [km s^{-1}Mpc^{-1}]	67.32	67.36 ± 0.54	67.39 ± 0.54	+0.1	67.37 ± 0.54
Ω_m	0.3158	0.3153 ± 0.0073	0.3142 ± 0.0074	−0.2	0.3147 ± 0.0074
Age [Gyr]	13.7971	13.797 ± 0.023	13.805 ± 0.023	+0.4	13.801 ± 0.024
σ_8	0.8120	0.8111 ± 0.0060	0.8091 ± 0.0060	−0.3	0.8101 ± 0.0061
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$	0.8331	0.832 ± 0.013	0.828 ± 0.013	−0.3	0.830 ± 0.013
z_{re}	7.68	7.67 ± 0.73	7.61 ± 0.75	−0.1	7.64 ± 0.74
$100\theta_*$	1.041085	1.04110 ± 0.00031	1.04106 ± 0.00031	−0.1	1.04108 ± 0.00031
r_{drag} [Mpc]	147.049	147.09 ± 0.26	147.26 ± 0.28	+0.6	147.18 ± 0.29

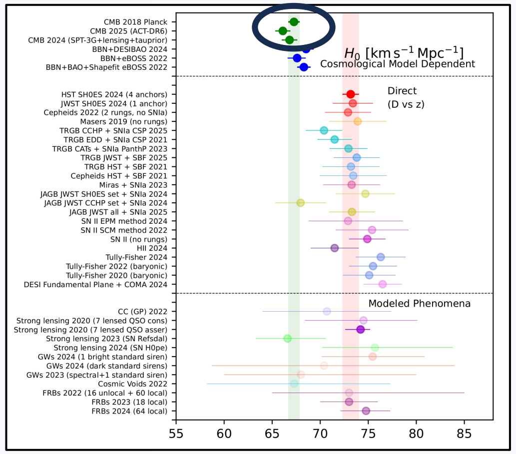
Planck Collaboration, A&A 641 (2020) A6

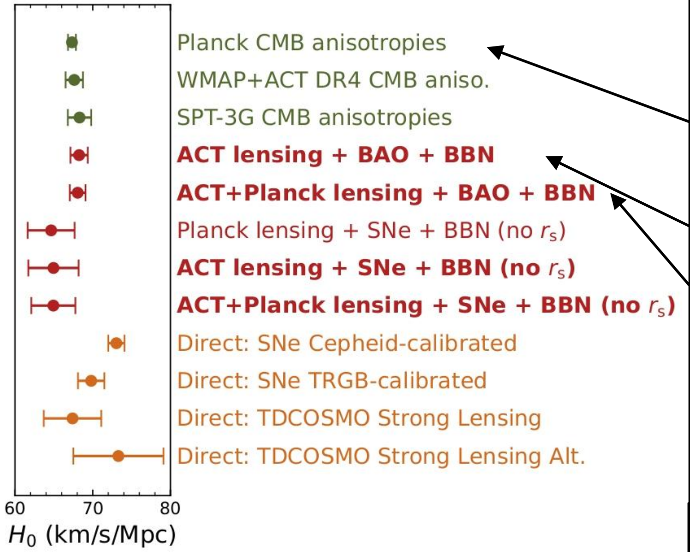
ΛCDM is a six parameter model:

- Baryon density ($\Omega_m h^2$)
- Cosmological dark matter density ($\Omega_c h^2$)
- Acoustic scale angle ($100\theta_{MC}$)
- Reionization optical depth (τ)
- Primordial power spectrum amplitude ($\ln(10^{10} A_s)$)
- Primordial spectral index (n_s)



Spectrum of CMB temperature anisotropies from Planck





ACT DR6 (2023)

$$H_0^{P18} = 67.4 \pm 0.5 \text{ km s}^{-1}\text{Mpc}^{-1}$$

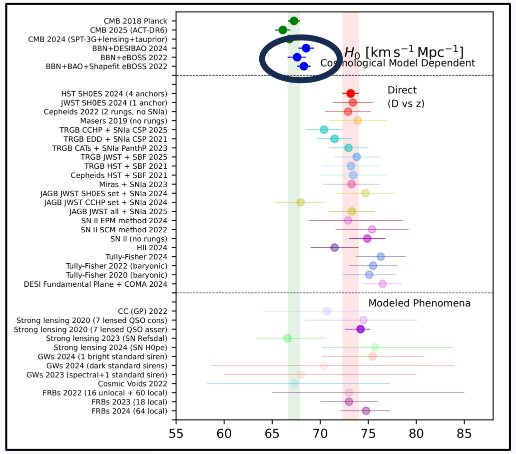
$$H_0^{ACT+BAO+BBN} = 68.3 \pm 1.1 \text{ km s}^{-1}\text{Mpc}^{-1}$$

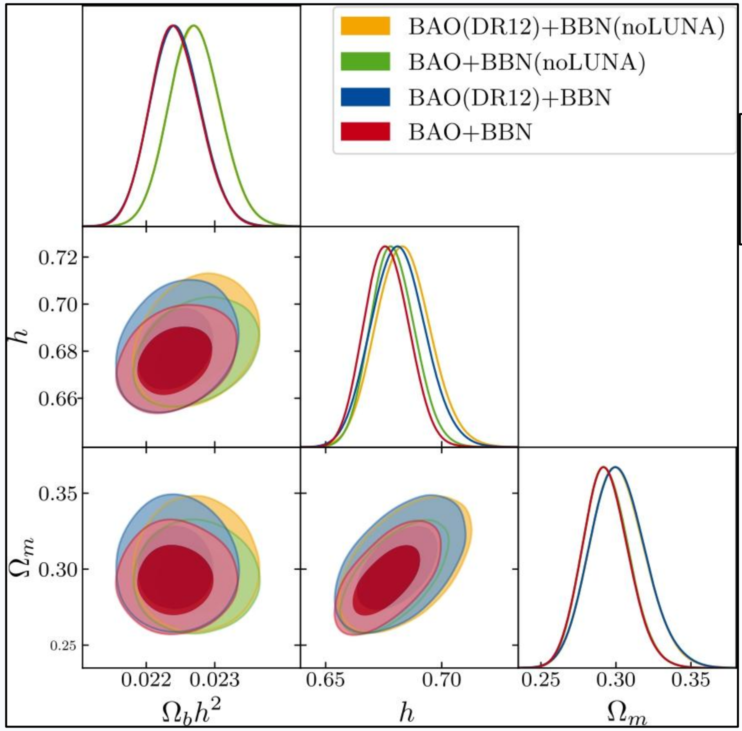
$$H_0^{ACT+P18+BAO+BBN} = 68.1 \pm 1.0 \text{ km s}^{-1}\text{Mpc}^{-1}$$

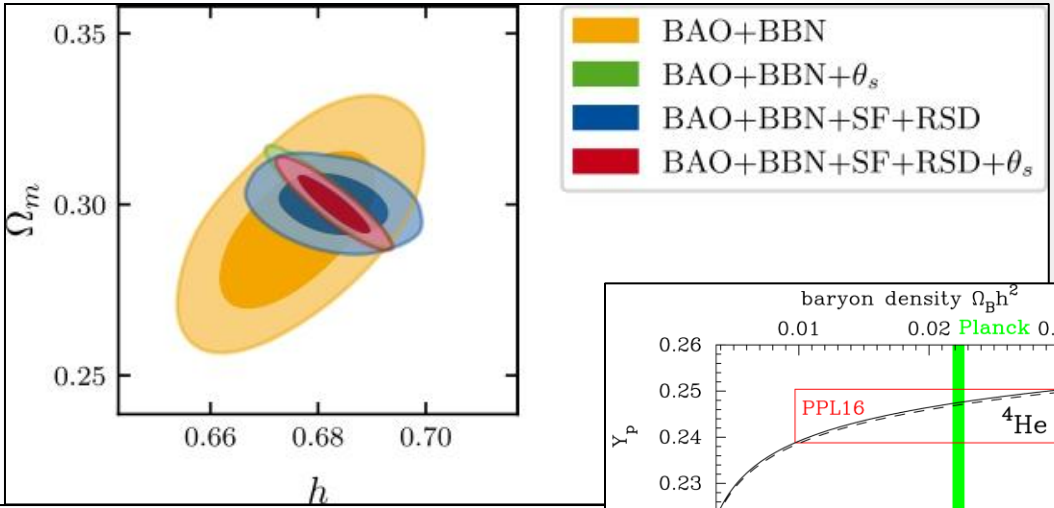
Cosmic Tensions: BBN

Data Sets	H_0 [km s^{-1} Mpc^{-1}]	$\Omega_{m,0}$
BAO (DR12)+BBN (noLUNA)	$68.36^{+1.13}_{-1.25}$	$0.302^{+0.018}_{-0.020}$
BAO+BBN (noLUNA)	$67.90^{+0.92}_{-1.03}$	$0.294^{+0.015}_{-0.016}$
BAO (DR12)+BBN	$68.14^{+1.13}_{-1.24}$	$0.302^{+0.017}_{-0.020}$
BAO+BBN	$67.64^{+0.97}_{-1.03}$	$0.293^{+0.015}_{-0.016}$

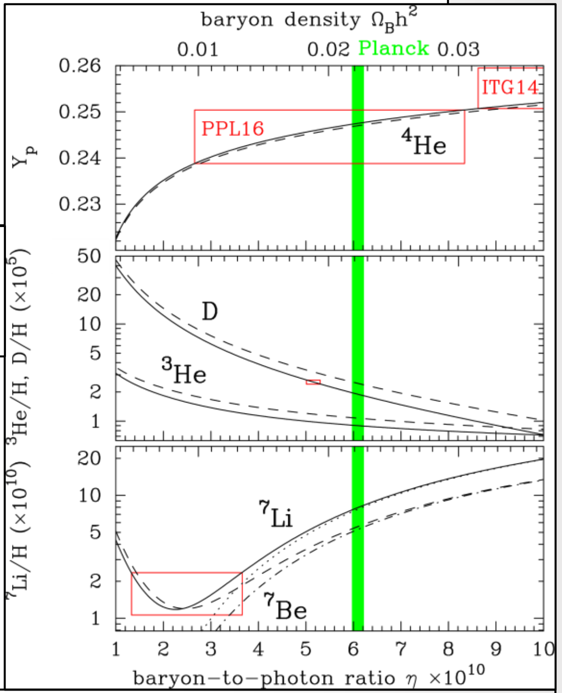
Schöneberg, N. et al
JCAP 11 (2022) 039





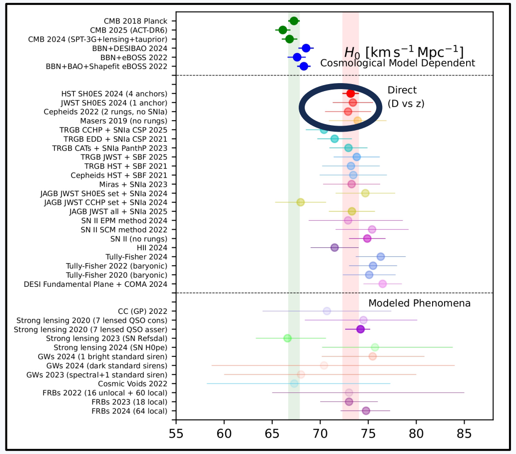


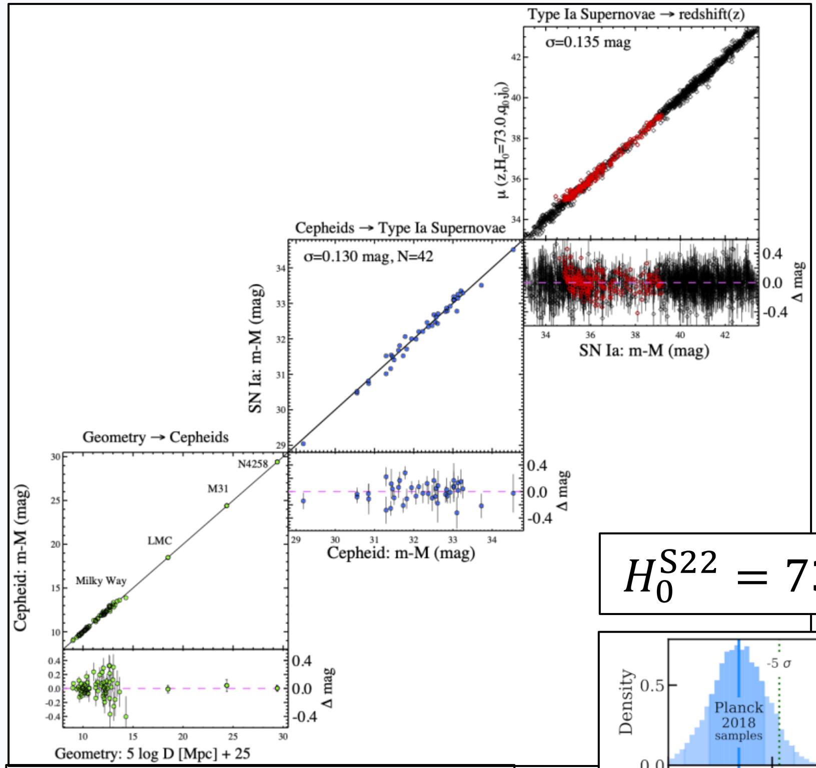
$$H_0^{\mathrm{BAO+BBN}+\theta_s} = 68.16^{+0.48}_{-0.49}\ \mathrm{km\ s^{-1} Mpc^{-1}}$$
$$\Omega_{\mathrm{m}}^{\mathrm{BAO+BBN}+\theta_s} = 0.3022^{+0.0062}_{-0.0064}$$



Sasankan, N. et al Phys. Rev. D 101 (2020) 123532

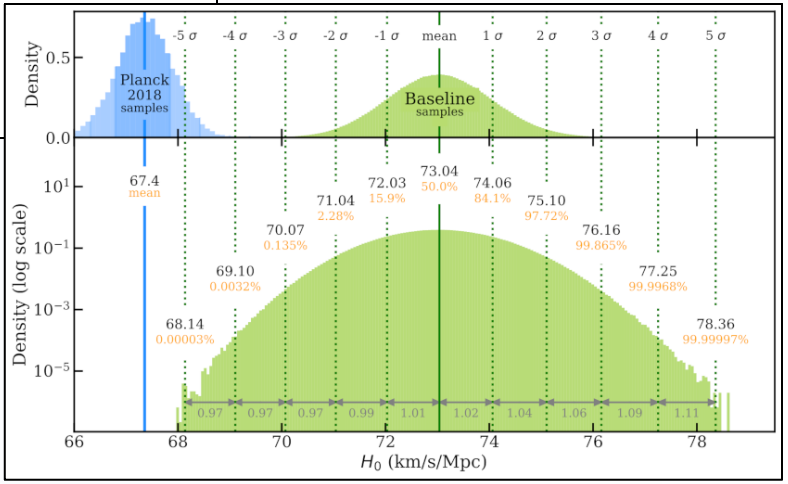
Cosmic Tensions: SH0ES Result



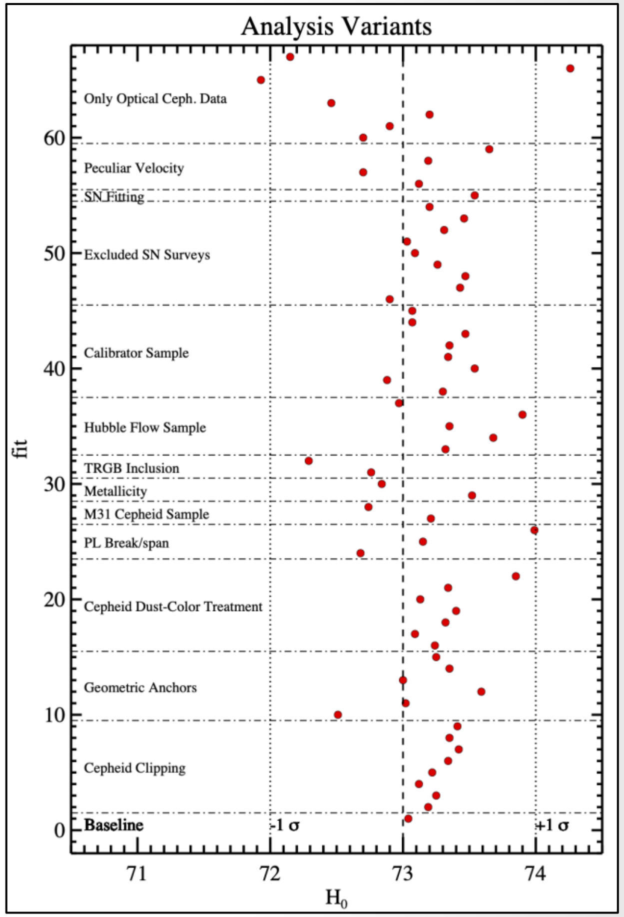


Riess, A. G. et al. ApJL 934 (2022) L7

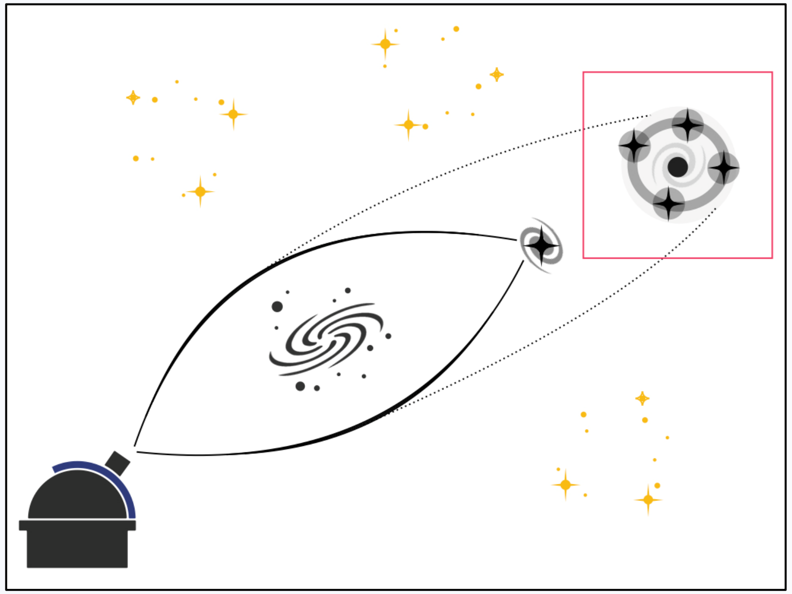
$$H_0^{S22} = 73.04 \pm 1.04 \text{ km s}^{-1}\text{Mpc}^{-1}$$



12 variants of analyses

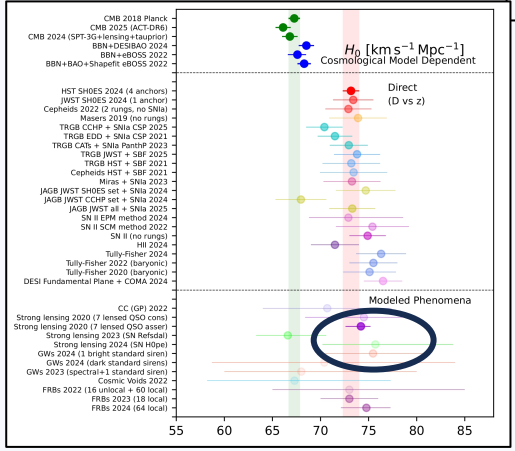


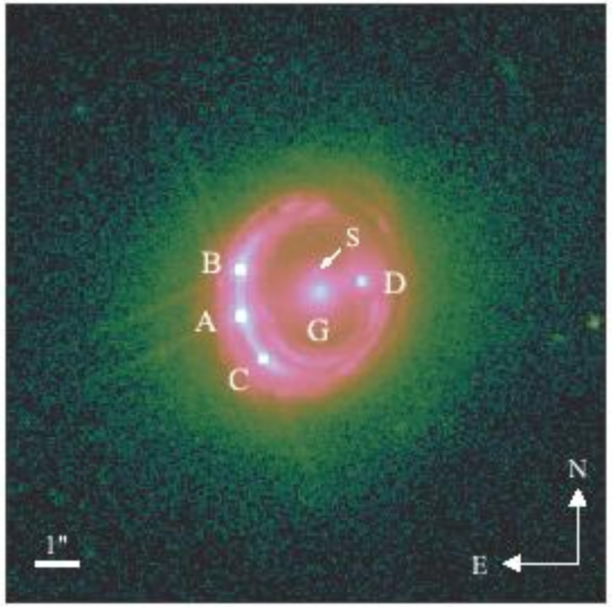
Cosmic Tensions: Strong Lensing



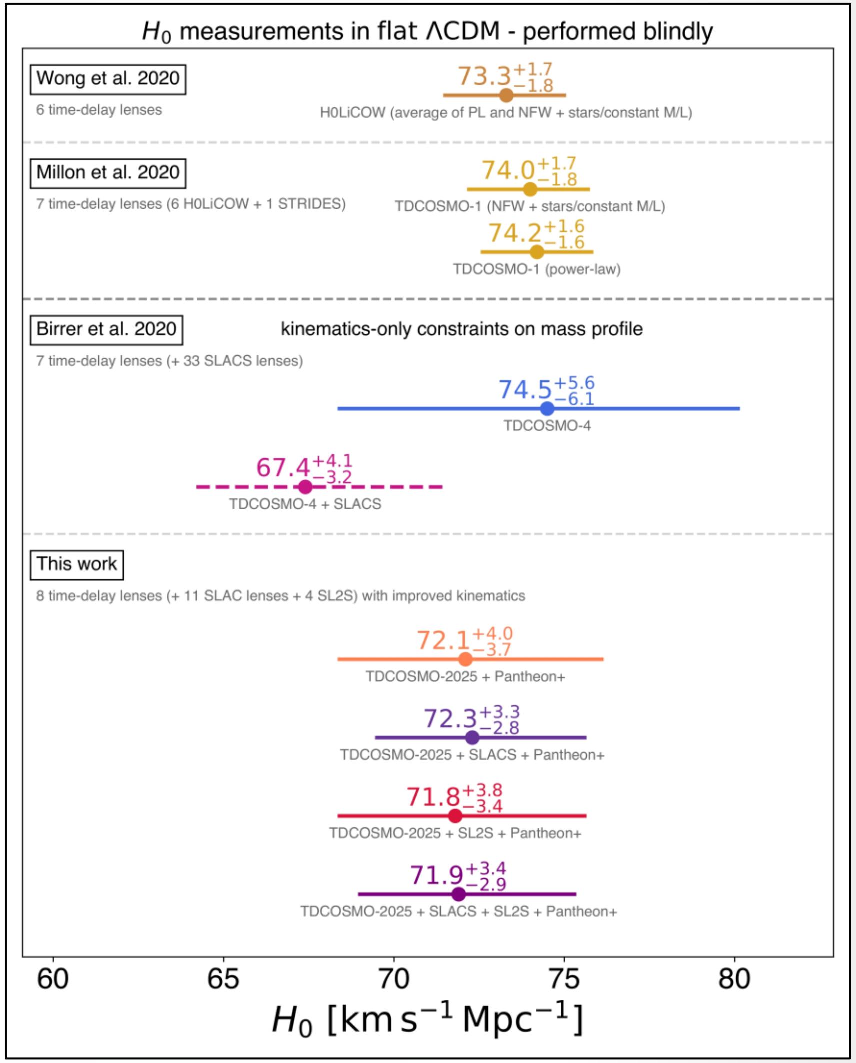
TDCosmo Collaboration

arXiv:2506.03023





RX J1131-1231 [Shajib, A. J., et al., A&A 673, A9 (2023)] (A, B, C, D – Lensed quasar, G – Lens, S – Disconnected satellite galaxy)



Cosmic Tensions in recent years

Uddin et al. (2023)
[Cepheide+TRGB+SBF+Sne Ia]

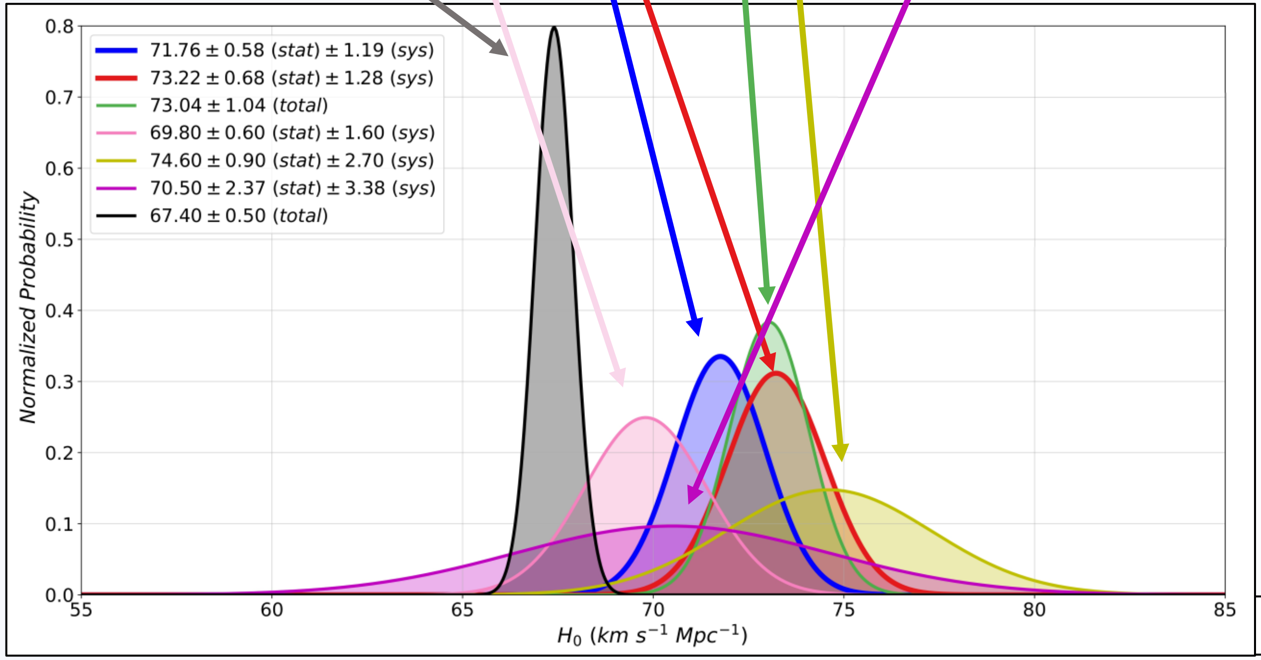
Riess et al. (2022)
[Cepheide+Sne Ia]

Freedman et al. (2021)
[TRGB]

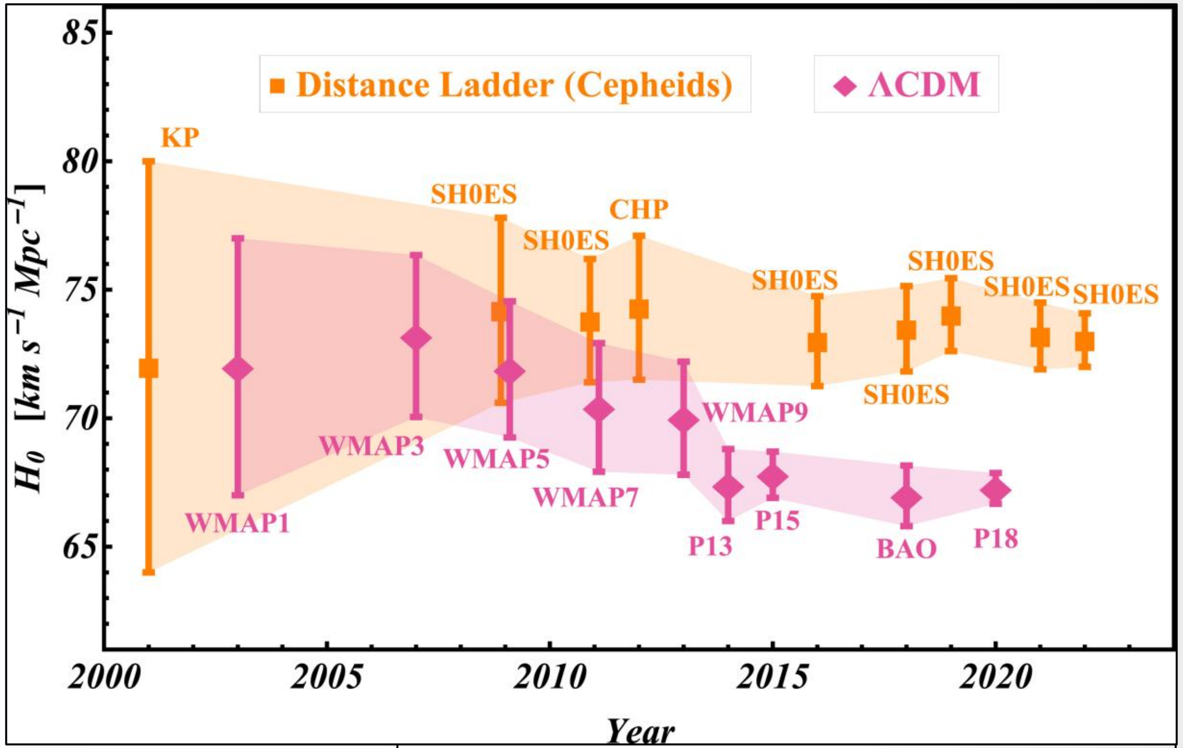
Garnavich et al. (2022)
[SBF+Sne Ia]

Planck Collaboration
legacy (2018) [CMB]

Khetan et al. (2021)
[SBF+Sne Ia]



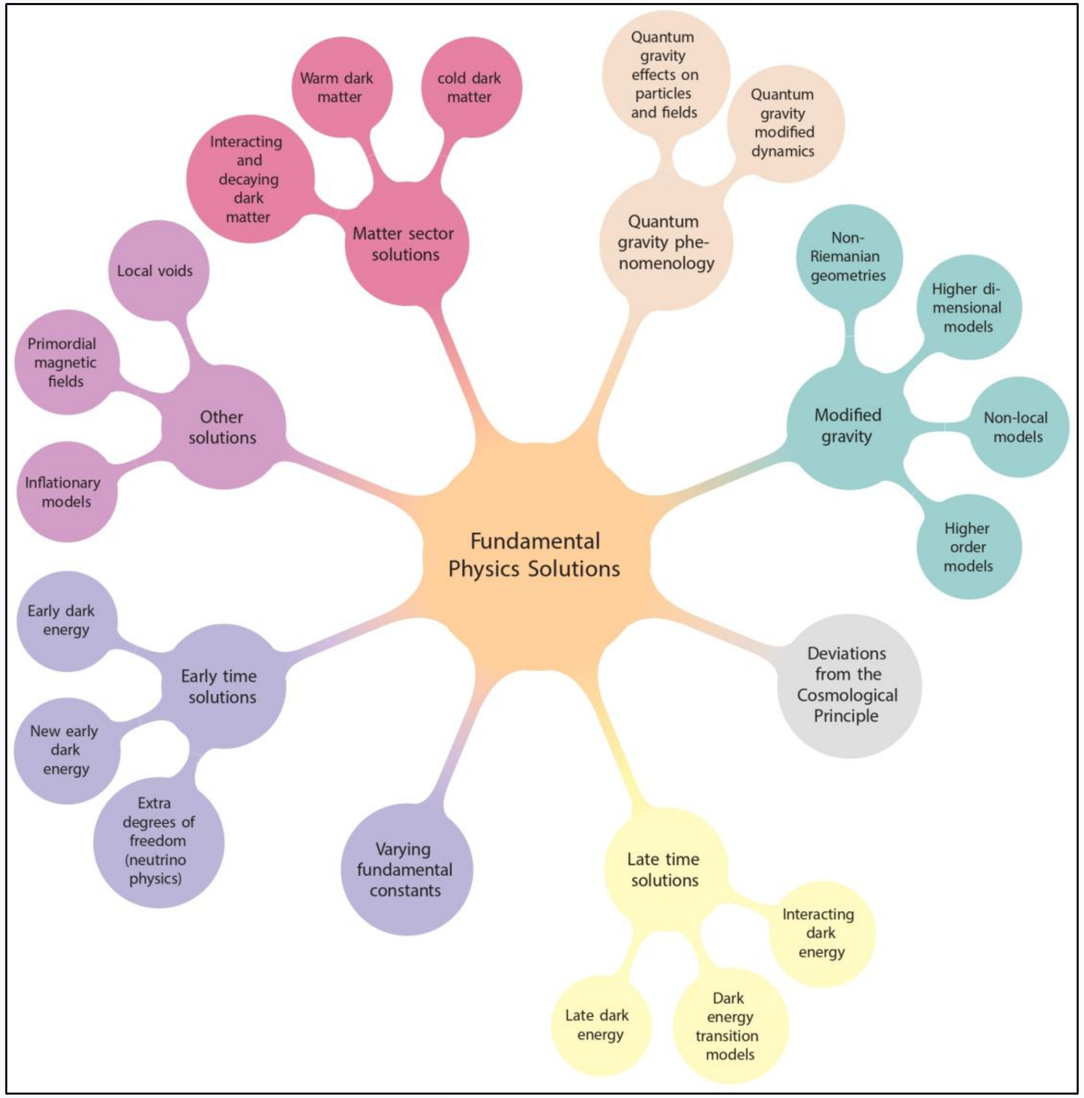
Uddin et al. Astrophys.J. 970 (2024) 1, 72

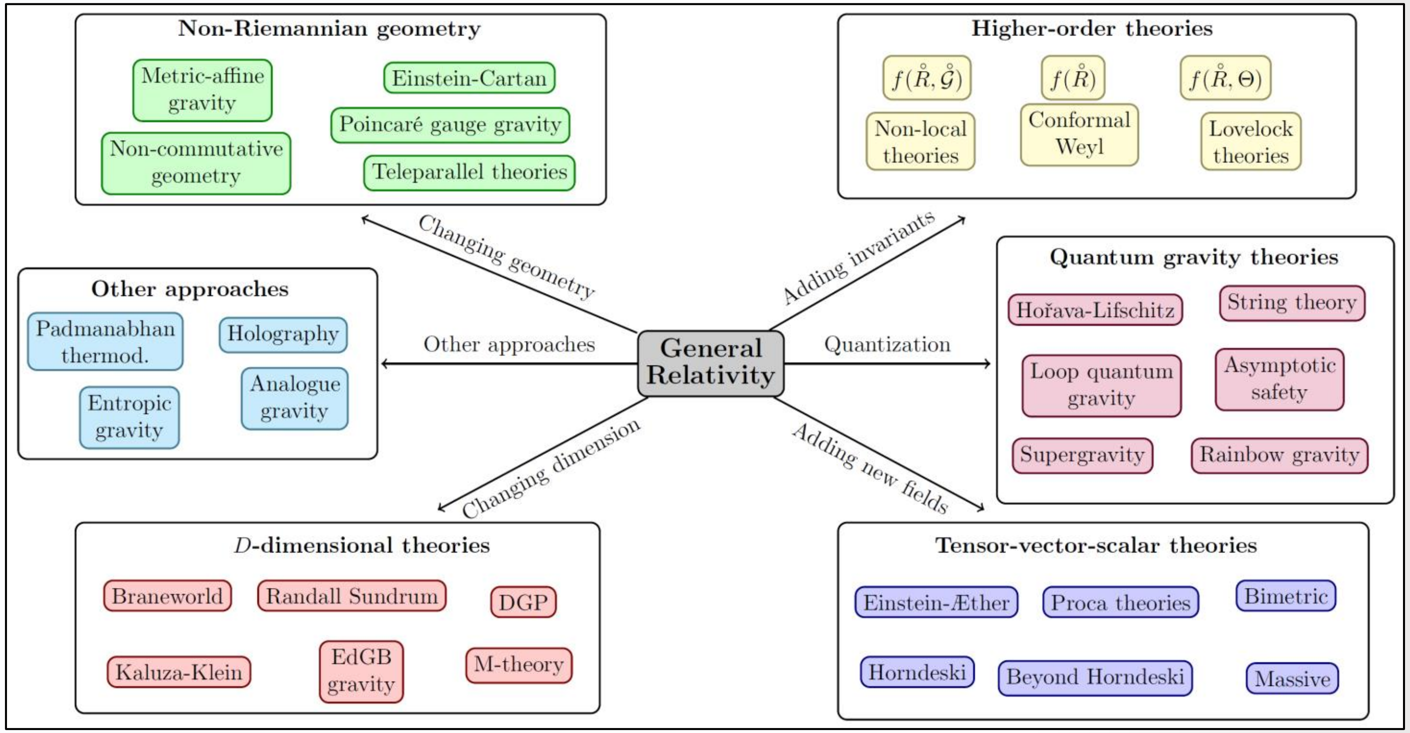


Perivolaropoulos, L.; Skara, F. Challenges for ΛCDM: An update. New Astron. Rev. 95 (2022) 101659.

What are possible solutions?

The Modified Cosmology Landscape





Modified Cosmology through Lovelock's Theorem

Lovelock's Theorem

For **second order** vacuum field equations $E_{\mu\nu} = 0$, if $E_{\mu\nu}$ is a function of the metric $g_{\mu\nu}$, then

$$E_{\mu\nu} \coloneqq G_{\mu\nu} + \Lambda g_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} + \Lambda g_{\mu\nu}$$

- Adding new fields (scalar, vector, tensor)
- Use more/less than four dimensions of spacetime
- Take an emergent form of gravity
- Changing the gravitational connection
- Consider non-local terms
- Adding more than second-order derivatives of the metric

Attempts at a solution

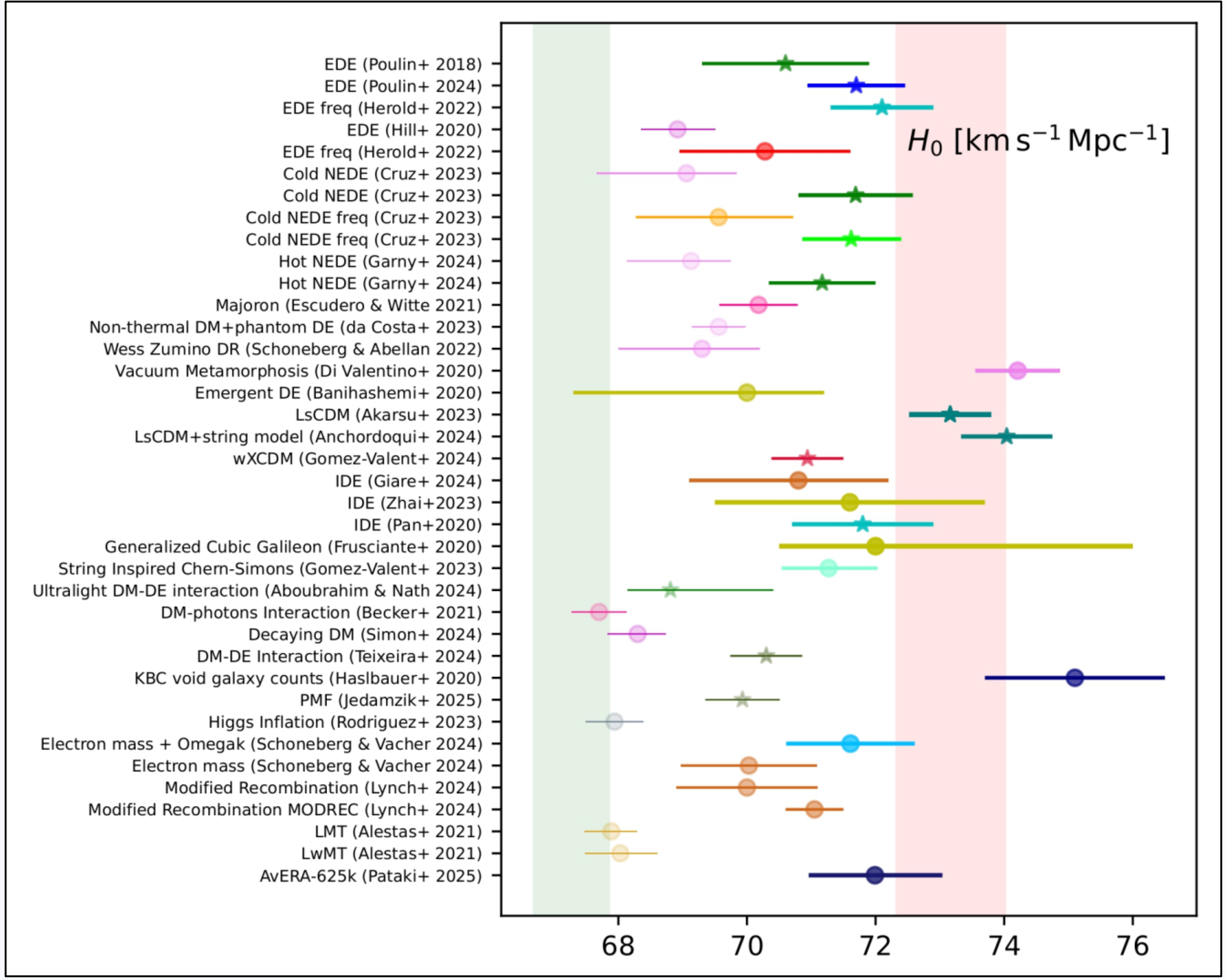
Model	$\Delta N_{\rm param}$	M_B	Gaussian Tension	$Q_{\rm DMAP}$ Tension		$\Delta\chi^2$	ΔAIC		Finalist
ΛCDM	0	-19.416 ± 0.012	4.4σ	4.5σ	✗	0.00	0.00	✗	✗
$\Delta N_{\rm ur}$	1	-19.395 ± 0.019	3.6σ	3.8σ	✗	-6.10	-4.10	✗	✗
SIDR	1	-19.385 ± 0.024	3.2σ	3.3σ	✗	-9.57	-7.57	✓	✓ (3)
mixed DR	2	-19.413 ± 0.036	3.3σ	3.4σ	✗	-8.83	-4.83	✗	✗
DR-DM	2	-19.388 ± 0.026	3.2σ	3.1σ	✗	-8.92	-4.92	✗	✗
SIν+DR	3	$-19.440^{+0.037}_{-0.039}$	3.8σ	3.9σ	✗	-4.98	1.02	✗	✗
Majoron	3	$-19.380^{+0.027}_{-0.021}$	3.0σ	2.9σ	✓	-15.49	-9.49	✓	✓ (2)
primordial B	1	$-19.390^{+0.018}_{-0.024}$	3.5σ	3.5σ	✗	-11.42	-9.42	✓	✓ (3)
varying m_e	1	-19.391 ± 0.034	2.9σ	2.9σ	✓	-12.27	-10.27	✓	✓ (1)
varying m_e+Ω_k	2	-19.368 ± 0.048	2.0σ	1.9σ	✓	-17.26	-13.26	✓	✓ (1)
EDE	3	$-19.390^{+0.016}_{-0.035}$	3.6σ	1.6σ	✓	-21.98	-15.98	✓	✓ (2)
NEDE	3	$-19.380^{+0.023}_{-0.040}$	3.1σ	1.9σ	✓	-18.93	-12.93	✓	✓ (2)
EMG	3	$-19.397^{+0.017}_{-0.023}$	3.7σ	2.3σ	✓	-18.56	-12.56	✓	✓ (2)
CPL	2	-19.400 ± 0.020	3.7σ	4.1σ	✗	-4.94	-0.94	✗	✗
PEDE	0	-19.349 ± 0.013	2.7σ	2.8σ	✓	2.24	2.24	✗	✗
GPEDE	1	-19.400 ± 0.022	3.6σ	4.6σ	✗	-0.45	1.55	✗	✗
DM $\to$ DR+WDM	2	-19.420 ± 0.012	4.5σ	4.5σ	✗	-0.19	3.81	✗	✗
DM $\to$ DR	2	-19.410 ± 0.011	4.3σ	4.5σ	✗	-0.53	3.47	✗	✗

The H_0 Olympics:

1. What tension does a model have with the SH0ES result using a baseline Planck 2018 + BAO + Pantheon best fit?
2. How does the inclusion of the SH0ES measurement impact this fit?
3. Does this inclusion make the best fit better than ΛCDM or worse?

Schöneberg, N. et al. Phys. Rept., 984 (2022) 1

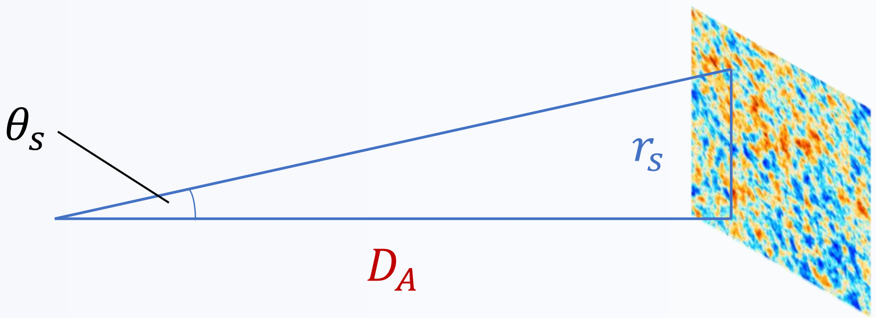
Status of Potential Solutions



How do mature cosmological models perform in the context of the Hubble tension?

CosmoVerse White Paper. arXiv:2504.01669

Early vs local measurement approaches



$$\theta_s = \frac{r_s(z_{\rm LS})}{D_A(z_{\rm LS})} = \frac{\int_{z_{\rm LS}}^{\infty} c_s(z,\rho_b) H^{-1}(z')dz'}{\int_0^{z_{\rm LS}} H^{-1}(z')dz'}$$

Early-Universe new physics ($\boldsymbol{r_s}$)

- Considering the angular size of the sound horizon

$$\theta_s \sim \frac{r_s}{1/H(z_{\rm late})} \sim r_s H_0$$

By decreasing $\boldsymbol{r_s}$, we can increase $\boldsymbol{H_0}$, or so one would expect

Late-Universe new physics ($\boldsymbol{D_A}$)

- Keep early Hubble evolution unchanged and modify late-time evolution of $H(z)$

This is very difficult to do provided BAO, SnIa and CC data

Late-Universe new physics

Possible late-Universe solutions with new physics (that give high H_0 values with CMB):

- Graduated Dark Energy Akarsu, Ö., Barrow, J. D., Escamilla, L. A., and Vazquez, J. A. 2020
- Late-time interacting dark sector Gariazzo, S., Di Valentino, E., Mena, O., and Nunes, R. C. 2022
- Decaying dark matter Vattis, K., Koushiappas, S. M., Loeb, A 2020
- Decaying dark energy Li, X., Shafieloo, A., Sahni, V., and Starobinsky, A. A. 2019
- Negative dark energy density Poulin V., Boddy, K. K., Bird, S., and Kamionkowski, M 2018
- Phenomenologically Emergent Dark Energy Li, X., and Shafieloo, A. 2020
- Running vacuum models Sola J., Gomez-Valent, A., and de Cruz Perez, J. 2017

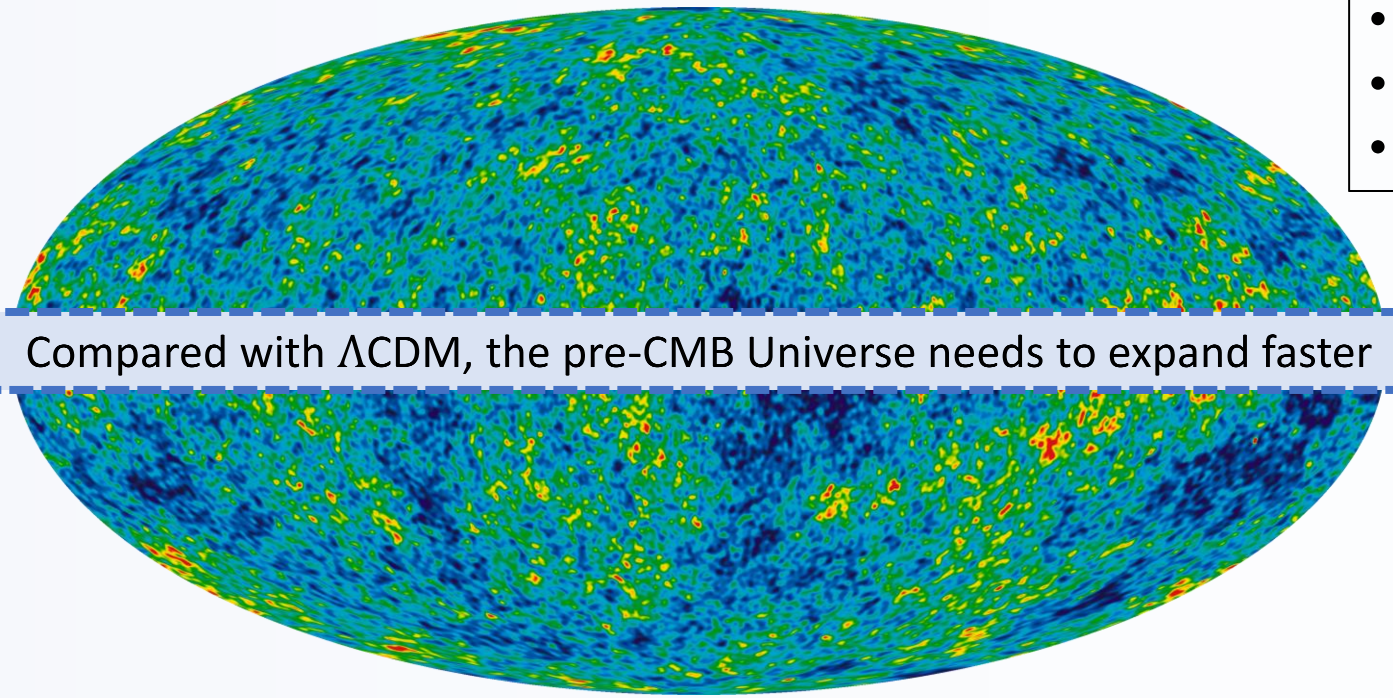
BAO constrain $\theta_s \sim r_s H_0$, anchoring r_s (early Universe) leaves few options for inferring H_0

Early-Universe new physics

Early-Universe physics concept:

- Fix θ_s (CMB peaks unchanged) so that $r_s \sim 1/H_0$
- Lower r_s which will increase pre-CMB expansion rate
- Do not change $D_A \propto 1/H_{\mathrm{Late}}(z)$, so modifications in the late Universe are not needed

- Recombination takes place sooner
- Sound waves travel a shorter distance (small r_s)
- The early Universe cools faster



Compared with ΛCDM, the pre-CMB Universe needs to expand faster

Early Universe Dark Energy (EDE)

- **Motivation**: Decrease the sound horizon by an early Universe dark component that is active up to roughly matter-radiation equality

- EDE continuity equation implies energy evolution

$$\rho_{\rm EDE}(a) = \rho_{\rm EDE,0}\, e^{3\int_a^1 [1+w_{\rm EDE}(a)]da/a}$$

  This defines the **EDE density parameter** $f_{\rm EDE} = \rho_{\rm EDE}/\rho_{\rm Crit}$

- This can be parametrized through the EoS

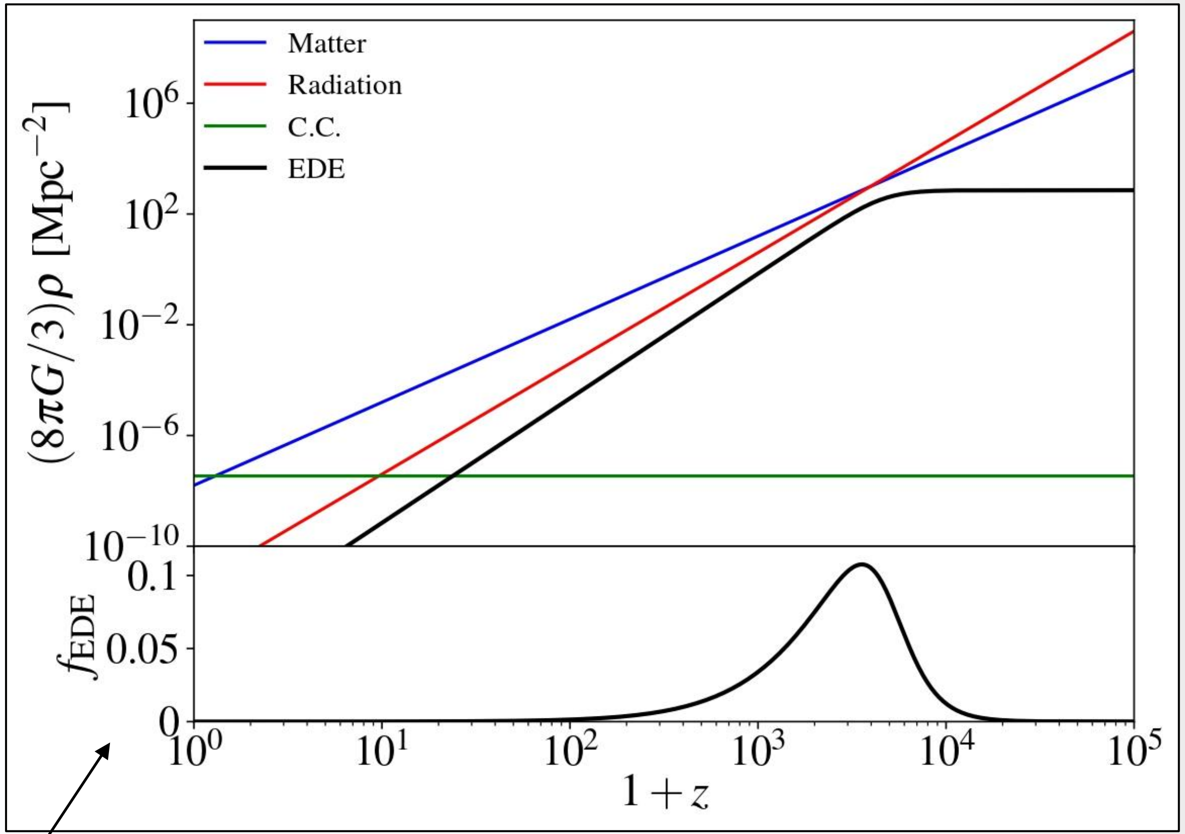
$$w_{\rm EDE}(a) = \frac{1+w_f}{1+(a_c/a)^{3(1+w_f)}} - 1$$

- The **critical scale factor** sets the scale for EDE:

  $a \ll a_c \ \rightarrow$ cosmic expansion with $w_{\rm EDE} \rightarrow -1$

  $a \gg a_c \ \rightarrow$ Dilutes as $a^{-3(1+w_f)}$

  Example: $V(\phi) = \phi^{2n} \Rightarrow w_f = (n-1)/(n+1)$



Representative example: $f_{\rm EDE,max} = 0.1$ at $z_c \simeq 3500$ ($w_{\rm EDE} \rightarrow 1/2$ afterwards)

Poulin, V., Smith, T. L., and Karwal, T. arXiv:2302.09032

EDE Models

- **Axion-like EDE (axEDE):**

$$V = m^2 f^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]^n$$

- **Rock ‘n Roll EDE (RnR EDE):**

$$V = V_0 \left(\frac{\phi}{M_{\rm Pl}}\right)^{2n} + V_\Lambda$$

- **Acoustic EDE (ADE):**

$$1 + w_{\rm ADE} = \frac{1 + w_f}{\left[1 + (a_c/a)^{3(1+w_f)/p}\right]^p}$$

- **New EDE (NEDE):**

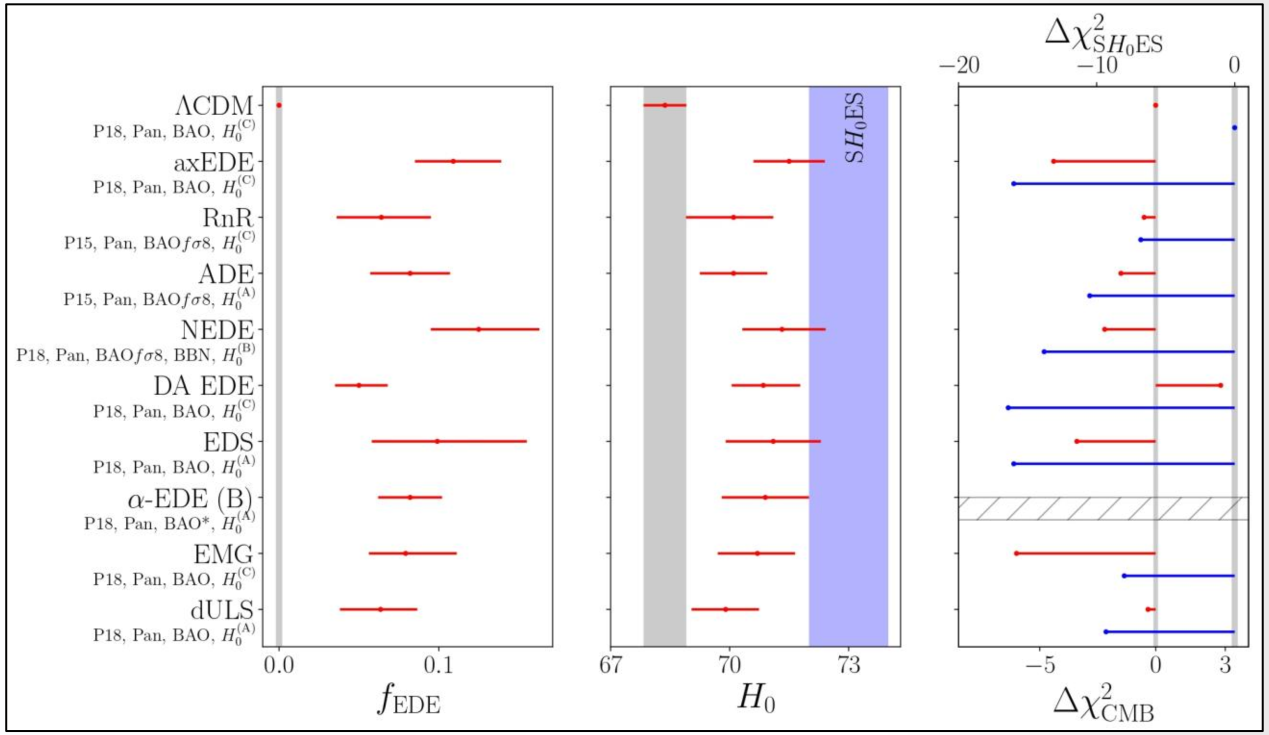
$$V(\psi, \phi) = \frac{\lambda}{4}\psi^4 + \frac{1}{2}\beta M^2 \psi^2 - \frac{1}{3}\alpha M \psi^3 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2}\gamma \phi^2 \psi^2$$

- **EDE coupled to DM (EDS):**

$$V(\phi, a) = V(\phi) + \rho_{\rm DM}(a)$$

- **α −attractors EDE (α −EDE):**

$$V = \Lambda + V_0 \frac{(1+\beta)^{2n} \tanh\left(\phi/\sqrt{6\alpha}M_{\rm Pl}\right)^{2p}}{\left[1 + \beta \tanh\left(\phi/\sqrt{6\alpha}M_{\rm Pl}\right)\right]^{2n}}$$



Klein-Gordon equation of motion:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0$$

$\Delta\chi^2_{\rm SH0ES}$
$\Delta\chi^2_{\rm CMB}$

Evolution of EDE

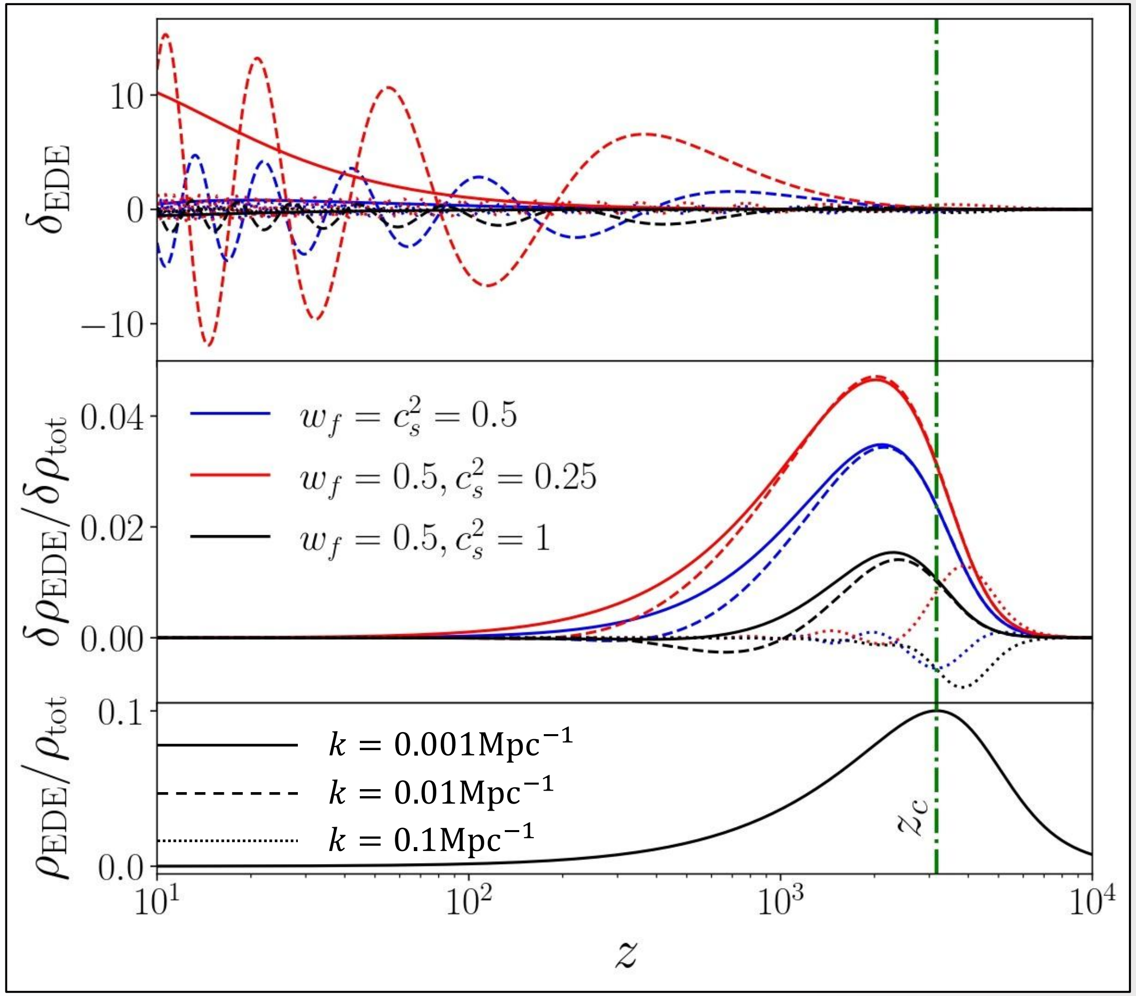
On **subhorizon scales**, the fluid equation takes the form

$$\frac{d^2}{d\eta^2}\left(\frac{\delta_{\rm EDE}}{1+w_{\rm EDE}}\right)$$
$$= -k^2\left(c_s^2\,\frac{\delta_{\rm EDE}}{1+w_{\rm EDE}}+\psi_{\rm N}\right)-\left(1-3c_a^2\right)\frac{a'}{a}\frac{d}{d\eta}\left(\frac{\delta_{\rm EDE}}{1+w_{\rm EDE}}\right)$$

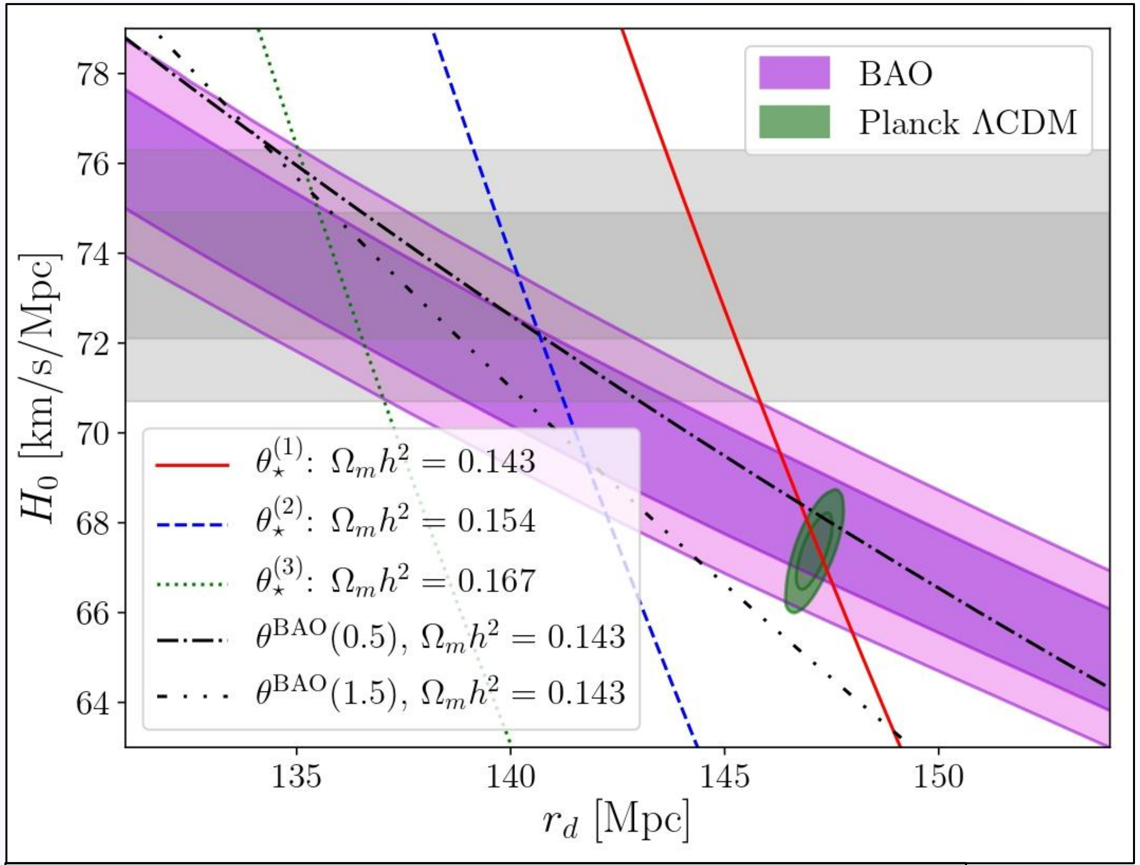
for the fractional EDE perturbation ($\delta_{\rm EDE}$), effective EDE sound speed (c_s^2), Newtonian potential ($\psi_{\rm N}$) and adiabatic EDE sound speed (c_a^2)

General features:

- Larger c_s^2 translates to more resistance in EDE collapse, while smaller c_s^2 give larger overall density perturbations. This sets the frequency of the oscillations
- The sign of $\left(1-3c_a^2\right)$ controls where the amplitude increases $(+)$ or decreases $(-)$
- EDE modes are counteracted by pressure within the horizon, with stable modes only entering the horizon at $w_{\rm EDE} \simeq -1$



The Problem with EDE



Jedamzik, K., Pogosian, L. and Zhao, G. B. Commun. Phys. 4 (2021) 123

CMB angular size at recombination:

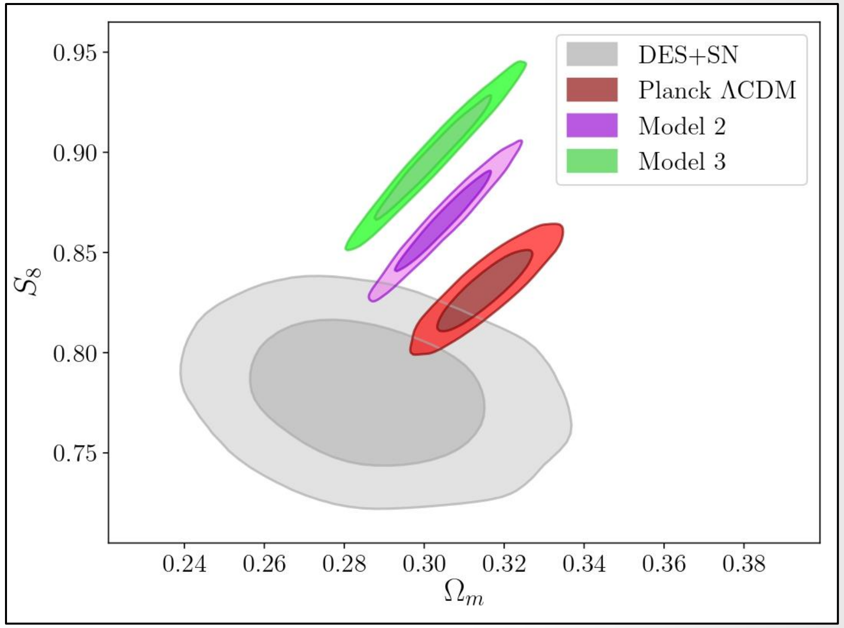
$$\theta_* = \frac{r_s(z_{\rm LS})}{D(z_{\rm LS})}$$

Transverse BAO angular scale:

$$\theta^{\rm BAO}(z_{\rm Obs}) = \frac{r_{\rm d}}{D(z_{\rm Obs})}$$

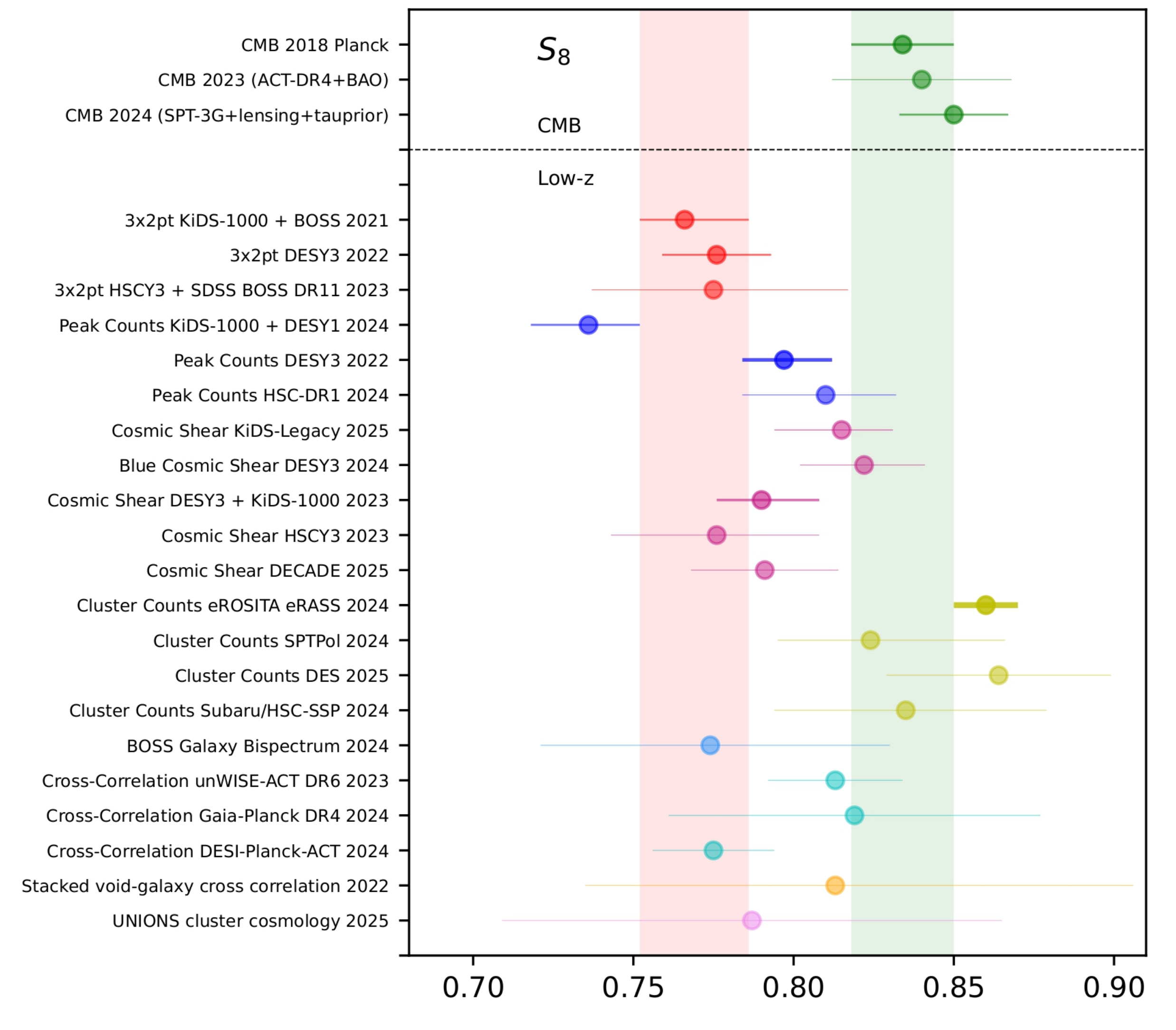
Model 2:
Fits BAO and CMB peaks at $\Omega_{\rm m}h^2 = 0.155$

Model 3:
Fits BAO, CMB peaks and SH0ES result at $\Omega_{\rm m}h^2 = 0.167$



What about other tensions on the rise?

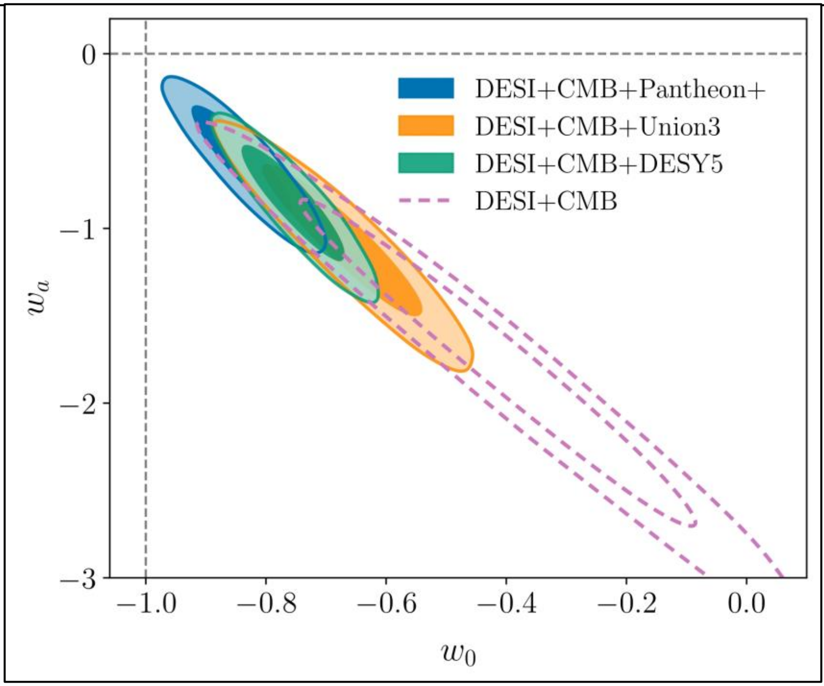
S_8 Tension



CosmoVerse White Paper. arXiv:2504.01669

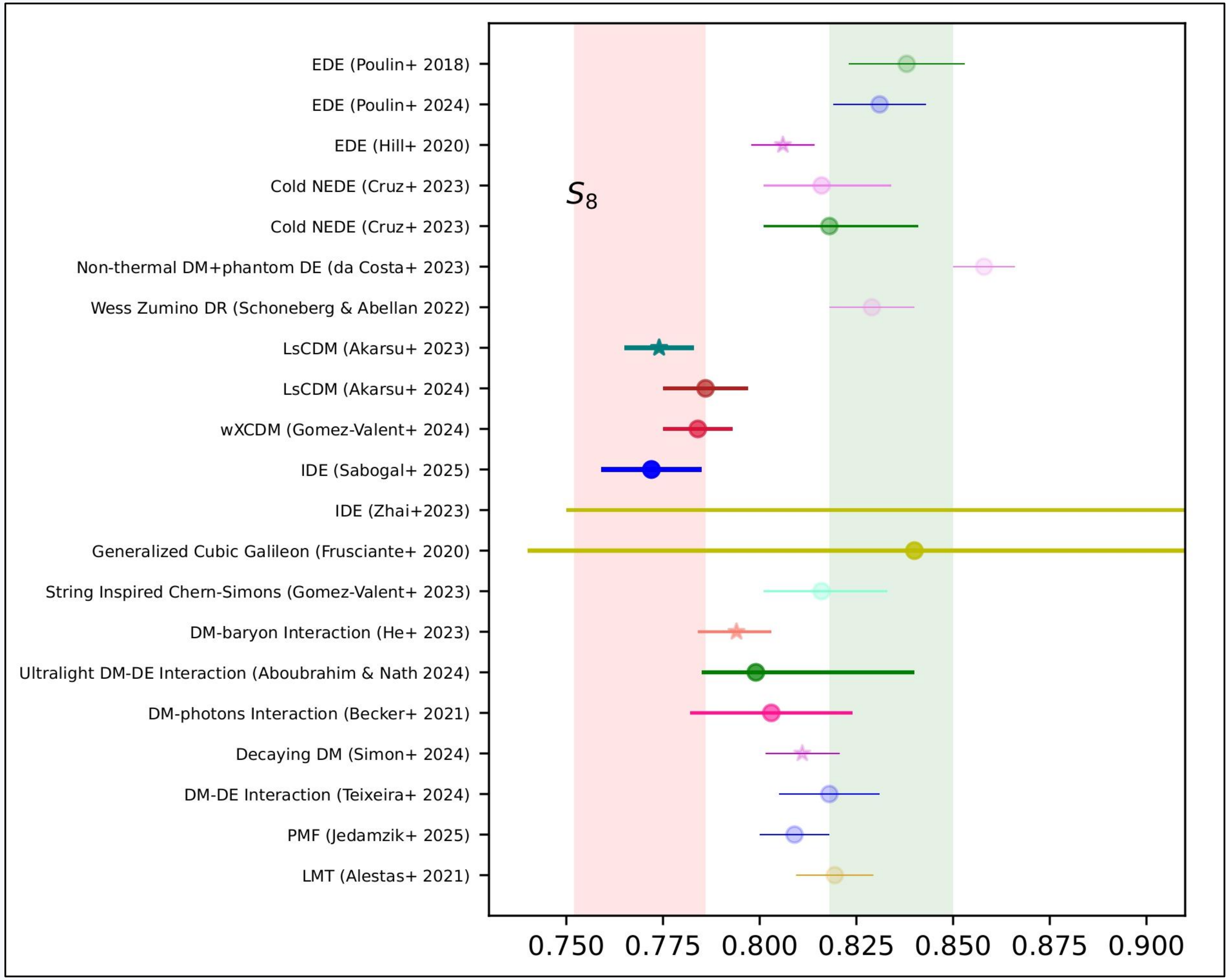
Large scale structure is well represented by S_8 which combines the matter density and matter density fluctuations on the scale of $8\ h^{-1}\mathrm{Mpc}$

$$S_{8,0} = \sigma_{8,0}\sqrt{\frac{\Omega_{\mathrm{m},0}}{0.3}}$$

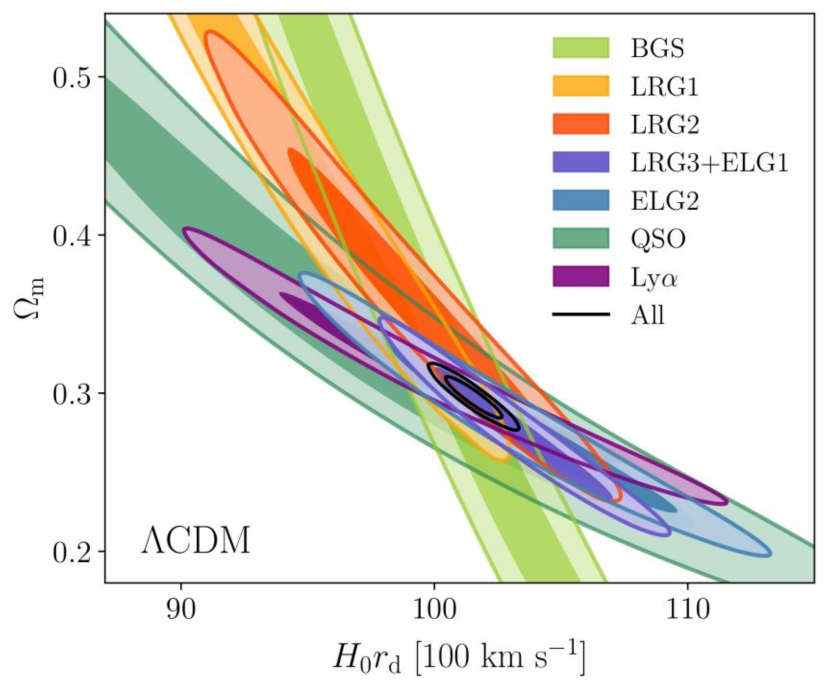


DESI Collaboration, arXiv:2503.14738 (2025)

Status of Potential Solutions



How do mature cosmological models perform in the context of the Hubble tension?

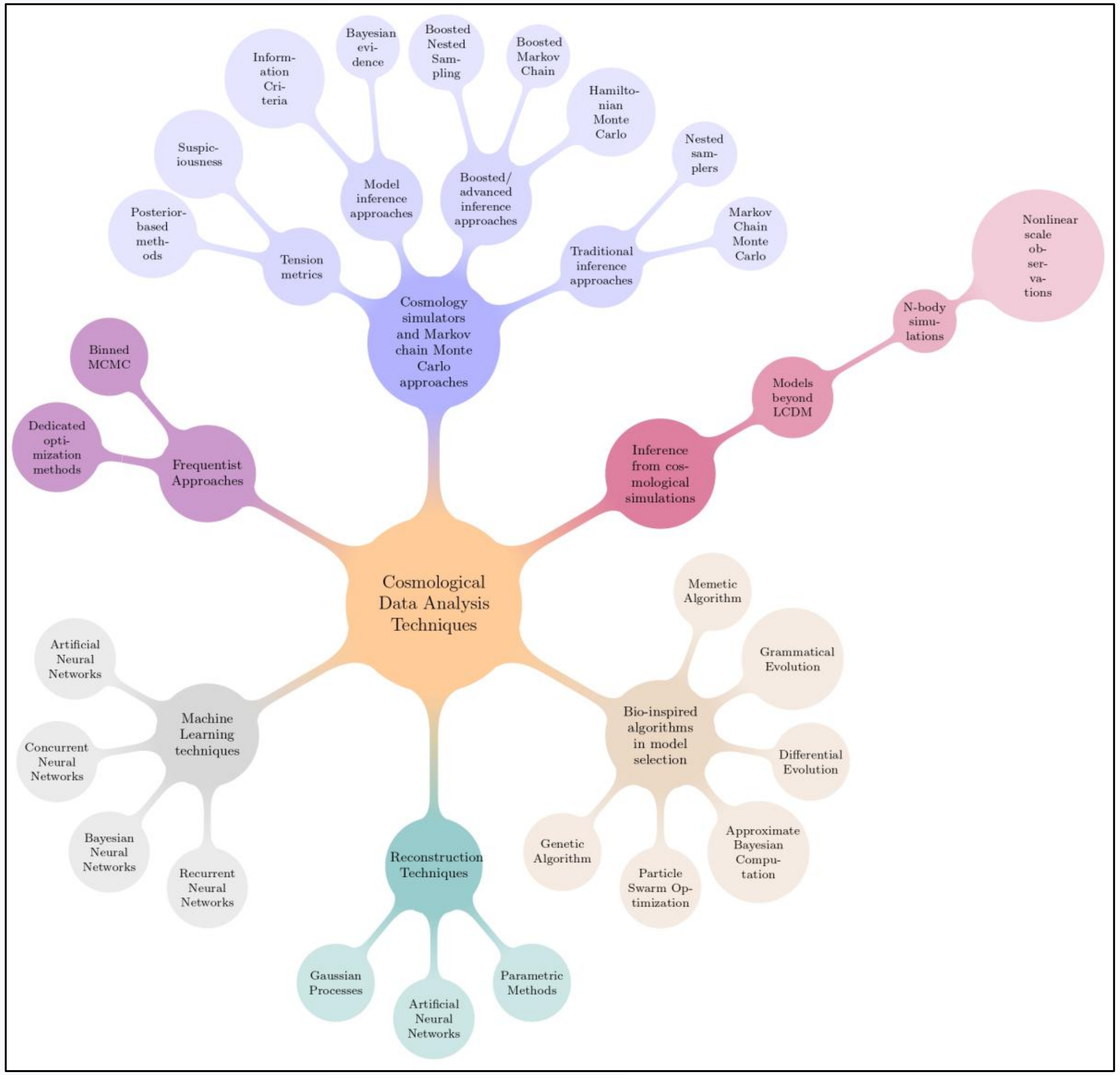


DESI Collaboration, arXiv:2503.14738 (2025)

CosmoVerse White Paper. arXiv:2504.01669

How can machine learning help?

Data Analysis Techniques



Gaussian Processes Regression

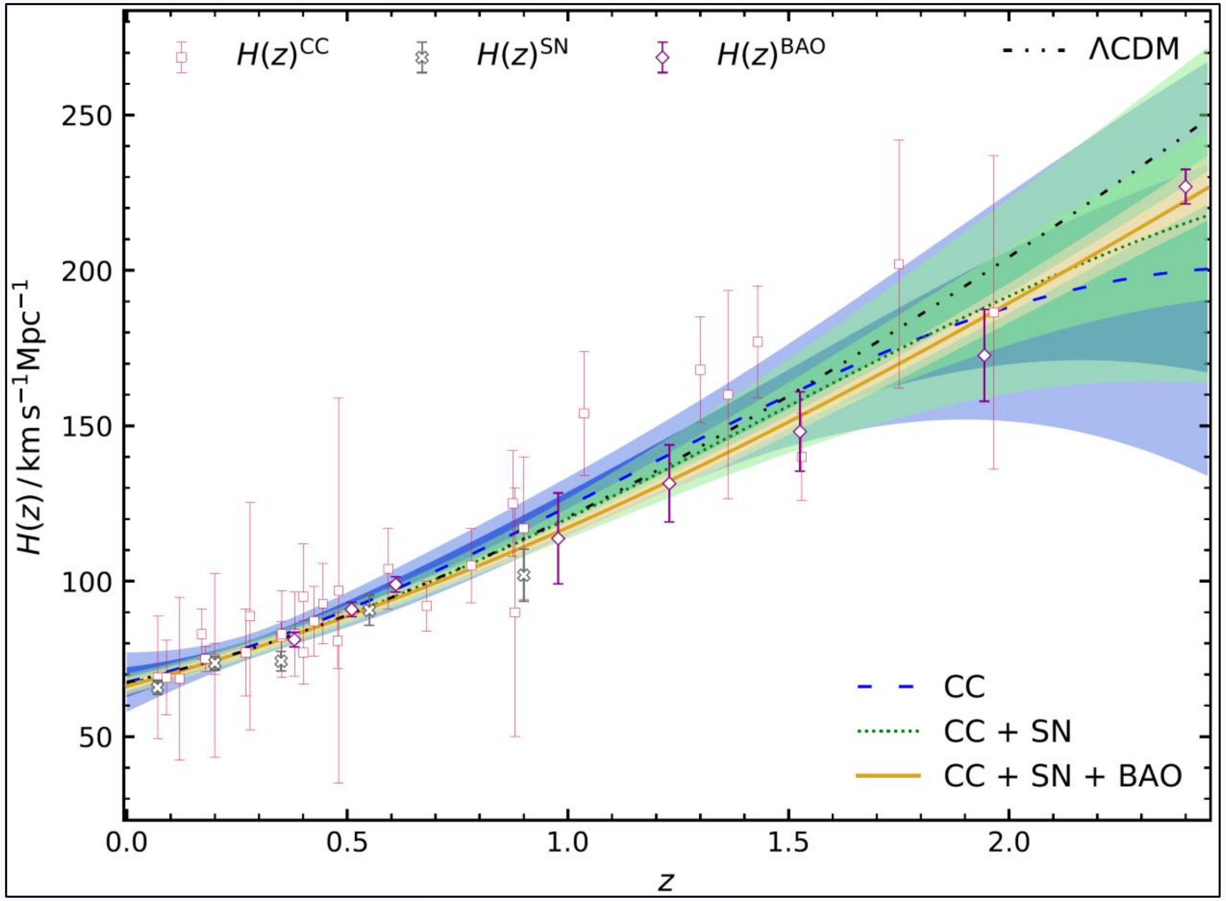
- The covariance function contains **non-physical hyperparameters** θ which define the distribution $k(\theta, x, x')$
- Iterating over these values using **Bayesian inference** (or others) can produce better hyperparameters
- The result is a (physics) **model independent reconstruction** of the behavior of some parameter
- This is superior to regular fitting because it is nonparametric and so **assumes no physical model** whatsoever

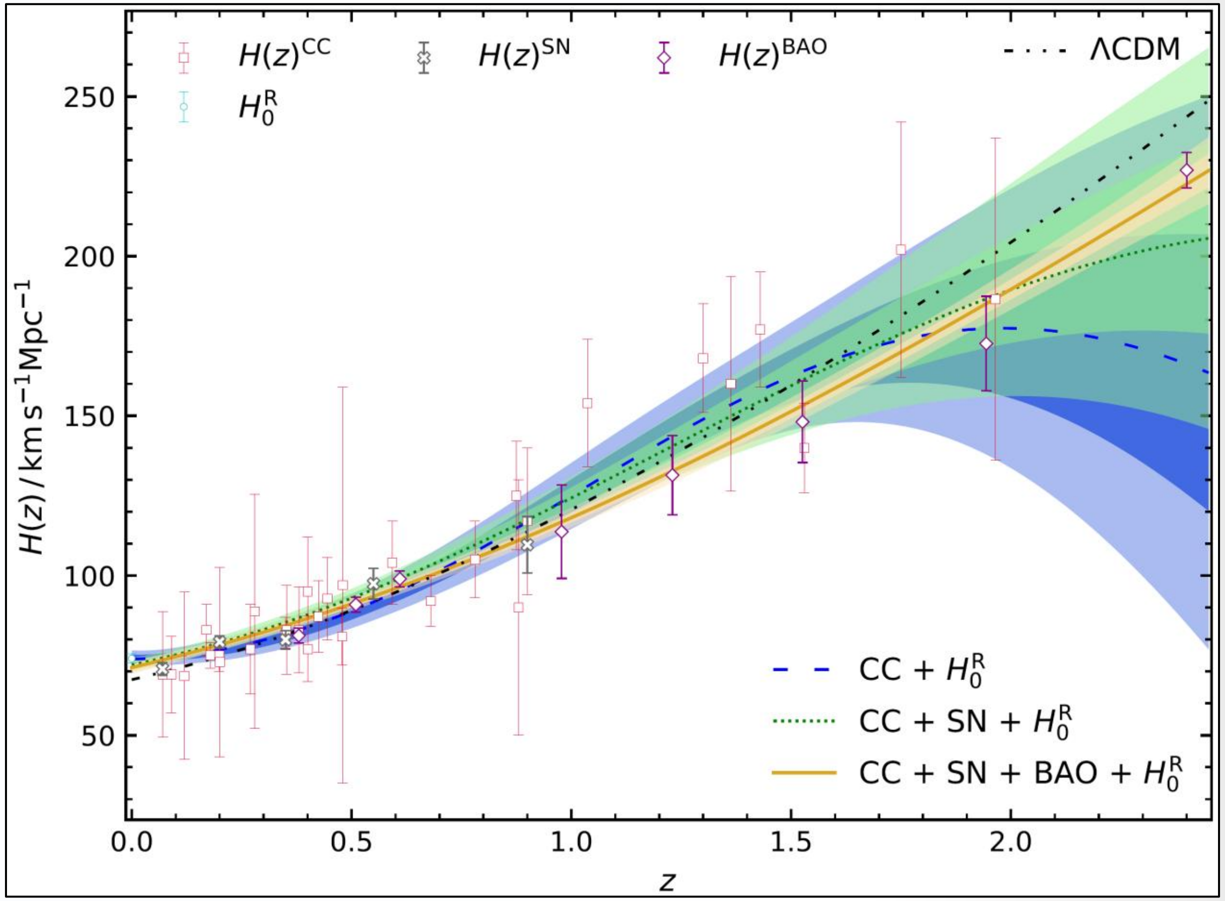
The Covariance Functions

Squared Exponential (Gaussian or RBF):

$$k(x, x') = \sigma_f^2 \, \mathrm{Exp}\left[-\frac{1}{2}\left(\frac{x - x'}{l_f}\right)^2\right]$$

Square Exponential H_0 GP



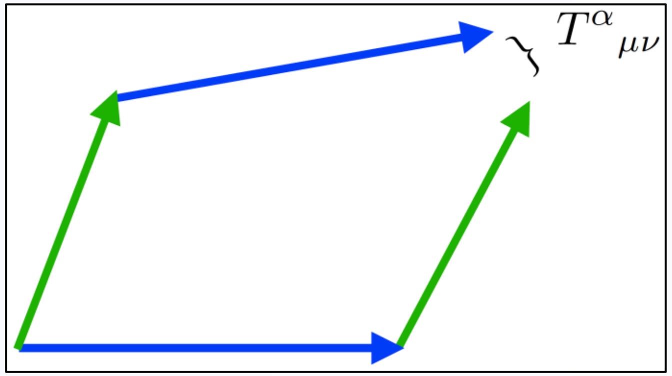


$H_0 = 67.539 \pm 4.772 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 67.001 \pm 1.653 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 66.197 \pm 1.464 \mathrm{km\ s^{-1} Mpc^{-1}}$

$H_0 = 73.782 \pm 1.374 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 72.022 \pm 1.076 \mathrm{km\ s^{-1} Mpc^{-1}}$
$H_0 = 71.180 \pm 1.025 \mathrm{km\ s^{-1} Mpc^{-1}}$

Toy Model: $f(T)$ Teleparallel Gravity

- Tetrad ($e^a{}_\mu$): Relate the tangent space ($g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$)
- Use the **teleparallel connection** ($\Gamma^\sigma_{\mu\nu} = e_a{}^\sigma \partial_\nu e^a{}_\mu + e_a{}^\sigma \omega^a{}_{\nu\mu}$) instead of the **Levi-Civita connection gives** $\mathcal{R} = -T + B$
- ***f*(*T*) Gravity:** $S = \frac{1}{16\pi G} \int d^4x\, e[-T + f(T)] + S_{\text{mat}}$
- Taking a flat (**FLRW**) cosmology: $g_{\mu\nu} = \text{diag}\left(-1, a(t)^2, a(t)^2, a(t)^2\right)$
- **Friedmann equations**:



$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3} f_T$$

$$\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 - f_T - 2T f_{TT}}$$

$$T = 6H^2 = 6\left(\frac{\dot{a}}{a}\right)^2$$

Bahamonde et al. RoPP 86 (2023) 026901

Boundary Conditions

ΛCDM (or $f(T) = \Lambda$) at works at late cosmological times

This implies that

$$f_T(z \simeq 0) \simeq 0$$

$$\Rightarrow f(z \simeq 0) = 6H_0^2\left(\Omega_{m_0} - 1\right)$$

Briffa et al. CQG 38 055007 (2020)

$$S = \frac{1}{16\pi G}\int d^4x\, e[-T + \boldsymbol{f(T)}] + S_{\text{matter}}$$

Propagating $f\big(T(z)\big)$

- The Friedmann equation contains f_T which **need to be eliminated finite difference** methods
- Using a **central differencing** approach (error $\sim \mathcal{O}\big(\Delta z^2\big)$), we can assume

$$f'(z_i) \simeq \frac{f(z_{i+1}) - f(z_{i-1})}{z_{i+1} - z_{i-1}}$$

- Therefore, we can remove the $f_T(T) = f'(z)/T'(z)$

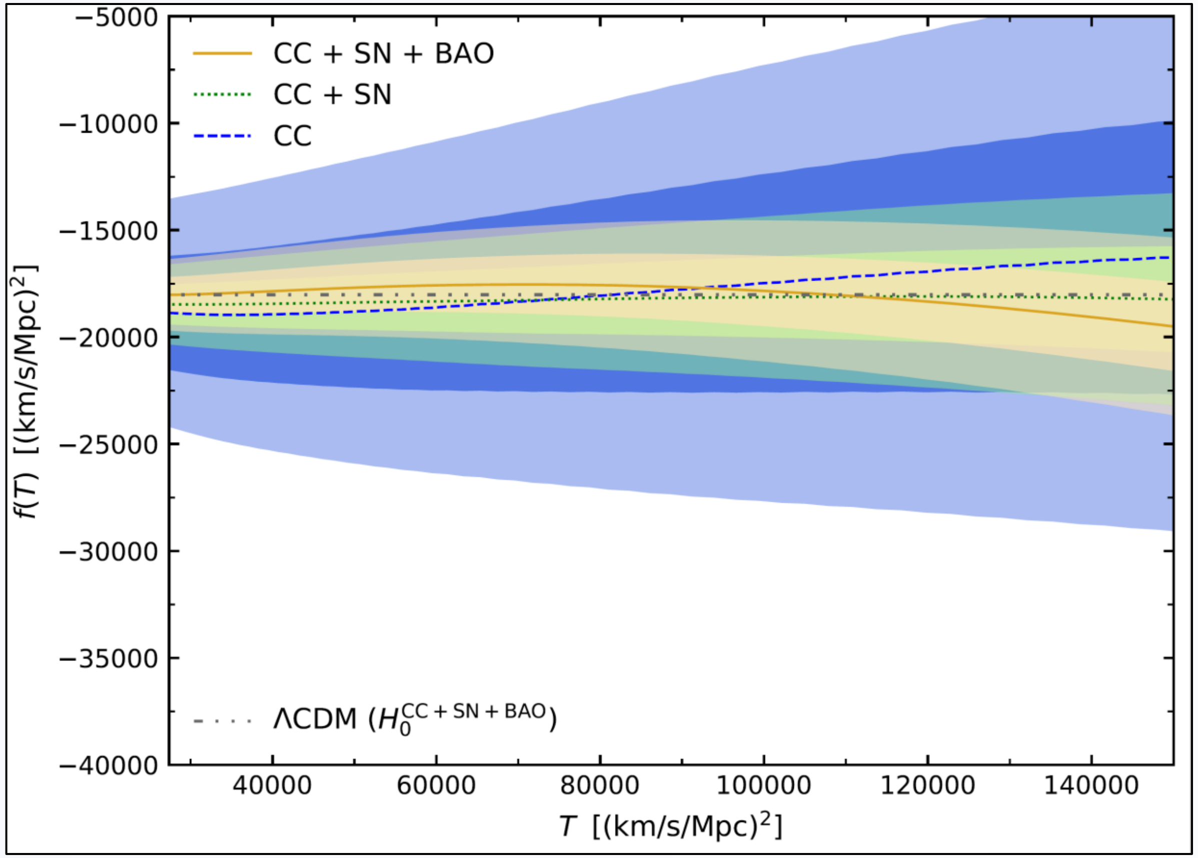
$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3}\boldsymbol{f_T}$$

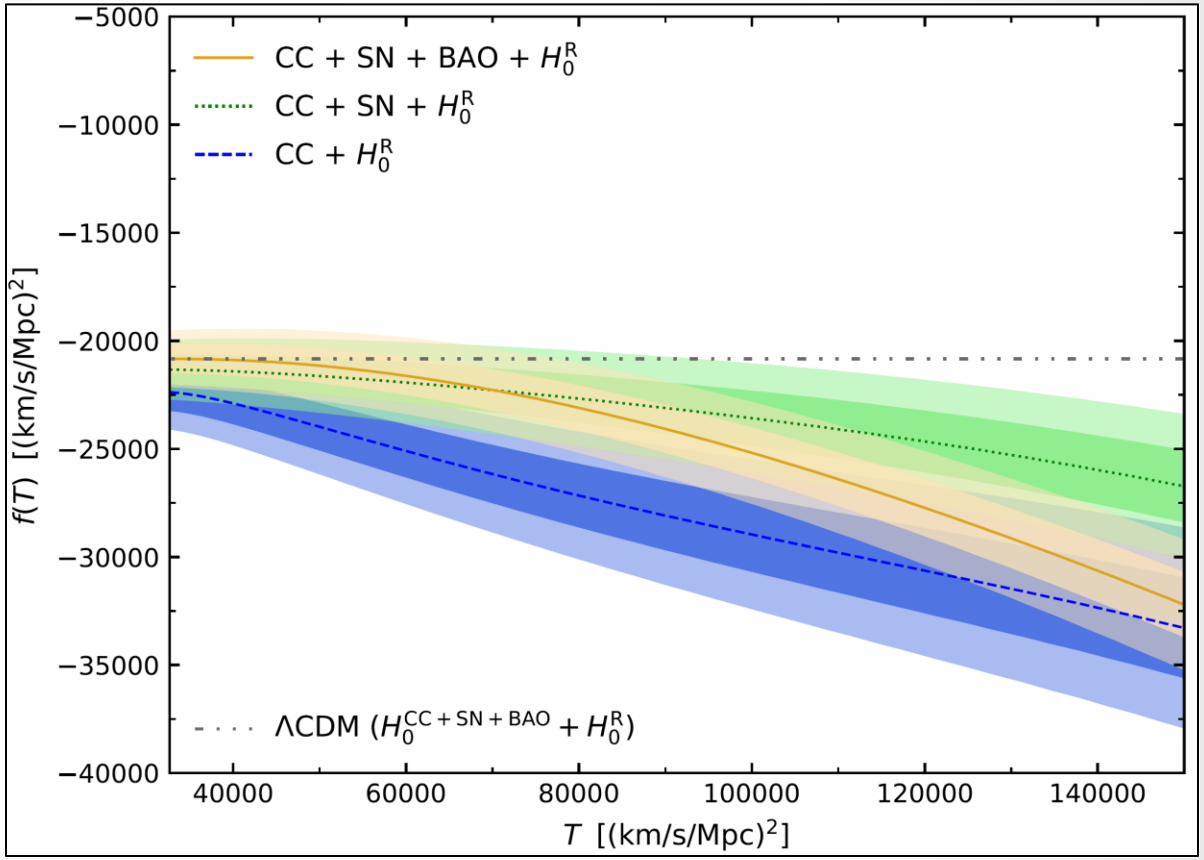
- This then gives a **propagation equation**

$$f(z_{i+1}) = f(z_{i-1}) + 2(z_{i+1} - z_{i-1})\frac{H'(z_i)}{H(z_i)}\left(3H(z_i)^2 + \frac{f(z_i)}{2} - 3H_0^2\Omega_{m_0}(1+z_i)^3\right)$$

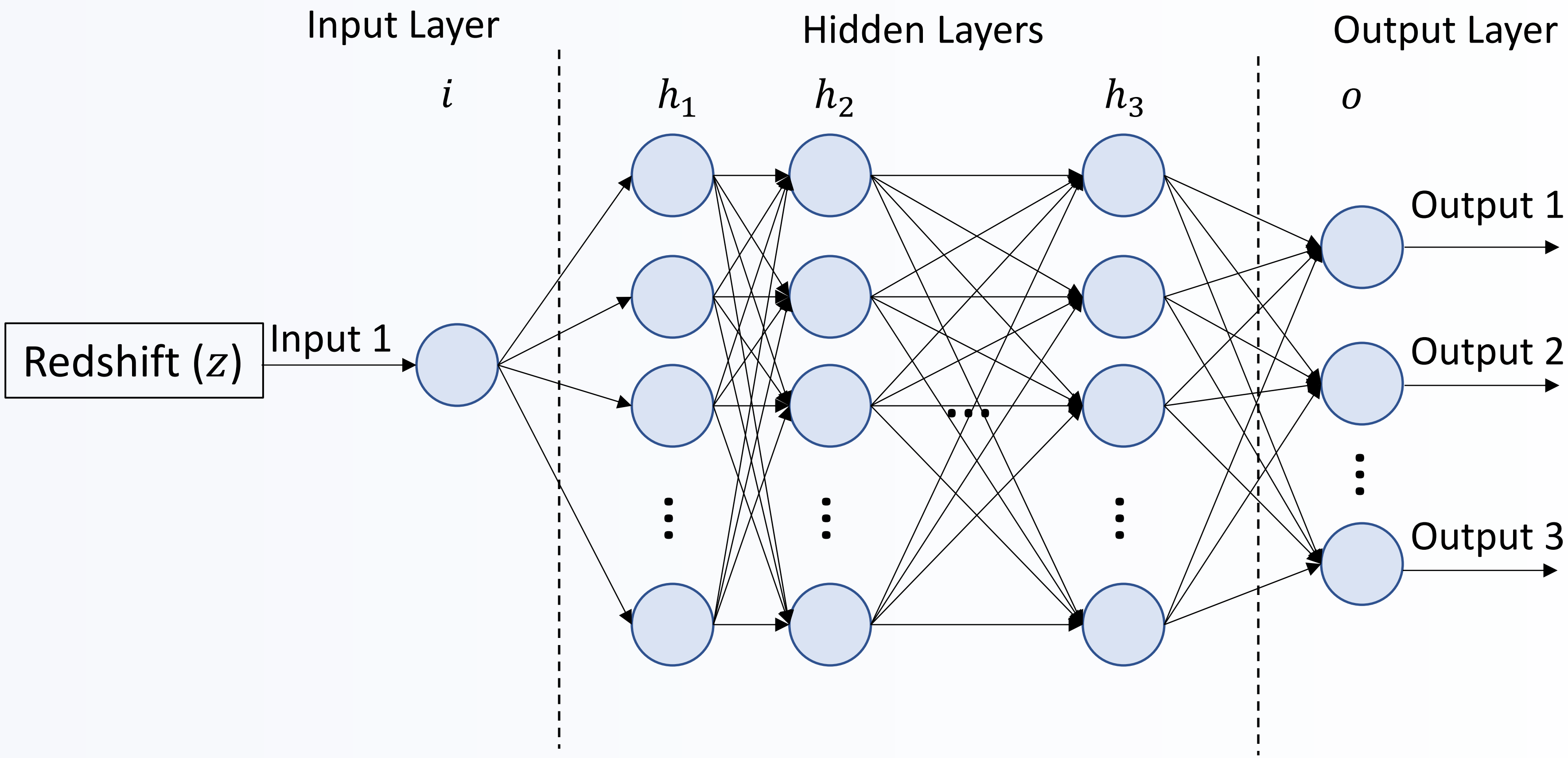
- Using **forward differencing**, we can produce a second boundary condition

Square Exponential $f(T)$ GP



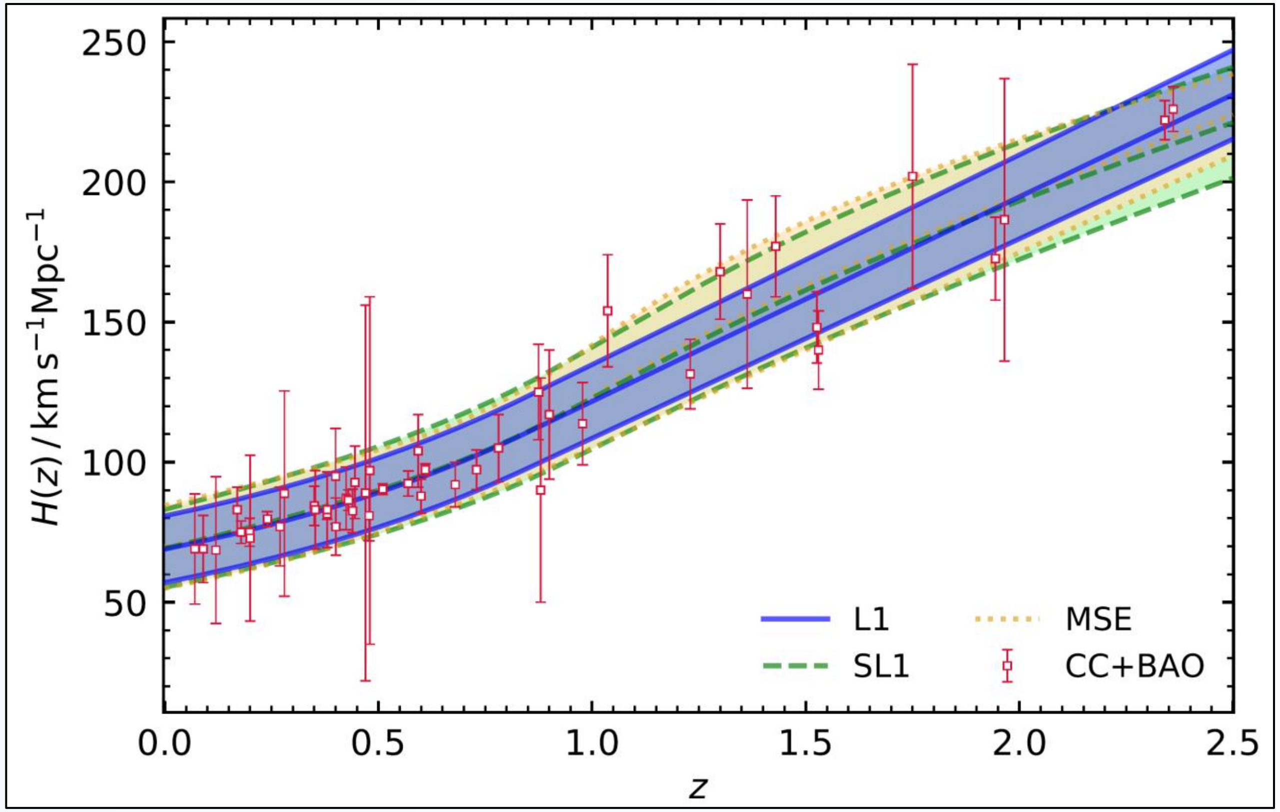


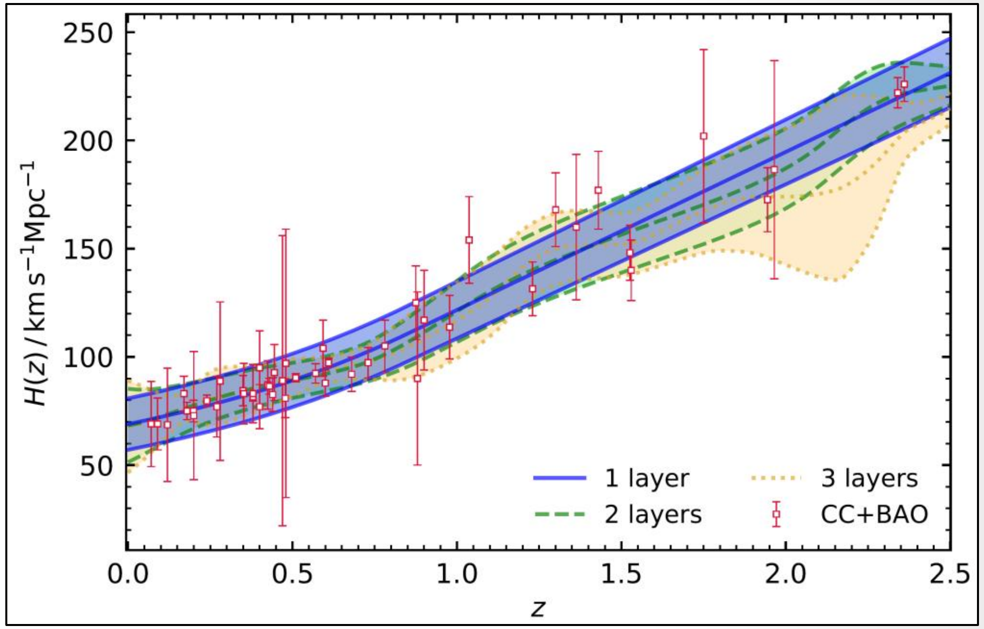
Artificial Neural Networks (ANNs)



Cosmological parameters (ex. $H(z), \sigma_H(z)$)

Using the ANN



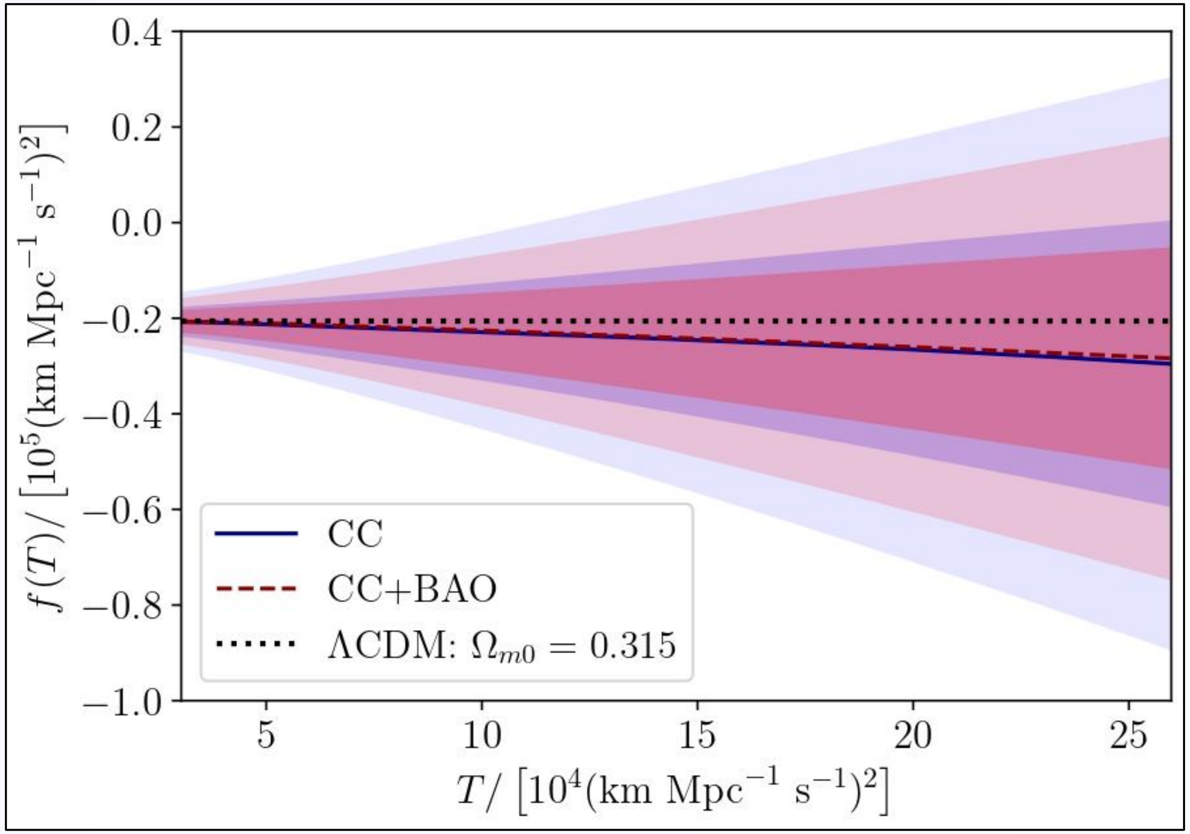


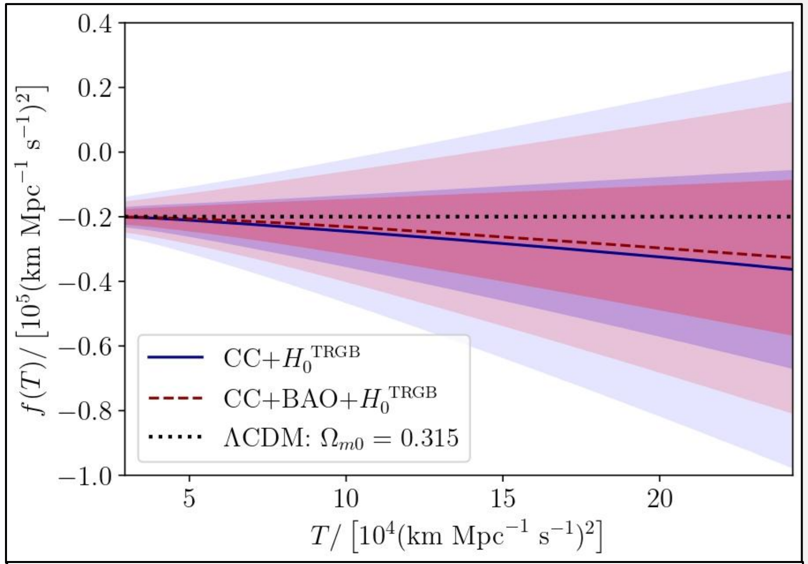
One layer is preferred

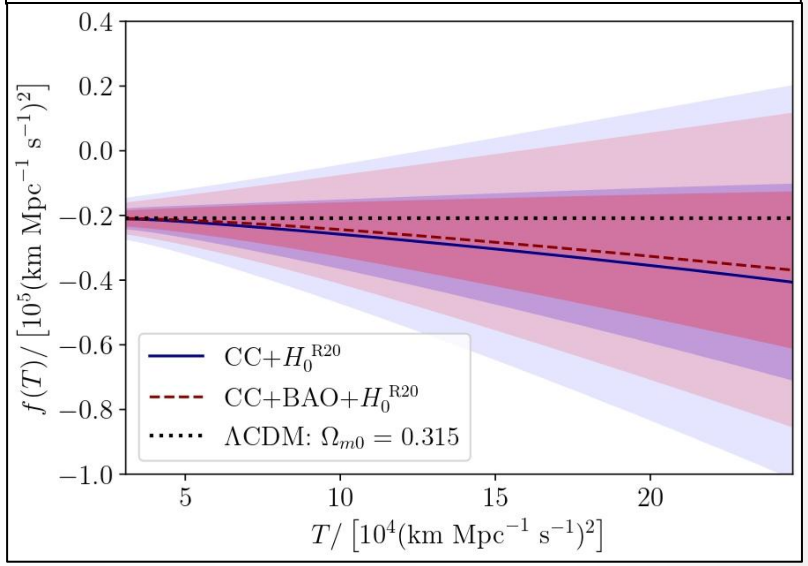
MSE: $H_0 = 69.76 \pm 14.82 \text{ km s}^{-1}\text{Mpc}^{-1}$
L1: $H_0 = 68.93 \pm 11.90 \text{ km s}^{-1}\text{Mpc}^{-1}$
SL1: $H_0 = 69.18 \pm 13.92 \text{ km s}^{-1}\text{Mpc}^{-1}$

Dialektopoulos, K. et al. JCAP 02 (2022) 023

Propagating f(T)CDM







$$H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} + \frac{T}{3}\boldsymbol{f_T}$$

Horndeski Gravity

Horndeski Gravity: Produces the most **general second-order theory** that contains only **one scalar field** (in **standard gravity**)

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right]$$

where

$$\mathcal{L}_2 = \mathrm{G}_2(\phi, X)$$
$$\mathcal{L}_3 = \mathrm{G}_3(\phi, X)\Box\phi$$
$$\mathcal{L}_4 = \mathrm{G}_4(\phi, X)R + \mathrm{G}_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$
$$\mathcal{L}_5 = \mathrm{G}_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}\mathrm{G}_{5,X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]$$

Example classes of models

Quintessence models

$$G_2 = X - V(\phi), G_3 = C,$$
$$G_4 = 1/2, G_5 = 0$$

Background equations:

$$3H^2 = \rho + \frac{\dot{\phi}^2}{2} + V(\phi)$$
$$2\,\dot{H} + 3H^2 = -p - \frac{\dot{\phi}^2}{2} + V(\phi)$$
$$\ddot{\phi} + 3H\,\dot{\phi} + V'(\phi) = 0$$

Equation of State parameter:

$$w_\phi = \frac{\dot{\phi}/2 - V}{\dot{\phi}/2 + V}$$

Designer Horndeski models

$$G_2 = K(X), G_3 = G(X),$$
$$G_4 = 1/2, G_5 = 0$$

Background equations:

$$3H^2 = \rho - K(X) + 2XK_X + 3H\dot{\phi}^2 G_X$$
$$2\,\dot{H} + 3H^2 = -p - K(X) + 2X\,\ddot{\phi}G_X$$

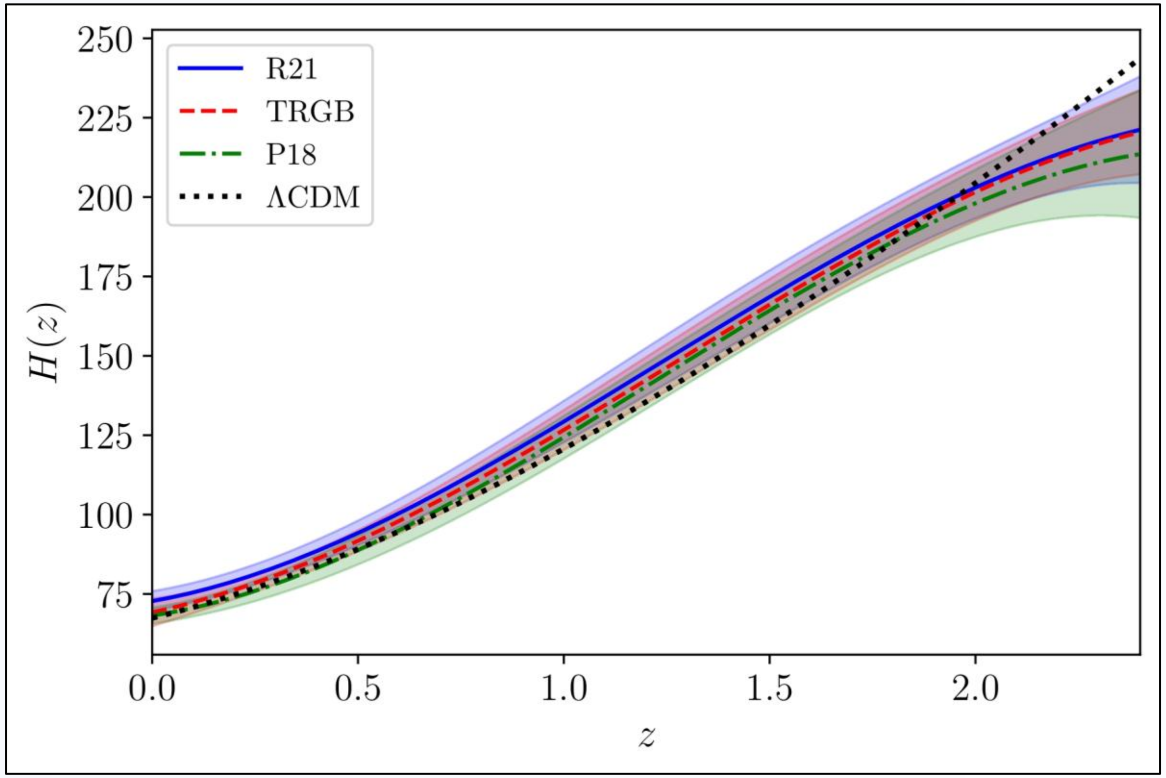
$$\ddot{\phi}\left[\dot{\phi}\left(3H\left(G_{XX}\,\dot{\phi}^2 + G_X\right) + K_{XX}\,\dot{\phi}\right) + K_X\right]$$
$$+ 3\,\dot{\phi}\left(G_X\,\dot{H}\,\dot{\phi} + 3G_X H^2\,\dot{\phi} + HK_X\right) = 0$$

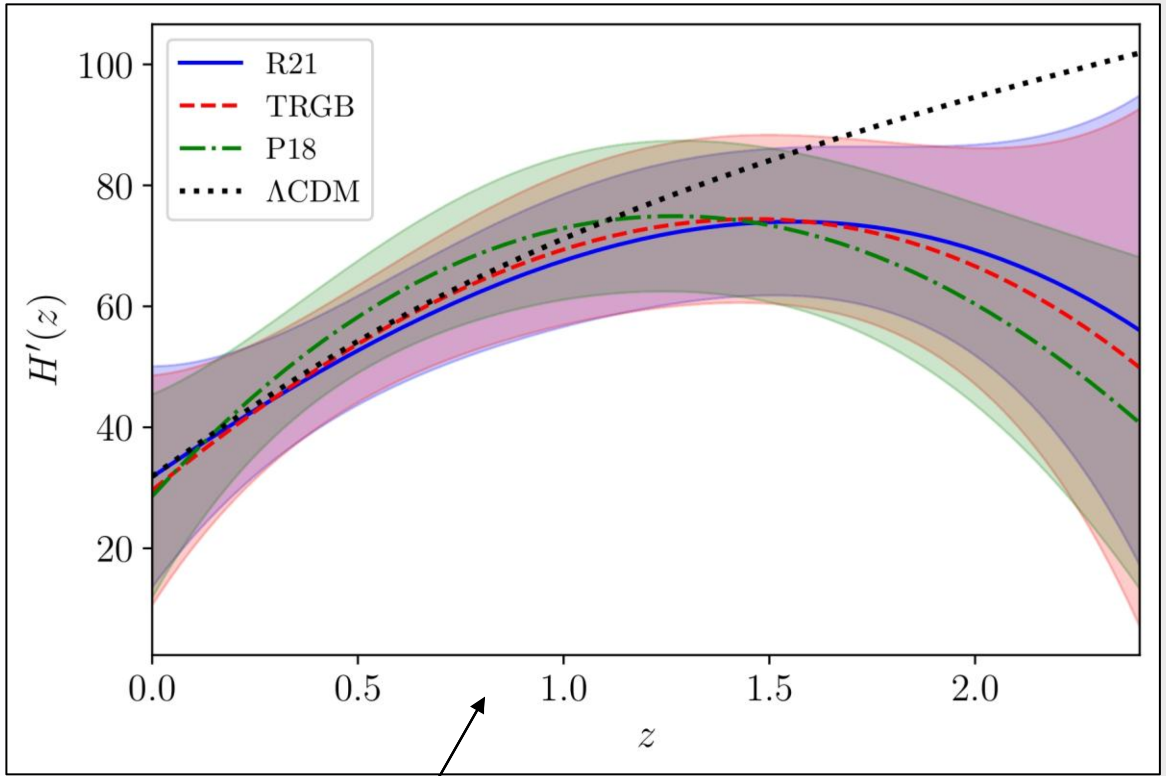
Equation of State parameter:

$$w_\phi = -1 + \frac{J\sqrt{2X}\left(H^2 - H_0^2(1-\Omega_m)\right)}{3H_0^4\Omega_m(1-\Omega_m)} - \frac{2J\sqrt{2X}\left(\dot{\phi}K_X + 3H\,\dot{\phi}^2 G_X\right)(1+z)HH'}{9H_0^4\Omega_m(1-\Omega_m)}$$

where $J = \dot{\phi}K_X + 3H\,\dot{\phi}^2 G_X$

Using the ANN



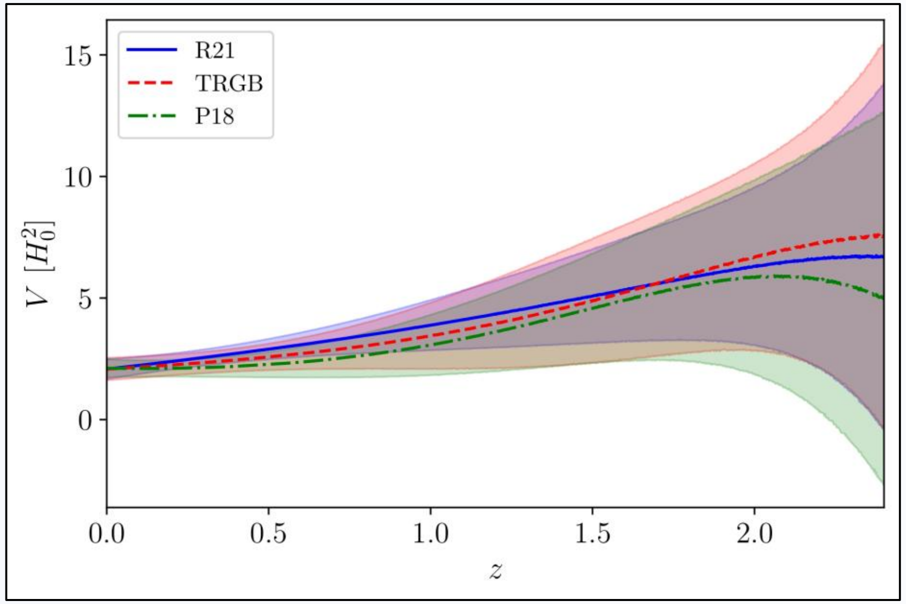


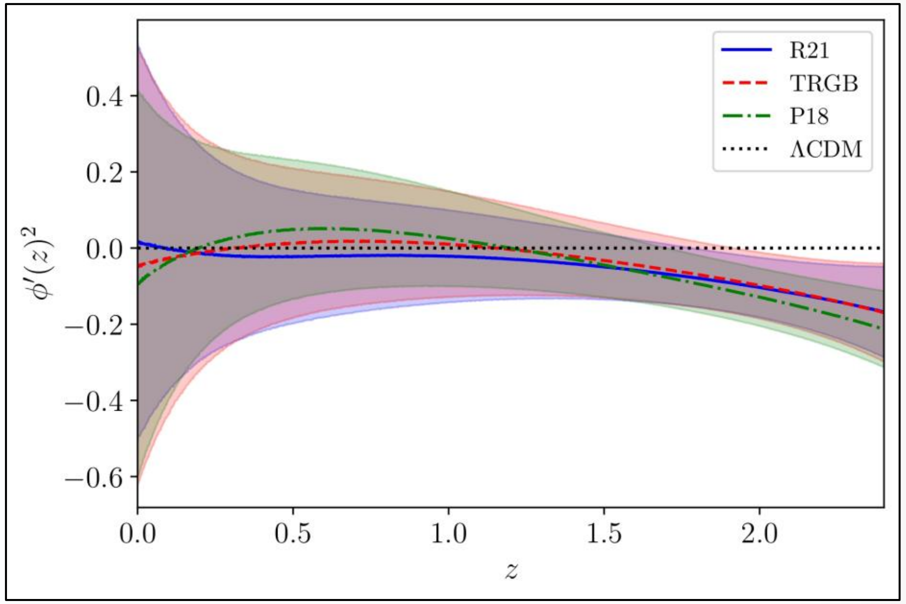
One layer is preferred

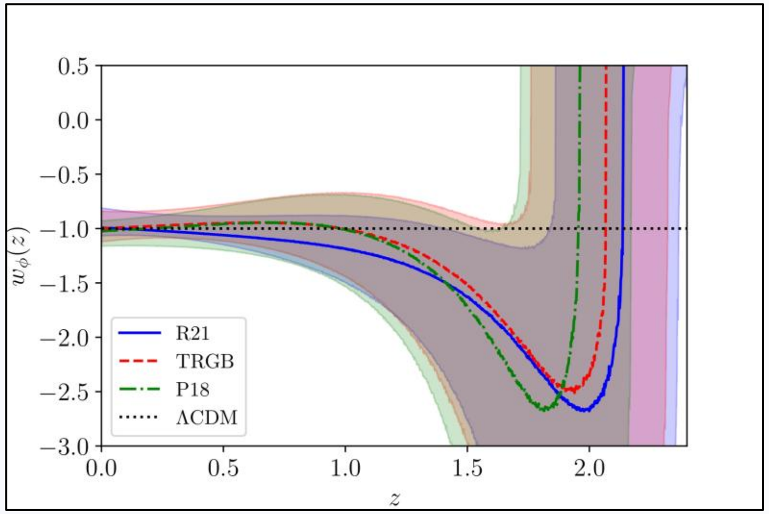
Monte Carlo routine used to determine uncertainties on $H'(z)$

Dialektopoulos, K. et al. Phys. Dark Univ. 43 (2024) 101383

Quintessence Models

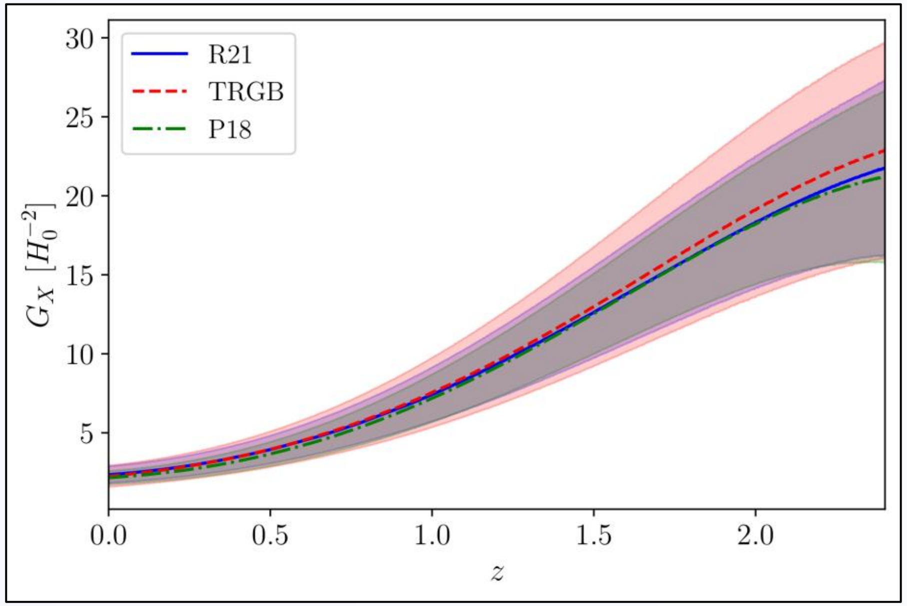


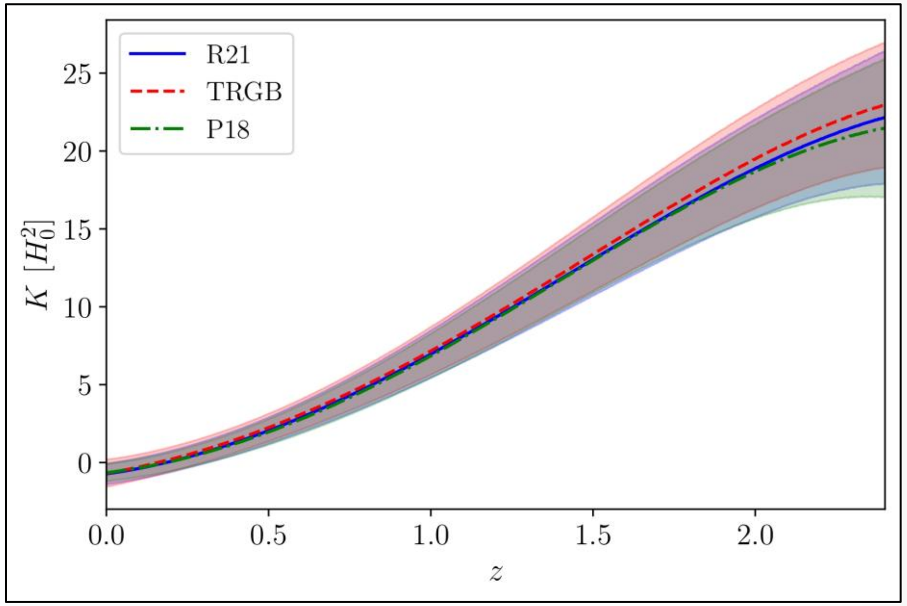


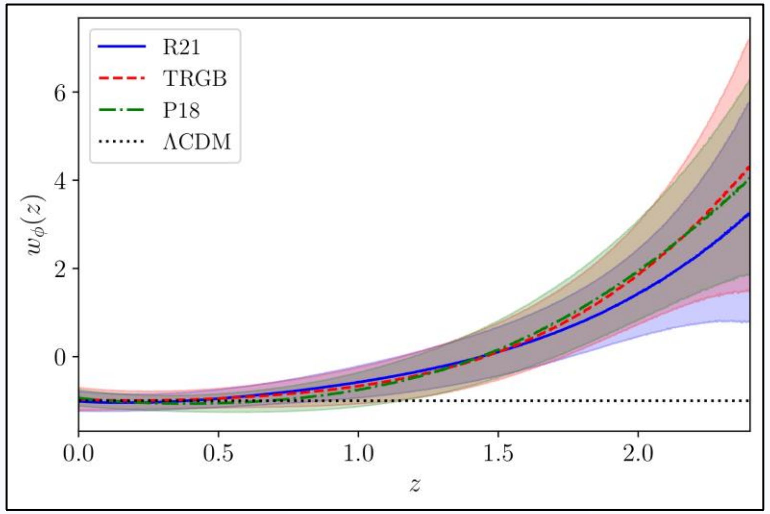


$$V(\phi) = \dot{H} + 3H^2 - \frac{\rho - p}{2}$$
$$\dot{\phi}^2 = -2\dot{H} - (\rho - p)$$

Designer Horndeski Models





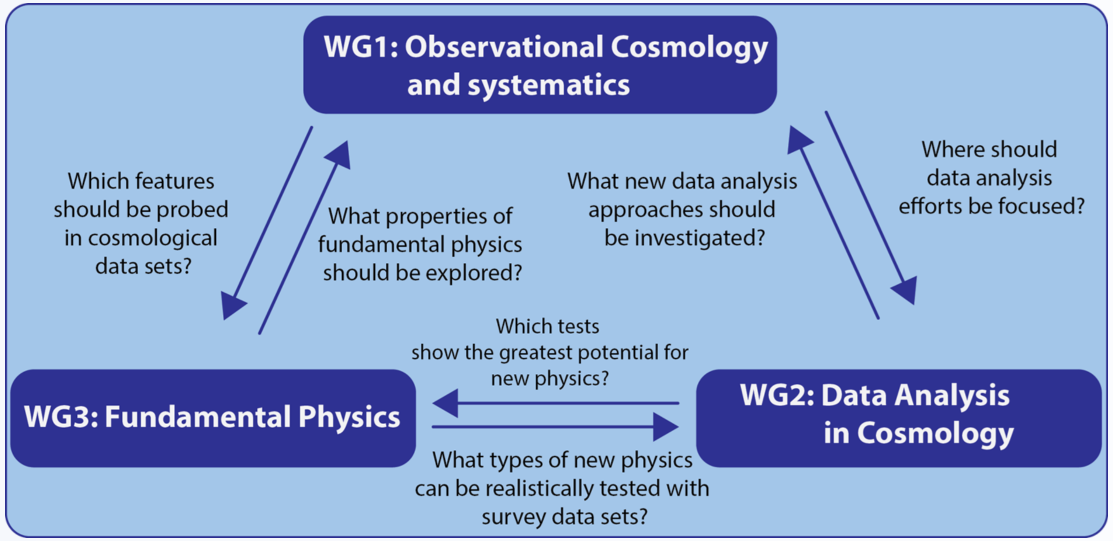


$$K = -3H_0^2(1-\Omega_m) + \frac{J\sqrt{2X}H^2}{3H_0^2\Omega_m} - \frac{J\sqrt{2X}(1-\Omega_m)}{\Omega_m}$$

$$G_X = -\frac{2JH'(X)}{3H_0^2\Omega_m}$$

CosmoVerse Activities

CosmoVerse Events 2025



CosmoVerse@Istanbul 2025 – 24-26 June

CosmoVerse White Paper

CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics

(Dated: September 5, 2024)

arXiv > astro-ph > arXiv:2504.01669

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[Submitted on 2 Apr 2025]

The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics

Eleonora Di Valentino, Jackson Levi Said, Adam Riess, Agnieszka Pollo, Vivian Poulin, Adrià Gómez-Valent, Amanda Weltman, Antonella Palmese, Caroline D. Huang, Carsten van de Bruck, Chandra Shekhar Saraf, Cheng-Yu Kuo, Cora Uhlemann, Daniela Grandón, Dante Paz, Dominique Eckert, Elsa M. Teixeira, Emmanuel N. Saridakis, Eoin Ó Colgáin, Florian Beutler, Florian Niedermann, Francesco Bajardi, Gabriela Barenboim, Giulia Gubitosi, Ilaria Musella, Indranil Banik, Istvan Szapudi, Jack Singal, Jaume Haro Cases, Jens Chluba, Jesús Torrado, Jurgen Mifsud, Karsten Jedamzik, Khaled Said, Konstantinos Dialektopoulos, Laura Herold, Leandros Perivolaropoulos, Lei Zu, Lluís Galbany, Louise Breuval, Luca Visinelli, Luis A. Escamilla, Luis A. Anchordoqui, M.M. Sheikh-Jabbari, Margherita Lembo, Maria Giovanna Dainotti, Maria Vincenzi, Marika Asgari, Martina Gerbino, Matteo Forconi, Michele Cantiello, Michele Moresco, Micol Benetti, Nils Schöneberg, Özgür Akarsu, Rafael C. Nunes, Reginald Christian Bernardo, Ricardo Chávez, Richard I. Anderson, Richard Watkins, Salvatore Capozziello, Siyang Li, Sunny Vagnozzi, Supriya Pan, Tommaso Treu, Vid Irsic, Will Handley, William Giarè, Yukei Murakami, Adèle Poudou, Alan Heavens, Alan Kogut, Alba Domi, Aleksander Łukasz Lenart, Alessandro Melchiorri, Alessandro Vadalà, Alexandra Amon, Alexander Bonilla, Alexander Reeves, Alexander Zhuk, Alfio Bonanno, Ali Övgün, Alice Pisani, Alireza Talebian, Amare Abebe, Amin Aboubrahim, Ana Luisa González Morán, András Kovács, Andreas Papatriantafyllou, Andrew R. Liddle, Andronikos Paliathanasis, Andrzej Borowiec, Anil Kumar Yadav, Anita Yadav, Anjan Ananda Sen, Anjitha John William Mini Latha, Anne Christine Davis, Anowar J. Shajib, Anthony Walters, Anto Idicherian Lonappan et al. (438 additional authors not shown)

The standard model of cosmology has provided a good phenomenological description of a wide range of observations both at astrophysical and cosmological scales for several decades. This concordance model is constructed by a universal cosmological constant and supported by a matter sector described by the standard model of particle physics and a cold dark matter contribution, as well as very early-time inflationary physics, and underpinned by gravitation through general relativity. There have always been open questions about the soundness of the foundations of the standard model. However, recent years have shown that there may also be questions from the observational sector with the emergence of differences between certain cosmological probes. In this White Paper, we identify the key objectives that need to be addressed over the coming decade together with the core science projects that aim to meet these challenges. These discordances primarily rest on the divergence in the measurement of core cosmological parameters with varying levels of statistical confidence. These possible statistical tensions may be partially accounted for by systematics in various measurements or cosmological probes but there is also a growing indication of potential new physics beyond the standard model. After reviewing the principal probes used in the measurement of cosmological parameters, as well as potential systematics, we discuss the most promising array of potential new physics that may be observable in upcoming surveys. We also discuss the growing set of novel data analysis approaches that go beyond traditional methods to test physical models. [Abridged]

Conclusion

- **Tensions in cosmology** is robustly established and unlikely to be due to cross-experiment systematics
- **Fundamental Physics** offers a variety of ways to address but more work is needed for the models to mature
- **Machine-learning inspired methods** (GP, GA & ANN) offer unique **opportunities to probe new physics**

Thank You