"The Trans-Carpathian Seminar on Geometry & Physics"

(See also the Geometry & Physics @ DFT seminar homepage)

Date: Wednesday, March 20, 2024, 15:15 Bucharest time

Location: online via Zoom

Speaker: **Prof. Janusz Grabowski** (IMPAN)

Title: **Homogeneity and formalisms of mechanics**

Abstract: We will present the concept of graded bundles whose canonical examples are higher tangent bundles - the playground for ODEs and mechanics. The fundamental discovery is that graded bundles can be characterized by homogeneity structures, understood as actions of the monoid of multiplicative reals on manifolds, which substantially simplifies many concepts and proofs in differential geometry. In particular, vector bundles are fully characterized by the multiplication by reals; one can forget the addition. Consequently, the concept of compatibility of a geometric structure with the vector bundle structure finds an elementary description. As fundamental examples serve double vector bundles. We will show that homogeneity structures can be naturally lifted to the tangent and cotangent bundles of the manifold, which produces canonical examples of double vector bundles. The main model in mechanics is the so-called Tulczyjew Triple, which leads to the best and simplest geometric description of Lagrangian and Hamiltonian mechanics we know, with no assumption of regularity for Lagrangians. The rest of the talk will be the explanation of how dynamics is obtained from a (possibly singular) Lagrangian and how both formalisms are related (Legendre transformation). The description of the Tulczyjew Triple in terms of isomorphisms of symplectic double vector bundles can be easily generalized and leads, for instance, to geometric mechanics on algebroids - the subject of the next seminar talk.
The talk will last 2x45 min with a break in the middle.