Primordial Black Holes from Rapid Turns in Two-field Models

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(arXiv:2012.03705 [hep-th])

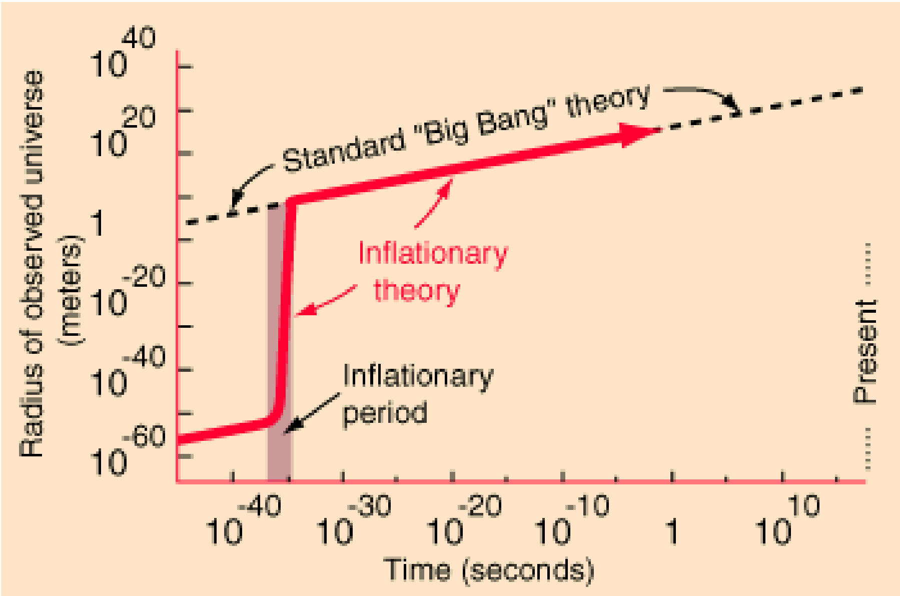
Primordial Black Holes (PBH)

Formed in Early Universe:

Large enough fluctuations during inflation can seed PBHs

Cosmological inflation: Period of very fast expansion of space in the Early Universe (faster than speed of light)

$\Rightarrow$ homogeneity and isotropy observed today



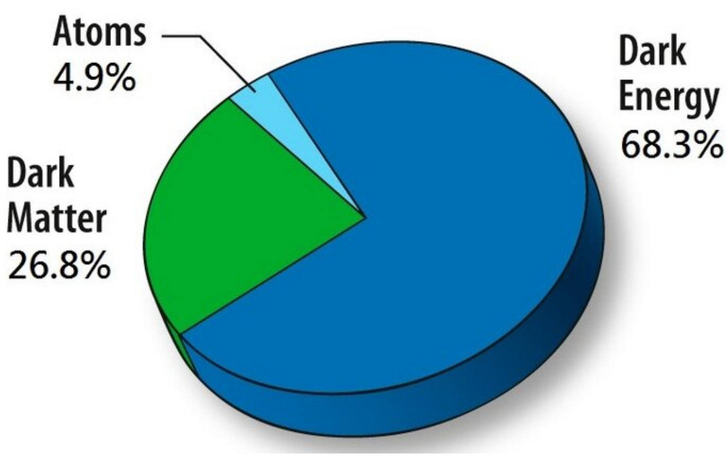
PBHs can contribute to Dark Matter (DM):

Sufficient abundance of PBHs: natural candidate for DM

(Depending on the model: from a fraction to all of DM...)

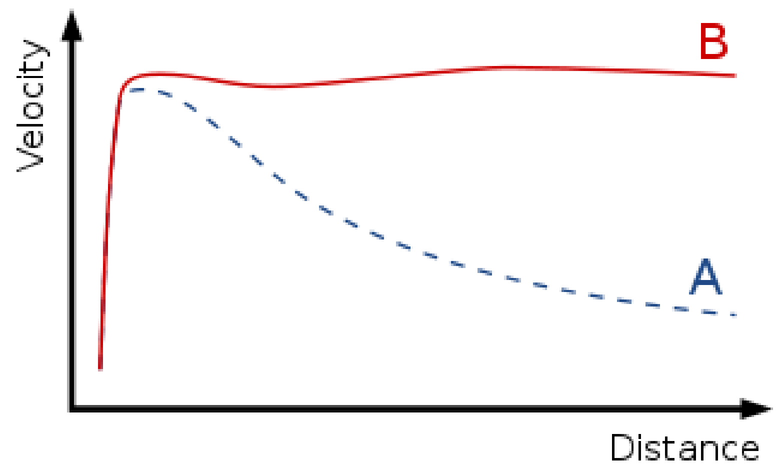
Composition of the Universe today

(from CMB observations and other sources)



Galactic rotation curves

(early evidence for DM)



[Orbital Velocity vs Distance from Galactic Center]

A - prediction due to visible matter only , B - observation

Observational evidence for PBHs?:

LIGO/Virgo Collaboration: (first detection: LIGO in 2015)

So far, observed Gravitational Waves (GW) from about 50 binary Black Hole (BH) mergers

From GW data: can determine characteristics of BHs

(mass, redshift, spin)

Analysis of the data:

arXiv:2008.12320 [astro-ph.CO], arXiv:2010.13811 [astro-ph.CO], arXiv:2102.03809 [astro-ph.CO],

arXiv:2105.03349 [gr-qc], arXiv:2106.13769 [astro-ph.CO], arXiv:2110.06815 [astro-ph.CO], ...

$\rightarrow$ indications that fraction of BHs are primordial

Theoretical understanding of PBH-generation:

- Single-field inflationary models:

  More conventional, but PBH-formation is a challenge...

- Multi-field cosmological inflation:

  – Motivated by quantum gravity

  (string compact.: even number of scalars ; swampland conjectures...)

  – Leads to new phenomena

Goal:

PBH-generation from certain class of two-field models

(due to solutions of the EoM, which exhibit sharp turns in field space)

Two-field Cosmological Models

Action:

$$S = \int d^4x \, \sqrt{-\det g} \left[\frac{R}{2} - \frac{1}{2} G_{IJ}(\phi) \, g^{\mu\nu} \, \partial_\mu \phi^I \, \partial_\nu \phi^J - V(\phi) \right] \; ,$$

$g_{\mu\nu}(x)$ - spacetime metric , $\mu, \nu = 0, ..., 3$

$G_{IJ}(\phi)$ - field space metric , $I, J = 1, 2$

Standard background Ansatze:

$$ds_g^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad , \quad \phi^I = \phi_0^I(t) \quad ,$$

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$ - Hubble parameter

Conceptual note:

In single-field models potential $V(\phi)$ plays key role:

Always: field redefinition $\rightarrow$ canonical kinetic term

(Can transfer complexity to the potential)

In multi-field models:

Cannot redefine away the curvature of G_{IJ}!

(I.e., kinetic term becomes important!)

$\Rightarrow$ Can have: - Genuine two (or multi-) field trajectories even when $\partial_{\phi^I} V = 0$ for some I

- New phenomena due to non-geodesic motion in field space

Characteristics of a background trajectory:

Background trajectory $(\phi_0^1(t), \phi_0^2(t))$ in field space:

Tangent and normal vectors: $\quad I, J = 1, 2$

$$T^I = \frac{\dot\phi_0^I}{\dot\phi_0} \quad , \quad \dot\phi_0^2 = G_{IJ}\dot\phi_0^I \dot\phi_0^J$$

$$N_I = (\det G)^{1/2} \epsilon_{IJ} T^J$$

(Note: $\quad N_I T^I = 0 \;\; , \;\; T_I T^I = 1 \;\; , \;\; N_I N^I = 1$)

Turning rate of the trajectory:

$$\Omega = -N_I D_t T^I \quad ,$$

$$D_t T^I \equiv \dot\phi_0^J \, \nabla_J T^I = \dot T^I + (\Gamma_G)^I_{JK} \, \dot\phi_0^J \, T^K$$

Characteristics of a background trajectory:

Equivalently, the turning rate:

$$\Omega^2 = G_{IJ}(D_t T^I)(D_t T^J) = ||D_t T^I||^2$$

Slow-roll parameters:

$$\varepsilon = -\frac{\dot{H}}{H^2} \quad , \quad \eta^I = -\frac{1}{H\dot{\phi}_0} D_t \dot{\phi}_0^I$$

Expand: $\eta^I = \eta_{||} T^I + \eta_\perp N^I \quad \rightarrow \quad \Omega = \eta_\perp H$

$\varepsilon \, , \eta_{||}$: same as for single-field inflation with inflaton $\phi_0(t)$

Slow roll: $\varepsilon \, , \eta_{||} \ll 1 \quad ; \quad$ Our interest: $\eta_\perp^2 \gg 1$

Perturbations around the background:

Decomposition:

Inflatons: $$\phi^I(t,\vec{x}) = \phi_0^I(t) + \delta\phi^I(t,\vec{x})$$

Spatial part of metric:

$$g_{ij}(t,\vec{x}) = a^2(t)\left[(1+2\zeta)\delta_{ij} + h_{ij}\right] \quad , \quad i,j = 1,2,3 \quad ,$$

$\zeta = \zeta(t,\vec{x})$ - curvature perturbation ,

$h_{ij} = h_{ij}(t,\vec{x})$ - tensor fluctuations

Expand: $$(\delta\phi)^I = (\delta\phi)_{\|} T^I + (\delta\phi)_{\perp} N^I \quad ,$$

$(\delta\phi)_{\|}$ - adiabatic pert. , $(\delta\phi)_{\perp}$ - entropic pert.

Perturbations around the background:

Gauge choice: $(\delta\phi)_{||} \equiv 0$

Only indep. scalar degrees of freedom: ζ , $(\delta\phi)_\perp$

Substitute (backgr.+pert.)-decomposition in Action:

$\rightarrow$ Effective Action for the perturbations

Important ingredients:

– interaction: $\dot{\zeta}\,(\delta\phi)_\perp$ (coeff. depends on backgr.)

[So $(\delta\phi)_\perp$ affects ζ and thus the density perturbations]

– mass m_s^2 for entropic pert. $(\delta\phi)_\perp$

Perturbations around the background:

Effective entropic mass:

$$m_s^2 \; = \; N^I N^J V_{;IJ} - \Omega^2 + \varepsilon H^2 \mathcal{R} \quad ,$$

$$V_{;IJ} \; = \; \partial_I \partial_J V - (\Gamma_G)^K_{IJ} \, \partial_K V \quad , \quad \partial_I \equiv \partial_{\phi_0^I} \quad ,$$

$\mathcal{R}$ - Ricci scalar of field-space metric G_{IJ}

Power spectrum of curvature perturbation:

(recall: $\eta_\perp = \Omega / H$)

$$\mathcal{P}_\zeta \; \sim \; \mathcal{P}_0 \, e^{c \, |\eta_\perp|} \quad , \quad c = const > 0$$

For PBH generation, need δt with: $\mathcal{P}_\zeta / \mathcal{P}_0 \sim 10^7$

Important remark: $\eta_\perp^2 \gg 1 \;\longleftrightarrow\; m_s^2 < 0$

$\rightarrow$ Period δt with $m_s^2 < 0 \;\Rightarrow\;$ desired enhancement of $\mathcal{P}_\zeta$!

(I.e., brief tachyonic instability $\Rightarrow$ PBH-generation !)

Rotationally-invariant scalar manifold

Take rotationally-invariant metric G_{IJ} : (recall: $I, J = 1, 2$)

$$ds_G^2 = d\varphi^2 + f(\varphi)d\theta^2 \quad ,$$

$$\phi_0^1(t) \equiv \varphi(t) \quad , \quad \phi_0^2(t) \equiv \theta(t) \quad , \quad f(\varphi) \geq 0 \;\; \forall\varphi$$

Can compute the turning rate $\Omega(t)$ and entropic mass $m_s^2(t)$ for every background trajectory $(\varphi(t), \theta(t))$:

- Turning rate:

$$\Omega = \frac{\sqrt{f}}{\left(\dot{\varphi}^2 + f\dot{\theta}^2\right)} \left[\dot{\theta}\partial_\varphi V - \frac{\dot{\varphi}}{f}\partial_\theta V\right]$$

Rotationally-invariant metric G_{IJ}:

- Entropic mass:

Too complicated, but simplifies for $\partial_\theta V = 0$:

$$m_s^2 = M_V^2 - \Omega^2 + \varepsilon H^2 \mathcal{R} \quad ,$$

$$M_V^2 \equiv \frac{f\dot{\theta}^2 \partial_\varphi^2 V + \frac{f'}{2f}\dot{\varphi}^2 \partial_\varphi V}{(\dot{\varphi}^2 + f\dot{\theta}^2)}$$

Note:

Even for $\partial_\theta V = 0$ there are genuine two-field trajectories

$(\varphi(t), \theta(t))$ in field space $\quad \rightarrow \quad$ important in following!

Background solutions

We will consider class of solutions of background EoMs obtained for hyperbolic field-space metric G_{IJ} (recall: $I, J = 1, 2$)

$\rightarrow$ Two-dimensional field space: hyperbolic surface

(Gaussian curvature $K_G = const < 0$)

Cosmological models of this type: α-attractors

Kallosh, Linde et al. (arXiv:1311.0472 [hep-th], arXiv:1405.3646 [hep-th], arXiv:1503.06785 [hep-th], arXiv:1504.05557 [hep-th])

Many numerical studies in the literature...

Several classes of exact solutions: Anguelova, Babalic, Lazaroiu, arXiv:1809.10563 [hep-th]

Exact solutions with Noether symmetry

Exact solutions of JHEP 1904 (2019) 148, arXiv:1809.10563 [hep-th] :

obtained by using Noether symmetry method

Imposing Noether symmetry is a powerful technical tool:

- can restrict:
  - form of potential V (expected)
  - value of Gaussian curvature K_G (unexpected!)

  (hence: may help for embedding in fundamental theory)

- can lead to simplified EoMs and thus facilitate finding exact solutions (as opposed to numerical ones)

Reduced action:

Substituting ansatz $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, $\phi^I = \phi_0^I(t)$:

$$\mathcal{L} = -3a\dot{a}^2 + a^3 \left[\frac{1}{2} G_{IJ} \dot{\phi}_0^I \dot{\phi}_0^J - V(\phi_0) \right]$$

→ classical mechanical action for $\{a, \phi_0^I\}$ ds.o.f.

Euler-L. eqs of $\mathcal{L}$ ≡ original EoMs, when imposing constraint:

$$E_{\mathcal{L}} \equiv \dot{a}\frac{\partial \mathcal{L}}{\partial \dot{a}} + \dot{\phi}_0^I \frac{\partial \mathcal{L}}{\partial \dot{\phi}_0^I} - \mathcal{L} = 0$$

Note: $E_{\mathcal{L}} = const$ on solutions of EL eqs., so Hamiltonian constraint → relation between integration constants

Noether symmetry:

Recall: $\mathcal{L} = -3a\dot{a}^2 + a^3\Big[\frac{1}{2}G_{IJ}\dot{\phi}_0^I\dot{\phi}_0^J - V(\phi_0)\Big]$

Denote $q^{\hat{I}} \equiv \{a, \phi_0^I\}$ - generalized coordinates on $\mathcal{M}$

Consider transformation $q^{\hat{I}} \to Q^{\hat{I}}(q)$:

– generated by: $X = X^a(a,\phi_0)\,\partial_a + X^I(a,\phi_0)\,\partial_{\phi_0^I}$

– induces transf. on tangent bundle $T\mathcal{M}$, generated by :

(with coord. $\{q^{\hat{I}}, \dot{q}^{\hat{I}}\}$)

$$\hat{X} = X + \dot{X}^a(a,\phi_0,\dot{a},\dot{\phi}_0)\,\partial_{\dot{a}} + \dot{X}^I(a,\phi_0,\dot{a},\dot{\phi}_0)\,\partial_{\dot{\phi}_0^I}$$

Symmetry condition: $L_{\hat{X}}(\mathcal{L}) = 0$

Noether symmetry: (Anguelova, Babalic, Lazaroiu, arXiv:1905.01611 [hep-th])

$L_{\hat{X}}(\mathcal{L}) = 0 \quad \Rightarrow$ coupled system of PDEs equivalent with:

$$X^a = \frac{\Lambda(\phi_0)}{\sqrt{a}} \quad , \quad X^I = Y^I(\phi_0) - \frac{4}{a^{3/2}} G^{IJ} \partial_J \Lambda \quad ,$$

where Λ and Y^I satisfy:

- $\nabla_I Y_J + \nabla_J Y_I = 0 \quad , \quad Y^I \partial_I V = 0$

  $\rightarrow$ Y^I - Killing vector preserving $V(\phi_0)$

- $\nabla_I \nabla_J \Lambda = \frac{3}{8} G_{IJ} \Lambda \quad , \quad G^{IJ} \partial_I V \partial_J \Lambda = \frac{3}{4} V \Lambda$

  $\rightarrow$ Λ - hidden symmetry (mixes a and $\{\phi_0^I\}$!)

Rotationally-invariant G_{IJ}: (recall: $I, J = 1, 2$)

Consider rot.-invariant metric G_{IJ} on $\mathcal{M}_0$ (with coord. $\{\phi_0^I\}$) :

$$ds_G^2 = d\varphi^2 + f(\varphi)d\theta^2$$

– Showed that Hessian equation $\nabla_I \nabla_J \Lambda = \frac{3}{8} G_{IJ} \Lambda$ implies:

$$K_G = -\frac{3}{8}$$

$\rightarrow$ Λ-symmetry requires hyperbolic $\mathcal{M}_0$!

– Found general Λ-solution for any rotationally-invariant hyperbolic surface

With known Λ: $G^{IJ} \partial_I V \partial_J \Lambda = \frac{3}{4} V \Lambda$ - equation for V

Exact solutions from separation of variables:

With separation-of-variables Ansatz, found V for three types of rotationally-invariant hyperbolic surfaces (arXiv:1809.10563 [hep-th])

To solve EL equations, transform to generalized coord., adapted to the symmetry: $(a, \varphi, \theta) \to (u, v, w)$, $\frac{\partial L}{\partial w} = 0$

[see arXiv:1809.10563 [hep-th] for the explicit expressions for: $a = a(u, v, w)$, $\varphi = \varphi(u, v, w)$, $\theta = \theta(u, v, w)$]

$\to$ easily solve EL eq. for cyclic variable : $w = w(t)$

$\to$ obtain simplified EL eqs. for $u = u(t)$, $v = v(t)$

$\Rightarrow$ many new exact solutions

Class of exact solutions:

(arXiv:1809.10563 [hep-th])

Take G_{IJ} - metric on Poincaré disk $\&$ impose hidden symmetry :

$$\Rightarrow \quad f(\varphi) = \frac{1}{q^2}\sinh^2(q\,\varphi) \;\;,\;\; V(\varphi,\theta) = V_0\cosh^2(q\,\varphi) \;\;,$$

$$q = \sqrt{\tfrac{3}{8}} \quad , \quad V_0 > 0$$

Poincaré disk metric:

(α-attractor notation)

$$ds_D^2 = 6\alpha \frac{dz d\bar{z}}{(1 - z\bar{z})^2} \quad ,$$

$$z = \rho e^{i\theta} \;\;,\;\; \rho \in [0,1) \;\;,$$

α - arbitrary parameter ; hid. sym.: $\alpha = \frac{16}{9}$

$$\rho = \tanh\left(\frac{\varphi}{\sqrt{6\alpha}}\right) \quad \Rightarrow \quad ds_D^2 = d\varphi^2 + f(\varphi)d\theta^2$$

Class of exact solutions:

(arXiv:1809.10563 [hep-th])

Then the background EoMs are solved by:

$$a(t) = \left[u^2 - \left(v^2 + w^2\right)\right]^{1/3} \quad ,$$

$$\varphi(t) = \sqrt{\frac{8}{3}}\,\text{arccoth}\left(\sqrt{\frac{u^2}{v^2 + w^2}}\right) \quad ,$$

$$\theta(t) = \text{arccot}\left(\frac{v}{w}\right) \quad ,$$

$$u(t) = C_1^u \sinh(\kappa\, t) + C_0^u \cosh(\kappa\, t) \quad , \quad \kappa \equiv \frac{1}{2}\sqrt{3V_0} \quad ,$$

$$v(t) = C_1^v\, t + C_0^v \qquad \text{and} \qquad w(t) = C_1^w\, t + C_0^w \quad ,$$

$$(C_1^v)^2 + (C_1^w)^2 = \kappa^2 \left[(C_1^u)^2 - (C_0^u)^2\right]$$

New results

Exact solutions with hidden symmetry:

- Proved that $\rho(t)$ can have at most two local extrema

  $\rightarrow$ Shape of trajectory: greatly restricted ;

  In particular: a single sharp rapid turn

- Showed the presence of the desired tachyonic instability

  – Sharp turn $\Rightarrow$ peak of $|\Omega|$ [Note: $\mathrm{sgn}\,(\Omega(t)) = const \;\; \forall t$]

  – $|\Omega|$-peak $\Rightarrow$ large and negative entropic mass m_s^2 ,

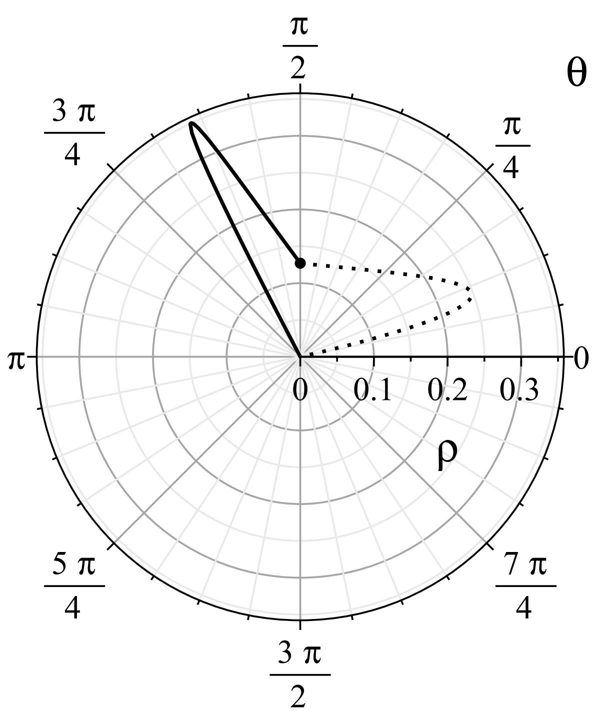
  i.e. brief tachyonic instability needed for PBH generation !

Examples of exact solutions:

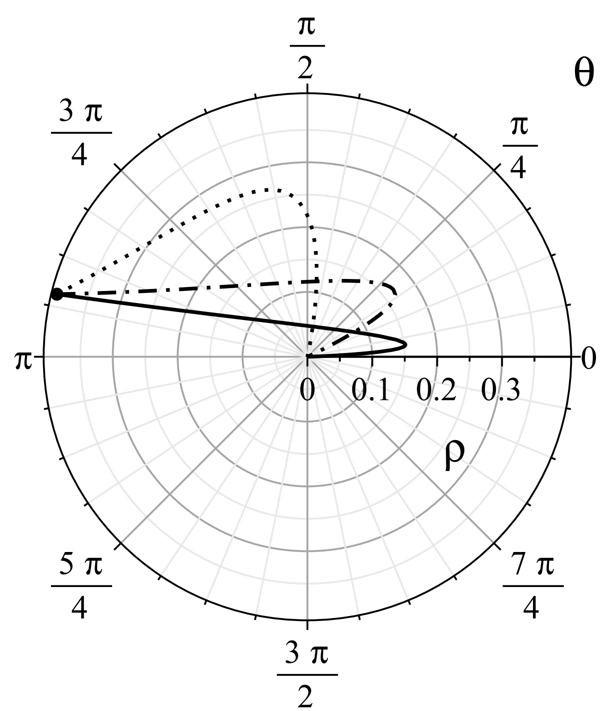
Illustration of all possible types of trajectories on Poincaré disk

[recall: radial variable $\rho \in [0, 1)$]

New result: $\rho(t)$ can have 0 , 1 or 2 local extrema



1 local extremum

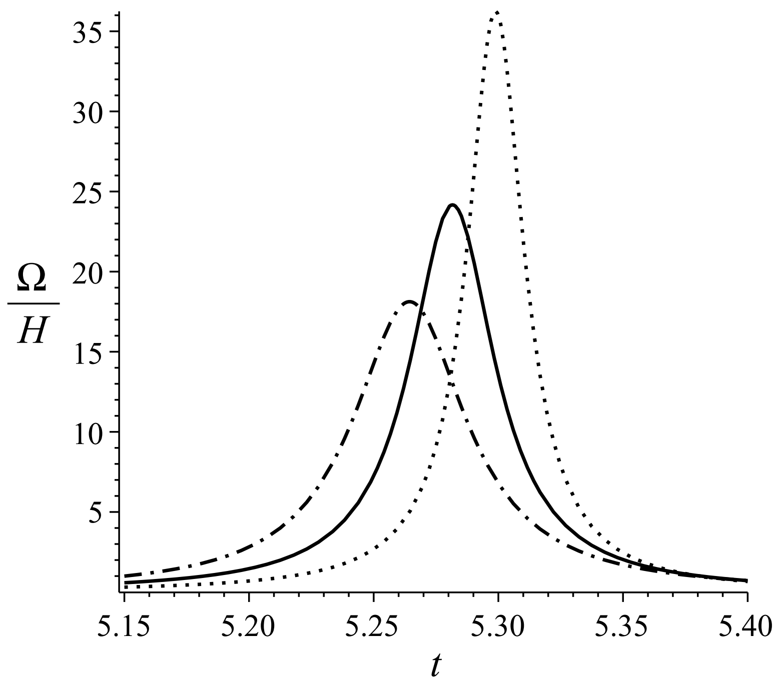


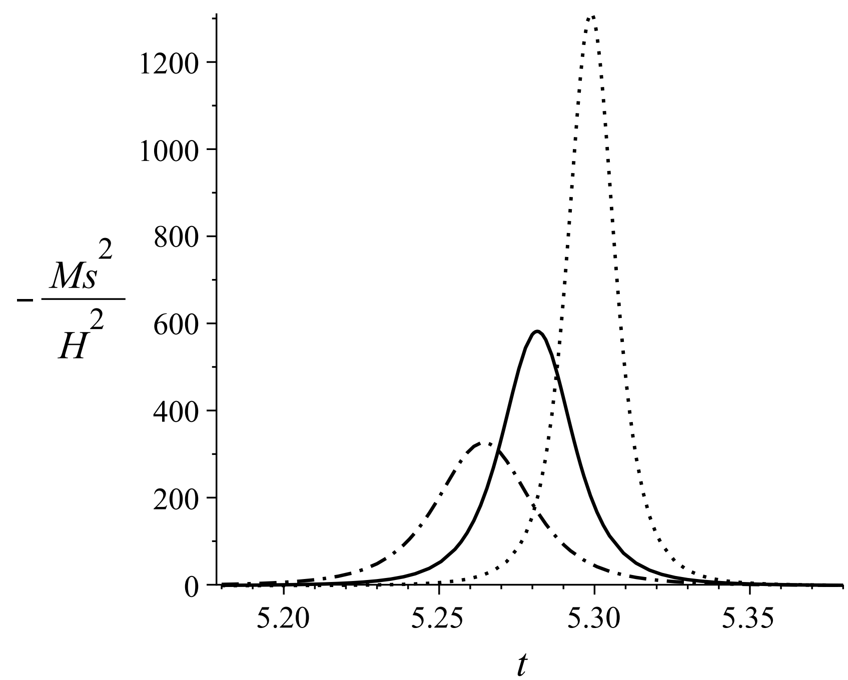
0 or 2 local extrema

Examples of exact solutions:

Illustration of behavior of $\eta_{\perp}(t) = \frac{\Omega}{H}$ and entropic mass $M_s^2(t)$

New result: transient tachyonic instability





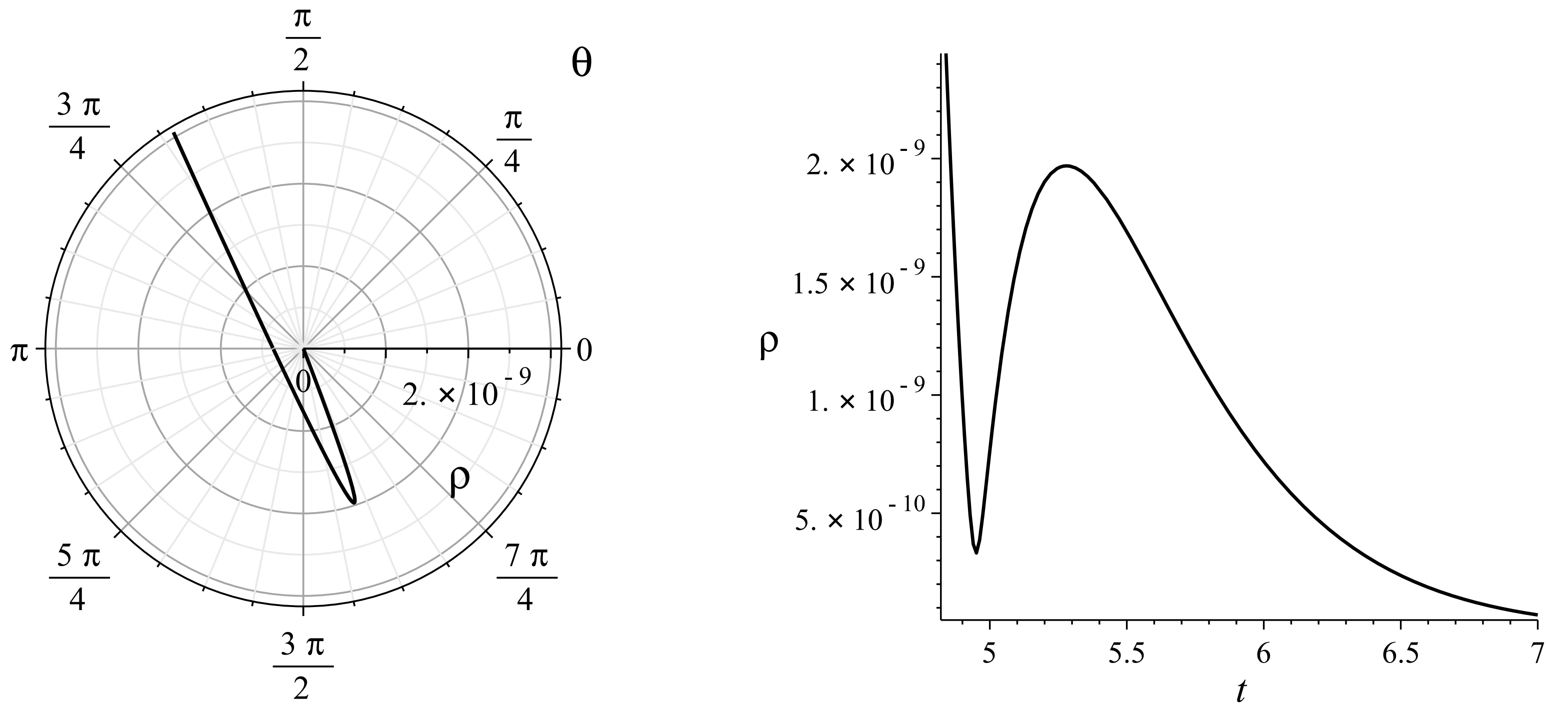
Three examples with number of e-folds $N = \int H dt$ at peak: $\sim$ 11

(PBH generation : $|\eta_{\perp}|_{peak} \sim 25$)

Examples of exact solutions:

Illustration of a typical slow-roll trajectory $(\rho(t), \theta(t))$

New result: $\varepsilon \ll 1$ (slow roll) occurs for $\rho \ll 1$ (equiv. $\varphi \ll 1$)



For comparison: In standard α-attractors slow roll occurs near boundary ($\varphi \to \infty$)

of Poincaré disk $\to$ super-super-Planckian excursions in field space

Modified solutions:

Obtained so far: small-field inflation and rapid turn

(Great for PBH-generation !)

BUT: Behavior of $\eta_{||}$-param. - problematic phenomenologically

(Recall: $\eta^I = \eta_{||} T^I + \eta_\perp N^I$, $\eta_{||} = -\ddot{\phi}_0 / (H \dot{\phi}_0)$, $\dot{\phi}_0^2 = G_{IJ} \dot{\phi}_0^I \dot{\phi}_0^J$)

On solutions of EoMs:

$$\eta_{||} = -\ddot{H} / (2H\dot{H}) \quad - \quad \text{Hubble } \eta\text{-parameter}$$

For pheno reasons: need $|\eta| \ll 1$ during inflation

But in hidden-sym. sols.: $\eta_{||} \to 3/2$ before and after turn

$\rightarrow$ Need to modify the hidden-symmetry solutions

Modified solutions:

New result: Modified solutions with additional parameter ;

for certain param. value: recover hidden symmetry ;

in general: do not respect the symmetry

- Preserve tachyonic instab. and small-field infl.
- Phase transition: ultra-slow-roll $\to$ slow-roll

Ansatz leading to modified solutions:

$$f(\varphi) = \frac{1}{q^2} \sinh^2(q\,\varphi) \quad , \quad V(\varphi, \theta) = V_0 \cosh^{6p}(q\,\varphi)$$

- $\{\varphi, \theta\}$ - manifold: still Poincaré disk ,
- BUT: no hidden symmetry

Modified solutions:

Lagrangian $\mathcal{L}(a, \varphi, \theta)$ simplifies under $(a, \varphi, \theta) \to (u, v, w)$:

$$
\begin{aligned}
u &= a^{\frac{1}{2p}} \cosh(q\,\varphi) \\
v &= a^{\frac{1}{2p}} \sinh(q\,\varphi) \cos\theta \qquad \text{and} \qquad q = 1/(\sqrt{24}\,p) \\
w &= a^{\frac{1}{2p}} \sinh(q\,\varphi) \sin\theta
\end{aligned}
$$

Hidden symmetry case: $p = 1/3$

Modified solutions with $p \gtrsim 2$: (preserve PBH-generation)

– before turn: $\eta \approx 3$, – after turn: $\eta \approx 1/(4p)$

$\to$ smooth transition: ultra-slow roll $\to$ slow roll

(for any $p \gtrsim 4$)

Summary

Found so far:

- Class of exact solutions with hidden symmetry exhibits tachyonic instability necessary for PBH generation
- Modified solutions with improved Hubble η-parameter

  [Transition between ultra-slow-roll and slow-roll phases]
- Hyperb. inflation: at small field values [unlike in α-attractors]

Open issues:

- More general hidden symmetries $\rightarrow$ PBH-generation ?...
- Small-field hyperbolic inflation in general ?...
- Transitions between other pairs of phases ?...

Thank you!