Gauge symmetries and their higher analogues

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Gauge theory is, on one hand, a coherent system of relations on the set of the elementary objects: intrinsically topological charges (carried by localised probes and fields), differential-geometric and cohomological gauge fields and equivalences on the space of states of the former induced by automorphisms of the latter. On the other hand, it is a theory of descent of dynamical models to spaces of equivalence classes of configurations determined by an essentially algebraic symmetry principle. In the classic picture, the probes are assumed pointlike, the symmetries are modelled on smooth Lie-group actions, the descent is that to (not necessarily smooth) orbispaces, and the gauge fields are locally presented by differential 1-forms with values in the Lie algebra of the symmetry group and with an affine transformation law under the automorphisms – thus, gauge theory becomes the theory of principal and associated bundles with compatible connections, and that of the corresponding automorphism invariants, including the differential characters which model propagation of the probes in the curved geometry of the fields. A nontrivial topology of the probes, a higher form-degree of the gauge fields or a more complex symmetry model (provided, *e.g.*, by a Lie groupoid or a more general category) each lead to a fundamental enrichment of the classic picture and give rise to what is customarily referred to as *higher gauge theory*. All three enhancements are actually built into the very definition of string theory, and so higher gauge theory can be regarded as *the* gauge theory of the emergent spacetime geometry of string theory and the effective field theory of excited strings scattering in it.

In the lectures, we give a lightning review of the classic (`*lower*') gauge theory, and subsequently use it as a springboard for a gentle introduction into higher gauge theory in the guise of the so-called gerbe theory. In so doing, due emphasis shall be laid on both: universal features of (any) gauge theory and peculiarities of its higher variant. As we storm forward, with an ever increasing level of formal complexity, we intend to get close to the frontline of the current robust higher gauge-theoretic research, where essentially novel notions are encountered, such as, *e.g.*, (*n*-)gerbe objects in the category of Lie (super)groups (in connection with superstring theory and M-theory), categorified Fourier-type transforms between higher geometries (in connection with mirror symmetry) and gerbe-twisted spectral noncommutative geometry (in connection with the emergent geometry of the superstring), and where intricate relations to various other important models of mathematical physics (*i.a.*, the 3d Chern-Simons theory and the Poisson-σ-model) are established.