EXAMPLE: $\sigma^\alpha \wedge \eta_{ab}\overline{\Gamma}^a_{\alpha\beta}\,\sigma^\beta \wedge e^b \equiv \chi \in Z^3_{dR}(G)^G$ of GS SUPERSTRING

$$[\eta_{ab}\overline{\Gamma}^a_{\alpha\beta}\,\sigma^\beta \wedge e^b] \neq 0 \text{ in } CaE^2(G) \iff [\eta_{ab}\overline{\Gamma}^a_{\alpha\beta}\,q^\beta \wedge p^b] \neq 0 \text{ in } CE^2(\mathfrak{g})$$

$$\Rightarrow \quad 0 \to \mathbb{R}^{0|D_{d,1}} \to \underset{\simeq\, \mathfrak{g}\oplus\langle Z^\alpha\rangle}{Y\mathfrak{g}} \to \overset{\text{sMink}}{\underset{\|}{}}\mathfrak{g} \to 0 \quad : \quad \{Q_\alpha, Q_\beta\} = \overline{\Gamma}^a_{\alpha\beta} P_a \,,\ [P_a, P_b] = 0$$

$$[Q_\alpha, P_a] = \overline{\Gamma}_{a\alpha\beta} Z^\beta \,,\ [Z^\alpha, \cdot\} = 0$$

GREEN ALGEBRA

$\int$

$$1 \to \mathbb{R}^{0|D_{d,1}} \to YG \xrightarrow{\pi_{YG}} G \to 1 \quad \text{w/} \quad \pi^*_{YG}\chi = d\big(-\underbrace{\pi^*_{YG}\sigma^\alpha}_{=\,\beta} \wedge \underbrace{\varpi_\alpha}_{\iota^* Z^\alpha}\big)$$

EXTENDED SUPERSPACE $\simeq G \times \mathbb{R}^{0|D_{d,1}}$

On $Y^{[2]}G \subset YG \times YG$: $[\Delta^{(1)}\beta] \neq 0$ in $CaE^2(Y^{[2]}G)$

$$\Rightarrow \quad 0 \to \mathbb{R} \to \mathfrak{l} \to Y^{[2]}\mathfrak{g} \to 0 \qquad \int \qquad 1 \to \mathbb{C}^\times \to L \xrightarrow{\pi_L} Y^{[2]}G \to 1 \quad \text{w/} \quad \pi_L^*\Delta^{(1)}\beta = dA_L$$

$\mathfrak{l} \simeq Y^{[2]}\mathfrak{g} \oplus \langle z \rangle$ $\qquad$ $L \simeq Y^{[2]}G \times \mathbb{C}^\times$ $\qquad$ $A_L \overset{*}{\sim} z$

FINALLY, $\mu_L = 1$: $pr^*_{1,2}L \otimes pr^*_{2,3}L \xrightarrow[\text{sLieGrp}]{\simeq} pr^*_{1,3}L$ over $Y^{[3]}G$

UPSHOT: GERBE / MURRAY DIAGRAM IN $\mathbf{sLieGrp}^{\circlearrowleft}$ =: CaE SUPER-GERBE [Suszek]

$$
\begin{tikzcd}
\Delta^{(3)}\mu_L = \mathrm{id} \arrow[d] & \mu_L : \Delta^{(2)}L \cong I_0 \arrow[d] & \mathbb{C}^\times \rightsquigarrow L, A_L \arrow[d, "\pi_L"] & (I_\beta) \arrow[d, phantom, "\parallel\!|"] & \\
Y^{[4]}G \arrow[r, "d_\bullet^{(3)}"'] & Y^{[3]}G \arrow[r, "d_\bullet^{(2)}"'] & Y^{[2]}G, \Delta^{(1)}\beta \arrow[r, "d_\bullet^{(1)}"'] & YG, \beta \arrow[d, "\pi_{YG}"] & \\
 & & & G, \chi &
\end{tikzcd}
$$

over $Y^{[\bullet+1]}G = N_\bullet \mathrm{Pair}_G(YG)$

w/ $\Delta^{(n)} = \sum_{k=0}^{n} (-1)^k d_k^{(n)*}$ DUPONT COBOUNDARIES

with SUPERSYMMETRY CATEGORIFIED :

$$
\Big\{ \Phi_{(\varepsilon,t)} \equiv \mathrm{id}_{\mathcal{G}} : \underset{\overset{!}{\cong}\,\mathcal{G}}{\ell^*_{(\varepsilon,t)}\mathcal{G}} \xrightarrow{\cong} \mathcal{G} \Big\}_{(\varepsilon,t)\in G}
$$

PoD for STUDY of :

* κ-SYMMETRY

** MAXYM SUSY DEFECTS

*** CATEGORIFICATION of İW CONTRACTION

STILL… (0) WHAT DOES 'GEOMETRISATION' MEAN? IS IT 'RESOLUTION' of A 'TOPOLOGY'?

ANSWER 1. CATEGORIFICATION of SUPERSYMMETRY PROVIDED by $\mathfrak{g}$ ITSELF.

ANSWER 2. LATTICE SUPERSYMMETRIC sFT LEADS to …

Q: $\dot{S}$ $\exists\, \Gamma \subset G$ 'DISCRETE' $\wedge$ $G/\!/\Gamma$ 'GENERALISED SUPERMANIFOLD' [Rothstein ↯]:

$$\Omega^{\bullet}_{dR}(G)^{\Gamma} \equiv \Omega^{\bullet}_{dR}(G)^{G} \,?$$

(NB: Γ-EQUIVARIANCE $\xrightarrow[\text{DISCRETELY}]{}$ Γ-INVARIANCE)

UPSHOT: (IF ✓)

$$\boxed{\Omega^{\bullet}_{dR}(G)^{G} \underset{\text{MODEL}}{\doteq} \Omega^{\bullet}_{dR}(G/\!/\Gamma)}$$

NEED: ABANDON (PURE) BLK PARADIGM

NESTED ROGERS-DEWITT ⋯→ JADCZYK-PILCH

NESTED $\mathrm{Yon}_G(\mathbb{R}^{0|L})$ ⋯$\xrightarrow[L]{\lim}$ $\mathrm{sMan}(B_\infty)$

ENTRAILS: $\mathbb{R}^{d+1|D_{d,1}} \underset{\text{REPLACE}}{\rightsquigarrow} B_L^{d+1|D_{d,1}} \equiv B_{L(0)}^{\times d+1} \times B_{L(1)}^{\times D_{d,1}}$, $B_L = \langle \beta^i \mid i \in \overline{1,L} \rangle$

w/ COÖRDS $\theta^\alpha \in B_{L(1)}$, $x^a \in B_{L(0)}$

LOOKING for $\Gamma_L < G_L$ AS above...

KOSTELECKÝ-RABIN (DISCRETE SUPERSYMMETRY) GROUP (AT LEVEL L)

UPSHOT: $\Gamma_{KR(L)} := \langle \mathbb{Z}\, \beta^{i_1}\beta^{i_2}\cdots\beta^{i_k} \mid 1 \leq i_1 < i_2 < \ldots < i_k \leq L,\ k \in \overline{1,L} \rangle$ DOES THE JOB! [Rabin]

$$\boxed{\Omega^\bullet_{dR}(G_L)^{G_L} \equiv \Omega^\bullet_{dR}(G_L)^{\Gamma_L}}$$

& NESTS $\Gamma_{KR(L)} < \Gamma_{KR(L'>L)}$ TRIVIALLY

$\Rightarrow$ MAY CONSIDER $G_L /\!/ \Gamma_{KR(L)} =: M_L$ w/ $\Omega^\bullet_{dR}(M_L) \underset{\text{MODEL}}{\doteq} \Omega^\bullet_{dR}(G_L)^{G_L}$

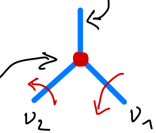
& 'PASS' to $\boxed{\varinjlim_L \bigcup_{n \leq L} M_n =: G /\!/ \Gamma_{KR}}$ RABIN-CRANE SUPERORBIFOLD

Categorification

Recall Thm [GSW] : $\mathrm{BGrb}_\nabla(M/\!\!/\Gamma) \simeq \mathrm{BGrb}_\nabla(M)^{\Gamma\text{-equiv}}_{\omega[\theta_i]=0}$ for $M \twoheadrightarrow M/\!\!/\Gamma$ principal

But... Γ-equivariance $\overset{\text{discretely}}{\rightsquigarrow}$ Γ-invariance $\Phi_\nu : \mathcal{G} \xrightarrow{\simeq} l^*_{\nu^{-1}}\mathcal{G}$

+ data of

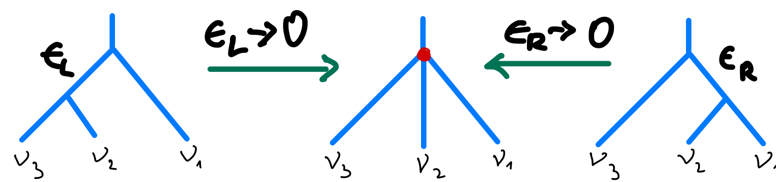


$\nu, \nu_1, \nu_2, \nu_3 \in \Gamma_{KR}$

$\varphi_{\nu_1,\nu_2} : l^*_{\nu_1^{-1}}\Phi_{\nu_2} \circ \Phi_{\nu_1} \overset{\simeq}{\Longrightarrow} \Phi_{\nu_1 \cdot \nu_2}$

+ [Associator 3-cocycle] = 0

for induction
via Associator Moves



For GS SUPERSTRING : (D) $\Phi_\nu \equiv \mathrm{id}_{\mathcal{G}} : \mathcal{G} \xrightarrow{\cong} \ell_{\nu^{-1}}^{*}\mathcal{G}$

$$\& \quad \varphi_{\nu_1,\nu_2} : \ell_{\nu_1^{-1}}^{*}\Phi_{\nu_2} \circ \Phi_{\nu_1} \xRightarrow{\cong} \Phi_{\nu_1\cdot\nu_2}$$

$$\ell_{\nu_1^{-1}}^{*}\Phi_{\nu_2} \circ \Phi_{\nu_1} \overset{(D)}{=} \ell_{\nu_1^{-1}}^{*}\mathrm{id}_{\mathcal{G}} \circ \mathrm{id}_{\mathcal{G}} \overset{\text{FUNCTORIALITY of PULLBACK}}{=} \mathrm{id}_{\ell_{\nu_1^{-1}}^{*}\mathcal{G}} \circ \mathrm{id}_{\mathcal{G}} = \mathrm{id}_{\mathcal{G}} \circ \mathrm{id}_{\mathcal{G}}, \qquad \Phi_{\nu_1\cdot\nu_2} = \mathrm{id}_{\mathcal{G}}$$

$$\varphi_{\nu_1,\nu_2} \equiv \Lambda^{L}_{\mathrm{id}_{\mathcal{G}}} \Leftarrow$$

CANONICAL 2-CELL in $\mathcal{B}\mathcal{G}rb_{\partial}(G)$, DETERMINED by $\mu_L = 1$

UPSHOT: $\boxed{\left(\mathcal{G}, \{\Phi_\nu = \mathrm{id}_{\mathcal{G}}\}_{\nu\in\Gamma_{KR}}, \{\varphi_{\nu_1,\nu_2} = 1\}_{\nu_1,\nu_2\in\Gamma_{KR}}\right)}$ is DESCENDABLE Γ_{KR}-EQUIVARIANT STRUCTURE

& $\mathcal{G}$ ITSELF MODELS A GERBE / $G/\!/\Gamma_{KR}$

The Grand sFT Finale:

Geometrisation of $\mathcal{X}$ given by CaE super-gerbe

defines RC superorbifold super-σ-model