Higher-spin symmetry and celestial holography

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Motivation

Black hole entropy obeys an area law: $S_{BH} = \frac{A}{4G_N} \sim L^2 M_{Pl}^2$ [Bekenstein '72; Hawking '72]

Entropy of local QFT obeys a volume law: $S_{QFT} \sim L^3 \Lambda^3$

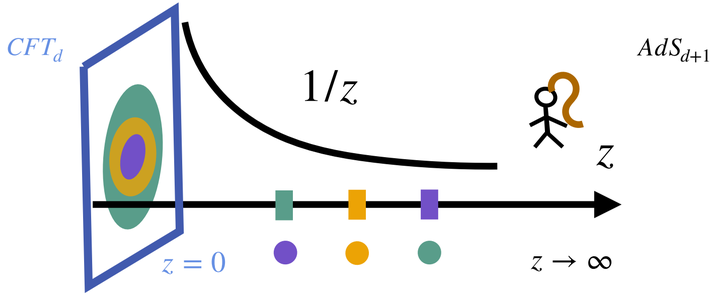
Theories of gravity in d−dimensional spacetimes have the same number of degrees of freedom as quantum field theories in (d−1)−dimensions $\Longrightarrow$ holographic principle ['t Hooft, Susskind '90s]

Motivation

Stringy "realization" in (asymptotically) negatively curved spacetimes $\Longrightarrow$ AdS/CFT correspondence

[Maldacena '98]

Conformal field theory (CFT)

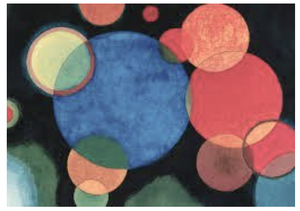


Anti-de-Sitter (AdS) gravity

$$ds^2 = R^2 \frac{dz^2 + d\vec{x}^2}{z^2}$$

$$z^{-2} \times (\text{size})_{CFT} = \text{bulk proper size}$$

Ultraviolet/short distances



Infrared/long distances

Motivation

Quantum gravity ~ CFT ??

- Very limited knowledge of either side, lacking independent definitions in general

- Example: $\mathcal{N}$ = 4 Super Yang−Mills ~ strings in $AdS_5 \times S_5$

Large number of colors Small string coupling & beyond through integrability

More generally, formulate
- (at least) for all Λ
- in any number of dimensions

Context and goals

Which aspects of gravity are captured by CFT?

Today:

$\Lambda = 0$ in the bulk

3+1 (bulk) dimensions

GR, gravitational waves, real word black holes, ..

Advantages:

- Scattering amplitude as building blocks of observables
- Lorentz symmetry in d−dimensions ~ conformal symmetry in (d−2)−dimensions
- In d = 4 Virasoro enhancement in bulk (asymptotically) !

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Holographic "dual" of gravity in AFS is a 2d CFT?

Context and goals

Which aspects of gravity are captured by CFT?

Today:

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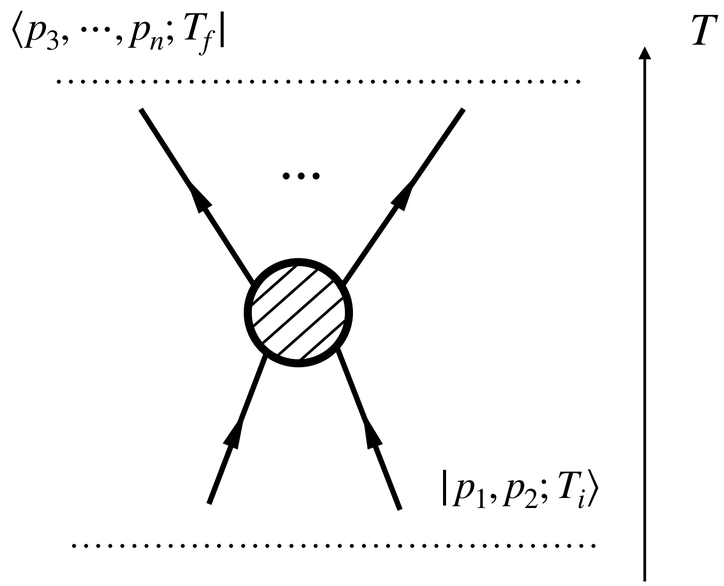
3+1 (bulk) dimensions

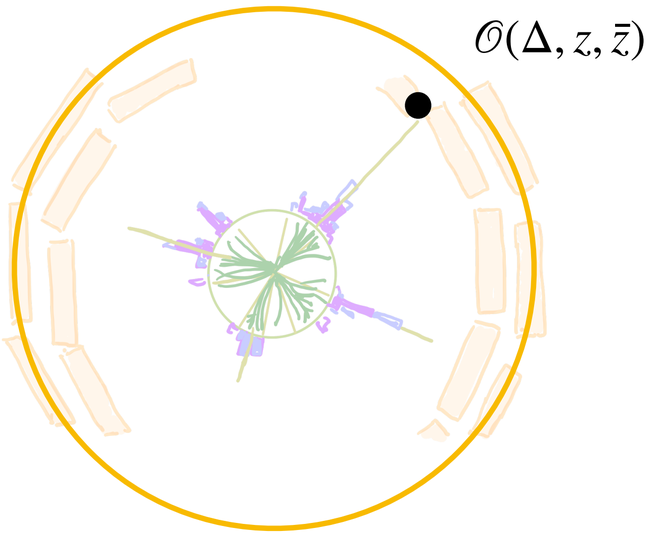
GR, gravitational waves, real word black holes, ..

Aims:

- Understand symmetries and properties of this 2d (celestial) CFT
- How does it fit into the broader holographic landscape?
- What can we learn about (quantum) gravity in 4d AFS?

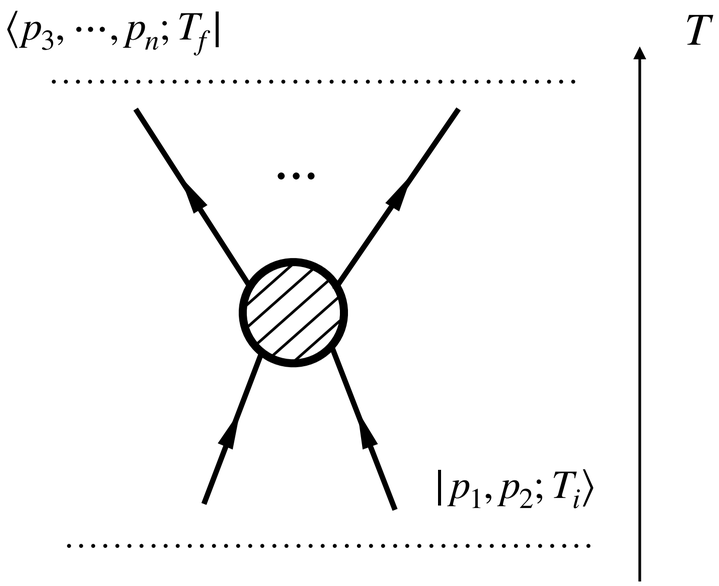
Celestial holography

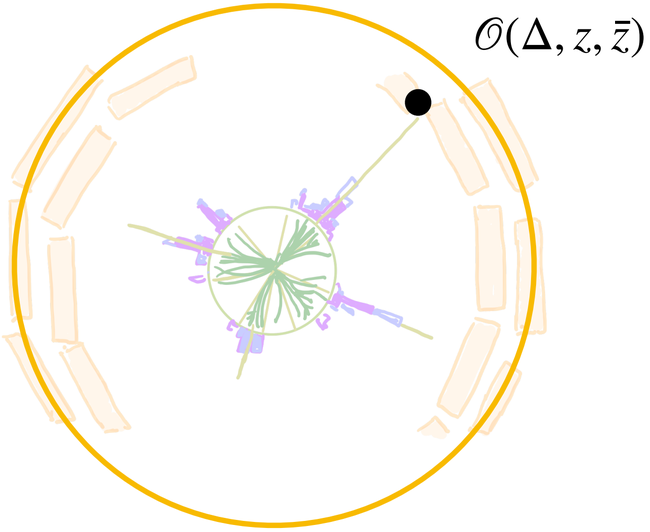




Massless particle states $\underbrace{|\omega, z, \bar{z}\rangle}_{\text{energy and 2 angles}}$ $\longrightarrow$ $|\Delta, z, \bar{z}\rangle \equiv \int_0^\infty \underbrace{d\omega\omega^{\Delta-1}}_{\text{diagonalizes 4d boosts} \sim \text{2d dilatations}} |\omega, z, \bar{z}\rangle$

Celestial holography





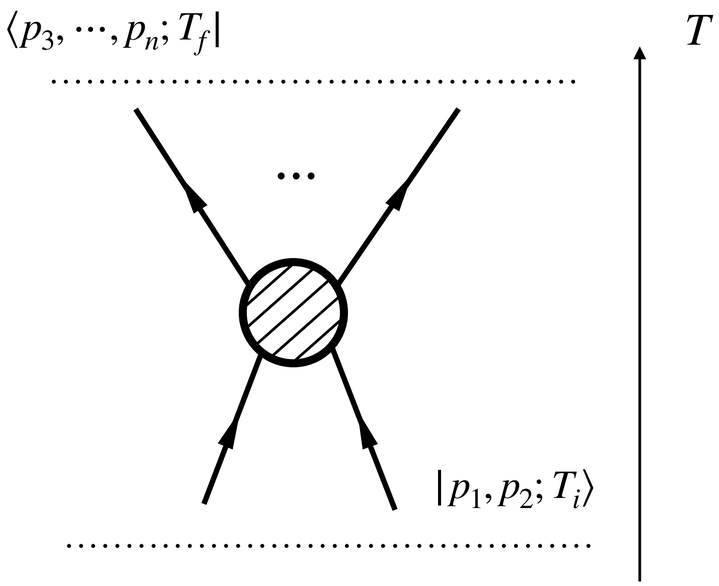
Scattering amplitude $\longrightarrow$ Celestial amplitude

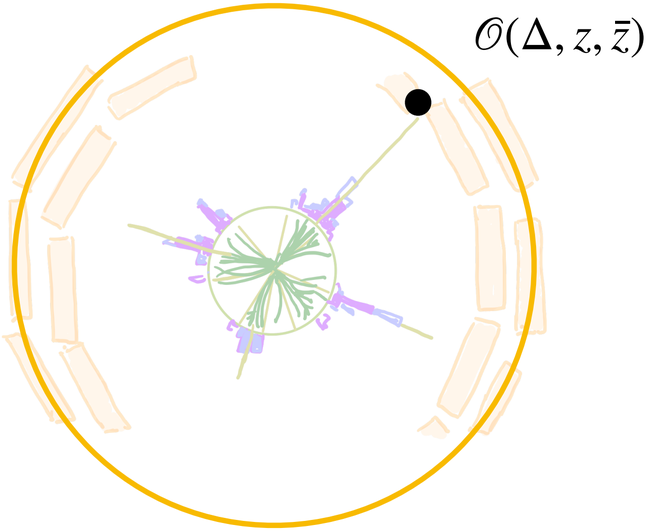
$$\mathcal{A} \sim \langle p_3, \cdots, p_n; T \to +\infty | p_1, p_2; T \to -\infty \rangle$$

$$\langle \mathcal{O}_{\Delta_1}(z_1) \ \cdots \ \mathcal{O}_{\Delta_n}(z_n) \rangle$$

subject to ∞ symmetries !

Celestial holography





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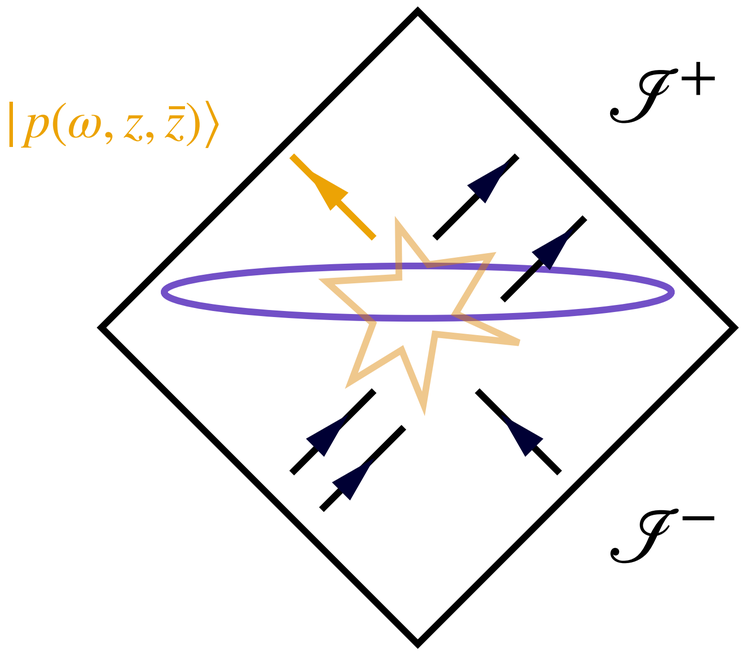
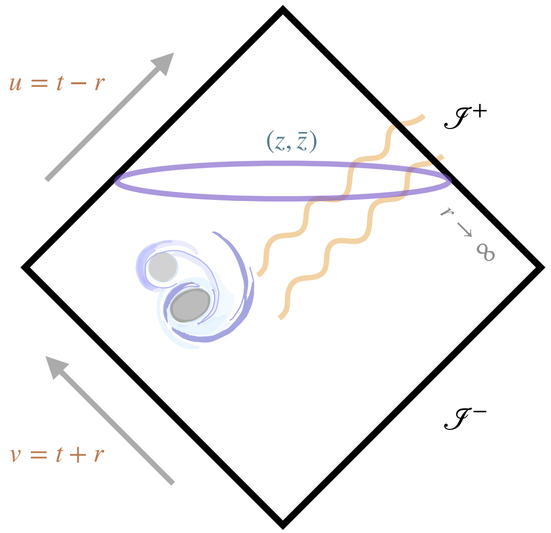
Spacetime symmetries and scattering



- spacetime ~ Minkowski + perturbations
- quantized metric perturbations ~ particles = gravitons
- infinite−dimensional asymptotic symmetry algebra

[Bondi, van der Burg, Metzner, Sachs '62]

Asymptotic symmetries

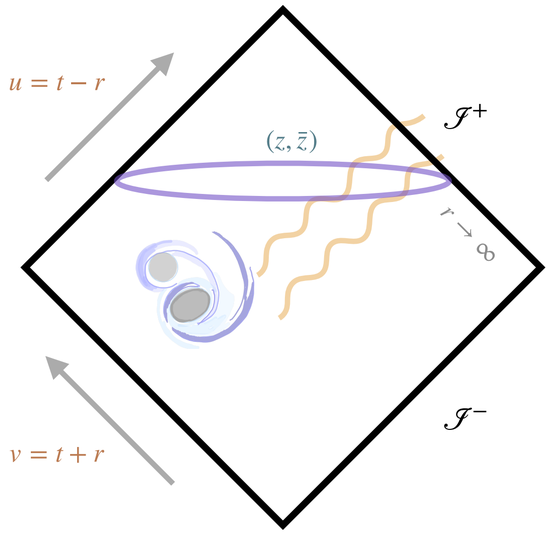


Minkowski metric in retarded coordinates $\quad ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$

Gauge choice well adapted to the study gravitational waves (Bondi gauge):

- waves propagate radially along family of null geodesics: $\quad g^{\mu\nu}\partial_\mu u \partial_\nu u = 0 \implies g^{uu} = 0$
- angular coordinates constant along null rays $\quad g^{\mu\nu}\partial_\mu u \partial_\nu x^A = 0 \implies g^{uA} = 0$
- constant area waveforms $\quad \partial_r \det\left(r^{-2}\gamma_{AB}\right) = 0$

Asymptotically flat spacetimes



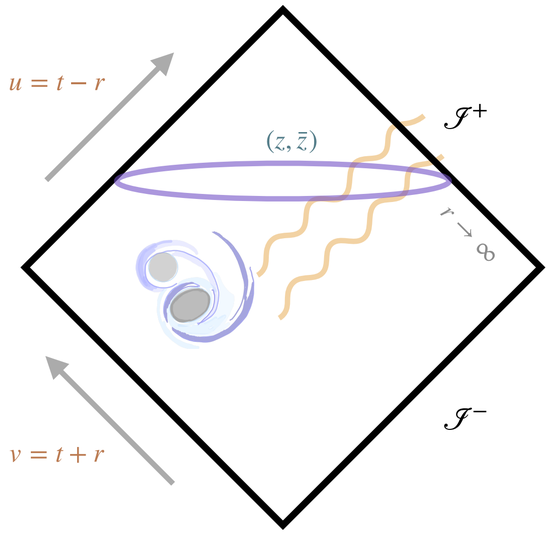
- Asymptotically flat Bondi metrics near $\mathscr{I}^+$ ~ Minkowski + perturbations that fall off with r:

$$ds^2 = -2e^{2\beta}du(dr + \Phi du) + r^2\gamma_{AB}\left(d\sigma^A - \frac{\Upsilon^A}{r^2}du\right)\left(d\sigma^B - \frac{\Upsilon^B}{r^2}du\right)$$

$$\Phi = F(\sigma^A) - \frac{M}{r} + \mathcal{O}(r^{-2}), \qquad \gamma_{AB} = q_{AB} + \frac{1}{r}C_{AB} + \frac{1}{r^3}T_{AB} + \mathcal{O}(r^{-4})$$

$$\begin{cases} F = \dfrac{1}{2} & \longrightarrow \quad q_{AB} \text{ sphere} \\ F = 0 & \longrightarrow \quad q_{AB} \text{ plane} \end{cases}$$

Asymptotically flat spacetimes



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$\partial_u C_{AB} = N_{AB}$ free data

$$\begin{cases} F = \frac{1}{2} & \longrightarrow \quad q_{AB} \text{ sphere} \\ F = 0 & \longrightarrow \quad q_{AB} \text{ plane} \end{cases}$$

- remaining functions determined in terms of C_{AB} by Einstein's equations

Einstein equations at large$-r$

- Radial equations ($G_{r\mu} = 0$) $\Longrightarrow$

$$\Phi = \Phi[\bar{R}, M]$$ mass aspect (energy lost through gravitational waves)

$$\Upsilon^A = \Upsilon^A[C_{AB}, N_A]$$ angular momentum aspect

$$\beta = \beta[C_{AB}]$$

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- Remaining equations at $\mathcal{O}(r^{-2}) \Longrightarrow$ time evolution of M, N_A, T_{AB}

  - $G_{uu} = 0 \Longrightarrow \partial_u M = \frac{1}{4} D^A D^B N_{AB} - \frac{1}{8} N_{AB} N^{AB}$

  - $G_{uA} = 0 \Longrightarrow \partial_u N_A = D_A M + \frac{1}{16} D_A (C_{AB} N^{AB}) - \frac{1}{4} N^{BC} D_A C_{BC} - \frac{1}{4} D_B \left(C^{BC} N_{AC} - N^{BC} C_{AC} \right) - \frac{1}{4} D_B D^B D^C C_{AC} + \frac{1}{4} D_B D_A D_C C^{BC}$

  - $G_{AB} = 0 \Longrightarrow \partial_u E_{AB} = \frac{1}{2} M C_{AB} + \frac{1}{3} D_{(A} N_{B)} - \frac{1}{6} \gamma_{AB} D_C N^C + \frac{1}{4} C_{AB} N_{CD} C^{CD} - \frac{1}{8} \epsilon_A{}^C C_{CB} \epsilon_{DE} D^E D_C C^{CD}$ [Nichols'18]

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  - $G_{uu} = 0 \Longrightarrow \partial_u M =$
  - $G_{uA} = 0 \Longrightarrow \partial_u N_A =$
  - $G_{AB} = 0 \Longrightarrow \partial_u E_{AB} =$

Greatly simplify after reorganizing asymptotic data according to symmetry!

[Nichols'18]

Asymptotic symmetries

Extended BMS ~ enhancement of Poincare including angle−dependent translations and Lorentz transformations

- Solve $\mathscr{L}_\xi g_{\mu\nu} = \mathcal{O}(r^{-\#}) \implies \xi(T,Y) = T(z,\bar{z})\partial_u + Y^A(z,\bar{z})\partial_A + \frac{1}{2}D\cdot Y\left(u\partial_u - r\partial_r\right) + \cdots$

Order dictated by large−r fall−off of metric components

$$\left.\begin{array}{l}\text{Supertranlsations}\\[2ex]\text{Superrotations}\end{array}\right\}\quad \text{generate eBMS}_4\text{ algebra}$$

[Barnich, Troessaert '09]

BMS primaries

Definition: $\Phi_{(h,\bar{h})}(z,\bar{z})$ is a conformal primary field of weights $(h,\bar{h})$ if it obeys

$$\delta_Y \Phi_{(h,\bar{h})} = \left(Y^A \partial_A + h \partial_z Y^z + \bar{h} \partial_{\bar{z}} Y^{\bar{z}} \right) \Phi_{h,\bar{h}}$$

- Metric components (M, N_A, T_{AB}) at fixed u cut on $\mathscr{I}$ are in general not primaries
- Can build combinations of them that are primaries at $u = 0$, eg. $\mathscr{M} = M + \frac{1}{8} C_{AB} N^{AB}$, $\qquad (h,\bar{h}) = \left(\frac{3}{2}, \frac{3}{2} \right)$

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Primary fields constructed from asymptotic data up to $\mathcal{O}(r^{-1})$:

$$(h,\bar{h}) \equiv \left(\frac{\Delta + J}{2}, \frac{\Delta - J}{2}\right)$$

Primaries	C	$\mathcal{N} \equiv \partial_u \hat{N}$	$\mathcal{J}$	$\mathcal{M}$	$\widetilde{\mathcal{M}}$	$\mathcal{P}$	$\mathcal{T}$
(Δ, J)	(1,2)	$(3,-2)$	$(3,-1)$	(3,0)	(3,0)	(3,1)	(3,2)

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$$C \equiv C_{AB} m^A m^B \qquad \mathcal{M}_{\mathbb{C}} \equiv \mathcal{M} + i\widetilde{\mathcal{M}}$$

$$\mathcal{N} \equiv \mathcal{N}^{AB} \bar{m}_A \bar{m}_B \qquad \mathcal{P} \equiv \mathcal{P}_A m^A$$

$$\mathcal{J} \equiv \mathcal{J}^A \bar{m}_A \qquad \mathcal{T} \equiv \mathcal{T}_{AB} m^A m^B$$

Frame fields: $m^A \bar{m}_A = 1$

Part of null spacetime tetrad $(l, n, m, \bar{m})$

Einstein equations at large−r, revisited

In terms of the BMS−covariant quantities, leading Einstein's equations collapse to:

$$(\dot{\mathscr{J}} = \tfrac{1}{2}D\mathscr{N}, \quad D = m^A D_A)$$

$$\mathscr{M}_{\mathbb{C}} \equiv Q_0, \quad \mathscr{P} \equiv Q_{s=1}, \quad \mathscr{T} \equiv Q_{s=2}$$

$$\left.\begin{aligned}
\dot{\mathscr{M}}_{\mathbb{C}} &= D\mathscr{J} + \frac{1}{4}C\mathscr{N} \iff \partial_u M = \cdots \\
\dot{\mathscr{P}} &= D\mathscr{M}_{\mathbb{C}} + C\mathscr{J} \iff \partial_u N_A = \cdots \\
\dot{\mathscr{T}} &= D\mathscr{P} + \frac{3}{2}C\mathscr{M}_{\mathbb{C}} \iff \partial_u T_{AB} = \cdots
\end{aligned}\right\}$$

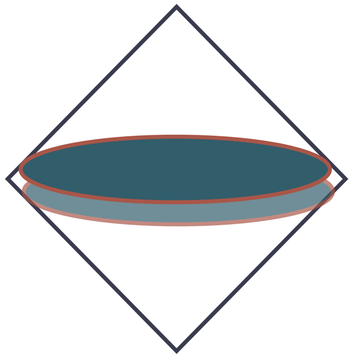
$$\boxed{\partial_u Q_s = DQ_{s-1} + \frac{s+1}{2}CQ_{s-2}, \quad s = 0,1,2}$$

+ boundary conditions $\quad Q_{-1} = \frac{1}{2}D\partial_u N, \quad Q_{-2} = \frac{1}{2}\partial_u N$

Einstein equations at large$-r$, revisited

$$\partial_u Q_s = DQ_{s-1} + \frac{s+1}{2} CQ_{s-2}, \quad s = 0,1,2$$

- Can solve order by order in the number of fields: $Q_s = Q_s^{(1)} + Q_s^{(2)} + \cdots$ (polynomial in field space)
- Expect to obtain conserved quantities near $\mathscr{I}^{\pm}_{\mp}$, but $\lim_{u \to -\infty} Q_s(u, z, \bar{z}) = \infty, \quad s \geq 1$
- Divergences can be systematically regularized by imposing no-flux condition $\Longrightarrow$ $q_s(z, \bar{z})$
- Assume matching across i_0

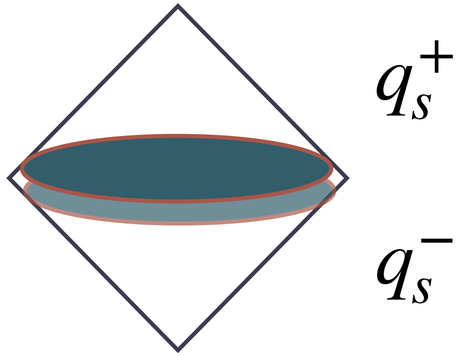


$$q_s^+ \quad \| \quad q_s^-$$

$\Longrightarrow$ spacetime conservation law

From conservation laws to soft theorems

$$\partial_u Q_s = DQ_{s-1} + \frac{s+1}{2} CQ_{s-2}, \quad s = 0,1,2$$



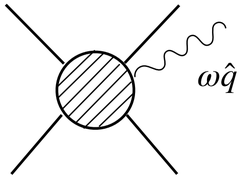
- Define pairing $\int_{S^2} \mathcal{F}(z,\bar{z}) q_s^{\pm}(z,\bar{z}) \equiv \mathcal{Q}_s$

- Matching $\Longrightarrow$ charges are conserved hence commute with the S−matrix:

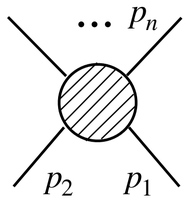
$$\langle \text{out} | \mathcal{Q}_s^+ \mathcal{S} - \mathcal{S} \mathcal{Q}_s^- | \text{in} \rangle = 0$$

- For s = 0, 1, 2 reproduce tree−level leading, subleading and subsubleading soft graviton theorems!

$$\lim_{\omega \to 0}$$



$$= \sum_{s=0}^{2} \omega^{s-1} S_{\pm}^{(s)} \times$$



From conservation laws to soft theorems

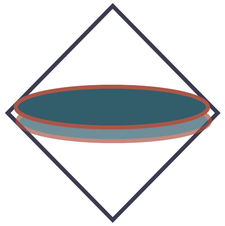
$$\partial_u Q_s = DQ_{s-1} + \frac{s+1}{2} C Q_{s-2}, \quad s \in \mathbb{N}$$

- Postulate extension for all positive integers s (can prove to linear order)

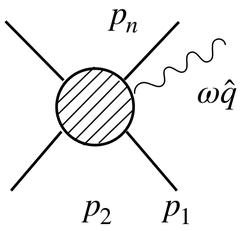
$$\begin{cases} q_s^1 \propto D^{s+2}\mathscr{M}_-(s), \quad \mathscr{M}_-(s) = \int du u^s N(u) \\ \text{(sub)}^s\text{−leading soft graviton} \end{cases}$$

- Use $\{N(u, x^A), C(u', x'^A)\} = \frac{\kappa^2}{2}\delta(u-u')\delta^{(2)}(x^A - x'^A)$ to show that:

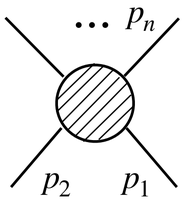
Conservation laws truncated to quadratic order $\langle \text{out} | [\mathcal{Q}_s^1, \mathcal{S}] | \text{in} \rangle = -\langle \text{out} | [\mathcal{Q}_s^2, \mathcal{S}] | \text{in} \rangle \implies$ (sub)^s−leading soft theorem!



$$\lim_{\omega \to 0}$$



$$= \sum_{s=0}^{\infty} \omega^{s-1} S_{\pm}^{(s)} \times$$



Higher-spin symmetry

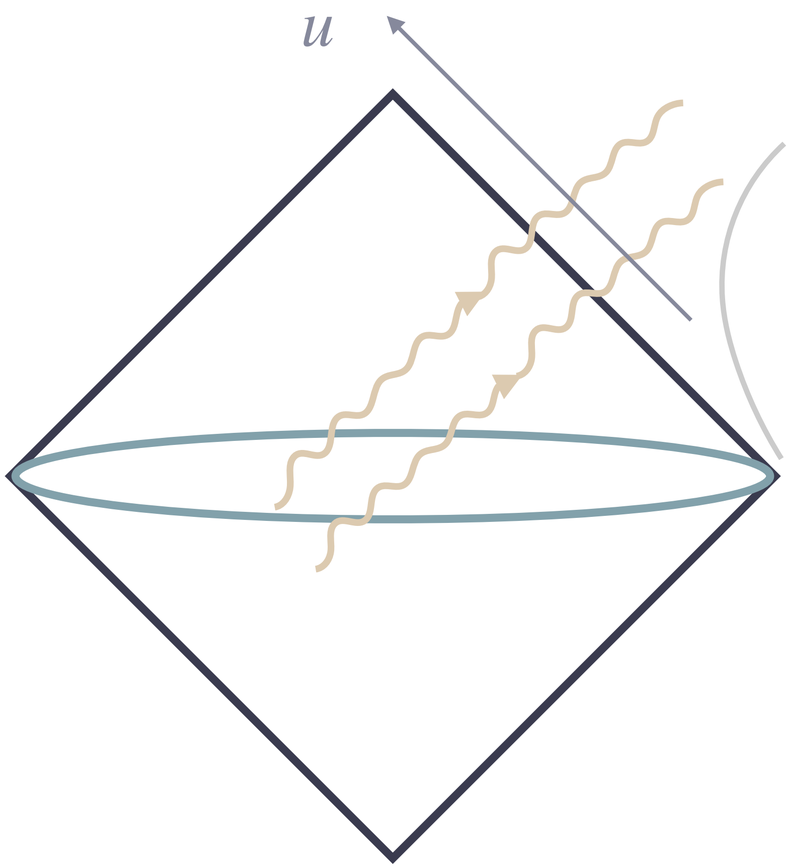
Use $\{N(u,x^A), C(u',x'^A)\} = \frac{\kappa^2}{2}\delta(u-u')\delta^{(2)}(x^A - x'^A)$ to show that (truncated) q_s realize a $w_{1+\infty}$ algebra on phase space:

$$\{q_s(z), q_{s'}(z')\}^1 = \{q_s^2, q_{s'}^1\} + (s \leftrightarrow s') = \frac{\kappa^2}{8}\left[-(s'+1)q^1_{s+s'-1}(z')D_z\delta(z,z') + (s+1)q^1_{s+s'-1}(z)D_{z'}\delta(z,z')\right]$$

[Freidel, Pranzetti, A.R '21]

- Similar structure (infinite symmetry algebra) in YM
- Same charge algebra at quadratic order upon including cubic terms in the truncation! [Freidel, Pranzetti, A.R '23]

Conformally soft gravitons



Charges well defined provided

$$N \sim |u|^{-1-s-\epsilon}, \quad u \to \pm\infty$$

IR condition

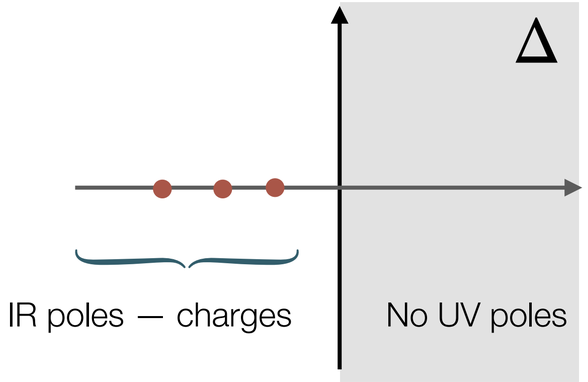
Assume exponential suppression as $\omega \to \infty$

$$\widetilde{N}(\omega) = \int du e^{i\omega u} \partial_u C^*(u)$$

UV condition

No poles in RH complex Δ plane

$$\widehat{N}(\Delta) = \int_0^\infty d\omega \omega^{\Delta-1} \widetilde{N}(\omega)$$



[Arkani-Hamed, Pate, A.R., Strominger '20]

Conformally soft gravitons = memory ``observables”

Tower of soft/memory operators:

$$\mathscr{M}(s) \equiv \operatorname*{Res}_{\Delta=-s} \widehat{N}(\Delta) = \frac{i^s}{s!}\int_{-\infty}^{+\infty} du u^s N(u), \quad s \in \mathbb{N}$$

completely determine gravitational signals that decrease rapidly as $\omega \to \infty$ (obey the UV condition)

$$\widetilde{N}_{\pm}(\omega) = \sum_{n=0}^{\infty} \omega^n \mathscr{M}_{\pm}(n)$$

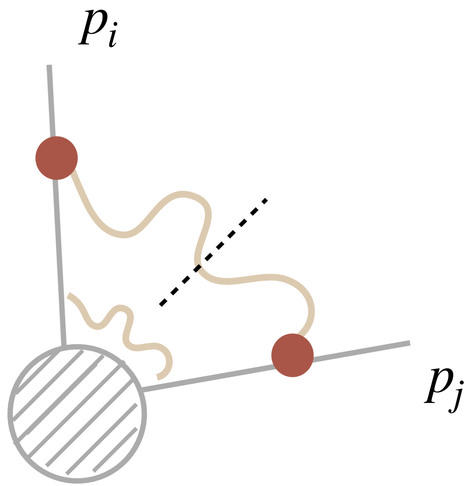
Tower of Goldstone operators

$$\mathcal{S}_{\pm}(n) \equiv \lim_{\Delta \to n} \widehat{N}_{\pm}(\Delta),\ n \in \mathbb{N}, \qquad \mathcal{S}_{+}(n) = D_z^{n+2}\mathcal{G}_{+}(n), \qquad \mathcal{S}_{-}(n) = D_{\bar{z}}^{n+2}\mathcal{G}_{-}(n)$$

- Involved in the all order generalization of Dirac−Faddeev−Kulish dressings:

[Dirac '31; Weinberg '65; Chung; Kibble; Faddeev, Kulish⋯]

$$\langle p | \mathcal{D}_{\pm} \equiv \langle p | \exp\left\{ \sum_{s=0}^{\infty} \frac{(-1)^s}{\pi\kappa^2} \int d^2z \left[Q_{\mp}(s,z;p)\mathcal{G}_{\pm}(s,z) - Q^*_{\mp}(s,z;p)\mathcal{G}^*_{\mp}(s,z) \right] \right\}$$



- Time signal reconstructed from Goldstones provided $C(u)\Big|_{u\to\infty} \propto |u|^{-1-n},\ \forall n > 0$ (IR condition)

$$C_{\pm}(u) = \frac{i}{2\pi} \sum_{n=0}^{\infty} \frac{(-iu)^n}{n!} \mathcal{S}_{\pm}(n)$$

UV & IR conditions $\Longleftrightarrow$ $N(u) \in \mathbf{S}$ = space of **Schwartz functions**

A discrete basis

$$\Theta \equiv \frac{2}{\kappa^2}\int du \int_S N\delta C$$

$$C_\pm(u) = \frac{i}{2\pi}\sum_{n=0}^{\infty}\frac{(-iu)^n}{n!}\mathcal{S}_\pm(n)$$

$$|\,n, z, \pm\,\rangle \equiv \frac{1}{\sqrt{\pi\kappa^2}}\mathcal{M}^\dagger_\pm(n,z)\,|\,0\rangle$$

$$|\,\hat{n}, z, \pm\,\rangle \equiv \frac{1}{\sqrt{\pi\kappa^2}}\mathcal{S}^\dagger_\pm(n,z)\,|\,0\rangle$$

- Orthogonality with respect to the symplectic product

$$\Theta = \Theta_+ + \Theta_- + \delta\alpha, \qquad \Theta_\pm = \frac{1}{i\kappa^2\pi}\sum_{n=0}^{\infty}\int_S d^2z\sqrt{q}\,\mathcal{M}_\pm(n)\delta\mathcal{S}^*_\pm(n)$$

- Completeness: $$P = \sum_{n=0,\epsilon\in\pm}^{\infty}\int_S |\,n, z, \epsilon\rangle\langle\hat{n}, z, \epsilon\,| = \sum_{n=0,\epsilon\in\pm}^{\infty}\int_S |\,\hat{n}, z, \epsilon\rangle\langle n, z, \epsilon\,|$$

- P coincides with the identity operator on the set of Schwartz functions!

$\mathcal{M}_\pm(n),\ \mathcal{S}_\pm(n)$ form a complete basis for gravitational signals in $\mathbf{S}$!

[Freidel, Pranzetti, A.R. '22]

Summary

Infinite tower of charges extracted from Einstein's equations

Matching condition ↔ conservation ↔ universal tower of tree−level soft theorems

Higher spin symmetry algebra in 2d!

Tower of memory/Goldstone modes forms a "discrete" basis for "sufficiently localized" wavepackets

Outlook: observational signatures of QG?

Theoretical: Asymptotic symmetry interpretation of the tower of soft operators? Higher spin symmetry from diffeos?!

Beyond tree level – loops, higher derivative corrections?

Meaningful constraints on scattering amplitudes and/or low energy EFT parameters?

Physical interpretation of IR-finite terms in dressings/soft S-matrix; higher dimensions?

Practical: Logarithmic soft theorems, tails?

Scattering amplitudes for GW: multipoles, relation to PN expansion, prospects for measuring tower of memories?

[…Grant, Nichols '22]

Matter, different backgrounds

Relation to shockwaves (beyond leading order) [Verlinde, Zurek]

Mulțumesc!