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Nonstandard Lagrangian in Cosmology - Constrained System

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Introduction

- We study real scalar field with non-standard Lagrangian density, imortant (at least for us) in Cosmological context.
- General space-time metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \quad \mu,\nu = 0,1,2,3$$

- Metric tensor: $\hat{g}$
- Components of metric tensor: $g_{\mu\nu}$
- Determinant of metric tensor: $g \equiv \det \hat{g}$

Introduction

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \quad \mu,\nu = 0,1,2,3$$

- Mostly used:
- Minkowski space-time (Special Relativity):

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = -dt^2 + d\vec{x}^2$$

- Friedmann-Robertson-Walker-Lemaitre (FRWL) flat space-time (General Relativity/Cosmology):

$$ds^2 = -N(t)^2 dt^2 + a^2(t) d\vec{x}^2$$

- Scale factor (how much the Universe changes its size): *a(t)*
- Lapse function: *N(t)*

Cosmology

- Real scalar field with non-standard Lagrangian density.
- General Lagrangian action:

$$S = \int d^4x \sqrt{-g}\, L(X(\partial\phi), \phi)$$

- Lagrangian (Lagrangian density) of the standard form:

$$L(\phi, \partial\phi) = X(\partial\phi) - V(\phi)$$

- Non-standard Lagrangian:

$$L_{tach}(T, X) = -V(T)\sqrt{1 + 2X(\partial T)} \qquad X = \frac{1}{2} g^{\mu\nu} \partial_\mu T \partial_\nu T$$

Cosmology

- The action:

$$S=\int d^4x\sqrt{-g}\,\mathcal{L}(X,T)$$

- In cosmology, scalar fields can be connected with a perfect fluid which describes (dominant) matter in the Universe.
- Components of the energy-momentum tensor:

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

$$T_{\mu\nu}=(P+\rho)u_\mu u_\nu-Pg_{\mu\nu}$$

Cosmology

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - Pg_{\mu\nu}$$

- Pressure, matter density and velocity 4-vector, respectively:

$$P(X,T) \equiv \mathrm{L}(X,T)$$

$$\rho(X,T) \equiv 2X\frac{\partial \mathrm{L}}{\partial X} - \mathrm{L}(X,T)$$

$$u_\mu \equiv \frac{\partial_\mu T}{\sqrt{2X}}$$

Cosmology

- Total action: term which describes gravity (Ricci scalar, Einstein-Hilbert action) plus term that describes cosmological fluid (scalar field minimally coupled to gravity):

$$S = \int d^4x \sqrt{-g}\,\left(R + L(X,T)\right)$$

- Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$$

Cosmological Perturbations

- Very important: cosmological perturbations around the classical background (give us information about Universe at the very beginning, i.e. CMB and cosmological structure formation).
- ADM decomposition of the metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + \gamma_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$S = \int d^4x \sqrt{-g} \left(\pi^{ij} \dot{\gamma}_{ij} - N \mathrm{H}_G - N_i \mathrm{H}_G^i + \mathrm{L}(X, T) \right)$$

Cosmological Perturbations

- Perturb everything:

$$\pi^{ij} = \pi^{ij}(t) + \delta\pi^{ij}$$

$$\gamma_{ij} = \gamma_{ij}(t) + \delta\gamma_{ij}$$

$$T = T(t) + \delta T \ldots$$

$$S = S^{(0)} + S^{(1)} + S^{(2)} + \ldots$$

- Action for classical background: $S^{(0)}$

$$S^{(1)} = 0$$

$$S^{(2)} = S^{(2)}_{QUAD} + S^{(2)}_{INT}$$

Cosmological Perturbations

- Path integral quantization of cosmological perturbations:

$$K \,\square \int D\mu \exp[iS^{(2)}_{QUAD} + iS^{(2)}_{INT}]$$

- Transition amplitude (tells us how the system evolve quantum-mehanically): K
- Path integral measure: $D\mu$
- If you are lucky, „standard“ Feynman measure (no constraints)
- If you are less lucky, Faddeev-Popov measure (first class constraints)
- No luck at all, Senjanovic measure (both first and second class constraints)

Cosmological Perturbations

- Background FLRW space-time:

$$ds^2_{FLRW} = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\vec{x}^2$$

- Friedmann equation (H-Hubble parameter):

$$H^2 = \frac{1}{3}\rho(X,T), \quad H = \frac{\dot{a}}{a}$$

- Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\left(\rho(X,T) + 3P(X,T)\right)$$

- Continuity equation:

$$\dot{\rho} = -3H\left(\rho(X,T) + P(X,T)\right)$$

Non-standard Lagrangian

- Effective field theory of non-standard lagrangian of DBI-type (tachyon scalar field):

$$\mathrm{L}_{tach} = -V(T)\sqrt{1+g_{\mu\nu}\partial^{\mu}T\partial^{\nu}T}$$

- EoM:

$$\left(g^{\mu\nu} - \frac{\partial^{\mu}T\partial^{\nu}T}{1-(\partial T)^2}\right)\partial_{\mu}T\partial_{\nu}T = -\frac{1}{V(T)}\frac{dV}{dT}(1-(\partial T)^2)$$

Non-standard Lagrangian

- Minisuperspace model for the background:

$$ds^2_{FLRW} = -N^2(t)dt^2 + a^2(t)d\vec{x}^2$$

- Total Lagrangian:

$$\mathrm{L} = \mathrm{L}_g(N, a, \dot{a}) + \mathrm{L}_{tach}(T, \dot{T}, N, a)$$

$$\mathrm{L}_g(N, a, \dot{a}) = -6\frac{a\dot{a}^2}{N} + 6kNa$$

$$\mathrm{L}_{tach}(T, \dot{T}, N, a) = -Na^3V(T)\sqrt{1 - N^{-2}\dot{T}^2}$$

Extension

- Minisuperspace model for the background :

$$ds^2_{FLRW} = -N^2(t)dt^2 + a^2(t)d\vec{x}^2$$

- Auxiliary field : σ
- Total Lagrangian:

$$\mathrm{L} = \mathrm{L}_g(N, a, \dot{a}) + \mathrm{L}_{ext}(T, \dot{T}, N, a, \sigma)$$

$$\mathrm{L}_g(N, a, \dot{a}) = -6\frac{a\dot{a}^2}{N} + 6kNa$$

$$\mathrm{L}_{ext}(T, \dot{T}, N, a, \sigma) = +Na^3[\frac{1}{2N^2}\frac{\dot{T}^2}{\sigma} - \frac{1}{2}\sigma V^2(T) - \frac{1}{2\sigma}]$$

Extension

- From the EoM for σ :

$$\sigma = f(T,\dot{T},N) = \frac{1}{V(T)}\sqrt{1-N^{-2}\dot{T}^2}$$

$$\mathrm{L}_{ext}(T,\dot{T},N,a,\sigma)\big|_{\sigma=f(T,\dot{T},N)} = \mathrm{L}_{tach}(T,\dot{T},N,a)$$

$$\mathrm{L} = -6\frac{a\dot{a}^2}{N} + 6kNa + \frac{a^3}{2N}\frac{\dot{T}^2}{\sigma} - \frac{1}{2}\sigma Na^3V^2(T) - \frac{Na^3}{2\sigma}$$

$$S^{(0)} = \int dt\mathrm{L}$$

Constraints

- We need to know what are the constraints in order to proceed with $S^{(2)}$.
- Conugated momenta:

$$p_a = \frac{\partial \mathrm{L}}{\partial \dot{a}} = -12\frac{a\dot{a}}{N} \quad \Rightarrow \quad \dot{a} = -\frac{N}{12a}p_a$$

$$p_T = \frac{\partial \mathrm{L}}{\partial \dot{T}} = \frac{a^3}{N}\frac{\dot{T}}{\sigma} \quad \Rightarrow \quad \dot{T} = \frac{N\sigma}{a^3}p_T$$

$$p_N = \frac{\partial \mathrm{L}}{\partial \dot{N}} = 0$$

$$p_\sigma = \frac{\partial \mathrm{L}}{\partial \dot{\sigma}} = 0$$

Constraints

- Primary constraints:

$$\phi_1 \equiv p_N - 0 \approx 0$$

$$\phi_2 \equiv p_\sigma - 0 \approx 0$$

- Canonical Hamiltonian:

$$\mathrm{H}_c \equiv N\mathrm{H}_0$$

$$\mathrm{H}_0 \equiv -\frac{1}{24}\frac{p_a^2}{a} - 6ka + \frac{\sigma}{2a^3}p_T^2 + \frac{1}{2}\sigma a^3 V^2(T) + \frac{a^3}{2\sigma}$$

- Total Hamiltonian (add all constraints to canonical Hamiltonian): $\mathrm{H}_T \equiv \mathrm{H}_c + \lambda_1\phi_1 + \lambda_2\phi_2$

Constraints

- Consistency conditions:

$$\dot{\phi}_1 = \{\phi_1, \mathrm{H}_T\} \approx 0$$

$$\dot{\phi}_2 = \{\phi_2, \mathrm{H}_T\} \approx 0$$

- Secondary constraints:

$$\chi_1 \equiv -\mathrm{H}_0 \approx 0$$

$$\chi_2 \equiv -N[\frac{1}{2a^3} p_T^2 + \frac{1}{2} a^3 V^2(T) - \frac{a^3}{2\sigma^2}] \approx 0$$

- Consistency conditions on secondary constraints:

$$\dot{\chi}_1 = \{\chi_1, \mathrm{H}_T\} \approx 0$$

$$\dot{\chi}_2 = \{\chi_2, \mathrm{H}_T\} \approx 0$$

Constraints

- (Reminder) First class constraints: their Poisson brackets with other constraints vanish:

$$\{\phi_{1st}, any\ constr.\} \approx 0$$

- (Reminder) Constraints that are not of the first class are called second class.
- (Reminder) The number of second class constraints is always even.
- For this extended model:
  - two first class constraints and
  - two second class constraints.

Conclusion

- There are both type of constraints.
- So, Senjanovic method/measure should be used.
- As already mentioned, extended model is classically equivalent to the initial one.
- As already mentioned, extended model is more suitable for path integral quantization.

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THANK YOU!

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