COSMOLOGY OF NONLOCAL GRAVITY

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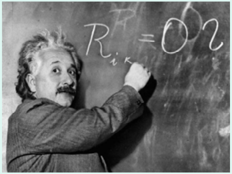
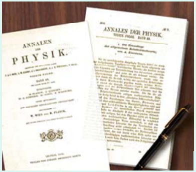
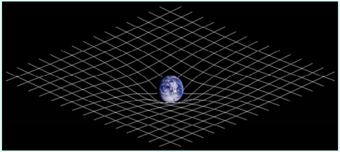
Contents

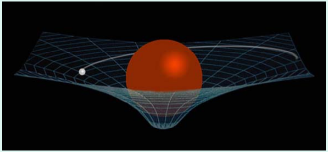
1. Introduction: General relativity

Einstein Theory of Gravity (1915)









$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

1. Introduction: General relativity

- General Relativity (GR) is Einstein theory of gravity, which is current theory of gravity
- Nice theoretical properties and remarkable phenomenological achievements
- Perfectly describes dynamics of Solar System
- Many significant predictions: deflection of light by the Sun, gravitational redshift of light, gravitational waves, gravitational lensing, black holes, Dark Energy (DE) and Dark Matter (DM), ...
- GR still serves as a starting point in understanding all gravitational phenomena.
- Modern cosmology is based on GR and started in 1917.

1. Introduction: Some problems

- There is no Quantum General Relativity: it is a non-renormalizabile quantum field theory.
- It predicts Dark Energy and Dark Matter which are mysterious and still without laboratory evidence.
- General Relativity has not been tested and confirmed at very large cosmic scales, hence its application to the Universe as a whole should be taken with caution.
- Cosmological solutions of GR mainly contain Big Bang singularity.
- It seems unnatural that Einstein GR is theory of gravity at all (spatial) scales: from Planck scale to the universe as a whole. (Physical theories differ at scales and complexity of systems.)
- There is a sense to search generalization of Einstein theory of gravity.
- There are many attempts to modify GR motivated by theoretical and phenomenological reasons.
- Here we consider some modifications of GR with respect to cosmology.

2. Modified Gravity: Cosmological motivations

- Einstein equations: $R_{\mu\nu} + \frac{1}{2}Rg_{\mu\nu} = 8\pi G\, T_{\mu\nu}$.
- FRW metric for homogeneous and isotropic space-time $ds^2 = -dt^2 + a^2(t)\big(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\big)$ and energy-momentum tensor $T_{\mu\nu} = diag(\rho, pg_{11}, pg_{22}, pg_{33})$.
- Friedmann equations

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \qquad \frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- If $\frac{\ddot a}{a} > 0 \Rightarrow \rho + 3p < 0$: Dark Energy. 68%
- Big velocities in spiral galaxies: Dark matter. 27%
- Standard cosmological model: 95% ΛCDM. Only 5% visible matter.
- There is no experimental evidence of DE and DM
- General rel. has not been verified for the whole universe.
- Initial (Big Bang) singularity.

2. Modified Gravity: Kinds of modification

- First modifications: Einstein 1917, ..., many modifications after 1998

Einstein-Hilbert action

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} R + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

modification

$$R \to f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...), \quad \Box = \nabla^{\mu}\nabla_{\mu} = \frac{1}{\sqrt{-g}}\partial_{\mu}\sqrt{-g}g^{\mu\nu}\partial_{\nu}$$

2. Modified Gravity: Kinds of modification

- $f(R)$ modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

- nonlocal modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R, \Box) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

3. Nonlocal Modified Gravity: Kinds of nonlocal gravity

- Nonlocal modified gravity with $\Box^{-n}$:

$$S = \frac{1}{16\pi G} \int \sqrt{-g}\,(R + L_{NL})\; d^4x,$$

with two typical examples:

1) $\quad L_{NL} = R\, f(\Box^{-1} R),$

2) $\quad L_{NL} = -\frac{1}{6} m^2 R \Box^{-2} R.$

3. Nonlocal Modified Gravity: Kinds of nonlocal gravity

The exact tree-level Lagrangian for effective scalar field φ which describes open *p*-adic string tachyon is

$$\mathcal{L}_p = \frac{m_p^D}{g_p^2}\,\frac{p^2}{p-1}\Big[-\frac{1}{2}\,\varphi\, p^{-\frac{\Box}{2m_p^2}}\,\varphi + \frac{1}{p+1}\,\varphi^{p+1}\Big]$$

where p is any prime number, $\Box = -\partial_t^2 + \nabla^2$ is the D-dimensional d'Alembertian and metric with signature $(- + \dots +)$.

3. Nonlocal Modified Gravity: Analytic approach

Action for a class of models:

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + P(R)\mathcal{F}(\Box)Q(R) \Big) \sqrt{-g}\; d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$, $P(R)$ and $Q(R)$ are some differentiable functions of R, Λ is cosmological constant.

For simplicity, we consider nonlocal modification without matter.

3. Nonlocal Modified Gravity: Analytic approach

Equations of motion:

$$
\begin{aligned}
& G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} P(R)\mathcal{F}(\Box)Q(R) + (R_{\mu\nu}\Phi - K_{\mu\nu}\Phi) \\
& + \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \Big(g_{\mu\nu} g^{\alpha\beta}\partial_\alpha \Box^l P(R)\partial_\beta \Box^{n-1-l} Q(R) \\
& - 2\partial_\mu \Box^l P(R)\partial_\nu \Box^{n-1-l} Q(R) + g_{\mu\nu}\Box^l P(R)\Box^{n-l} Q(R)\Big) = 0,
\end{aligned}
$$

where $K_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu}\Box$,
$\Phi = P'(R)\mathcal{F}(\Box)Q(R) + Q'(R)\mathcal{F}(\Box)P(R)$,
and $'$ denotes derivative on R.

3. Nonlocal Modified Gravity: Analytic approach

In homogeneous and isotropic spaces (FLRW metric) there are only two linearly independent equations of motion (Trace and 00-component):

$$4\Lambda - R - 2P(R)\mathcal{F}(\Box)Q(R) + (R\Phi + 3\Box\Phi)$$
$$+ \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\partial_\mu \Box^l P(R) \partial^\mu \Box^{n-1-l} Q(R) + 2\Box^l P(R)\Box^{n-l} Q(R)\right) = 0,$$

$$G_{00} + \Lambda g_{00} - \frac{1}{2} g_{00} P(R)\mathcal{F}(\Box)Q(R) + (R_{00}\Phi - K_{00}\Phi)$$
$$+ \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \big(g_{00} g^{\alpha\beta} \partial_\alpha \Box^l P(R) \partial_\beta \Box^{n-1-l} Q(R)$$
$$- 2\partial_0 \Box^l P(R) \partial_0 \Box^{n-1-l} Q(R) + g_{00}\Box^l P(R) \Box^{n-l} Q(R)\big) = 0.$$

4. Cosmological solutions

- Some cosmological solutions when $P = Q = (R + R_0)^m$, in action

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + P(R)\mathcal{F}(\Box)Q(R) \Big) \sqrt{-g}\, d^4x,$$

  where R_0 is a constant and $m \in \mathbb{Q}$,
- with an Ansatz

$$\Box (R + R_0)^m = p (R + R_0)^m, \qquad R = 6\Big(\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 + \frac{k}{a^2} \Big),$$

  where p is a constant and $\Box = -\partial_t^2 - 3H\partial_t$ is the d'Alembert operator in FLRW metric
- and scale factor of the form

$$a(t) = A\, t^n e^{\gamma\, t^2},$$

  where A and γ are some constants.

For the above Ansatz, scalar curvature R, $k = 0$, and $a(t) = A\, t^n e^{\gamma\, t^2}$ we get the following six equations:

$$
\begin{aligned}
&1)\quad 72m(1+2m-3n)n^2(-1+2n)^2 = 0,\\
&2)\quad 36n(-1+2n)(-np+2n^2p+mR_0-mnR_0+12m\gamma\\
&\quad +48mn\gamma-72mn^2\gamma) = 0,\\
&3)\quad 12n(-1+2n)(pR_0+12p\gamma+48np\gamma-6mR_0\gamma\\
&\quad +312m\gamma^2-192m^2\gamma^2-288mn\gamma^2) = 0,\\
&4)\quad pR_0^2+24pR_0\gamma+96npR_0\gamma+144p\gamma^2+576np\gamma^2\\
&\quad +3456n^2p\gamma^2+96mR_0\gamma^2+288mnR_0\gamma^2\\
&\quad +1152m\gamma^3+8064mn\gamma^3+13824mn^2\gamma^3 = 0,\\
&5)\quad 96\gamma^2(pR_0+12p\gamma+48np\gamma+6mR_0\gamma\\
&\quad +24m\gamma^2+96m^2\gamma^2+432mn\gamma^2) = 0,\\
&6)\quad 2304\gamma^4(p+12m\gamma) = 0.
\end{aligned}
$$

4. Cosmological solutions

The above system of equations has 5 solutions:

1. $p = -12m\gamma$, $n = 0$, $R_0 = -12\gamma$, $m = \frac{1}{2}$
2. $p = -12m\gamma$, $n = \frac{2m+1}{3}$, $R_0 = -28\gamma$, $m = \frac{1}{2}$
3. $p = -12m\gamma$, $n = 0$, $R_0 = -4\gamma$, $m = 1$
4. $p = -12m\gamma$, $n = \frac{1}{2}$, $R_0 = -16\gamma$, $m = 1$
5. $p = -12m\gamma$, $n = \frac{1}{2}$, $R_0 = -36\gamma$, $m = -\frac{1}{4}$

Ansatz solutions (1-4) also satisfy the corresponding equations of motion.

4. Cosmological solutions: 1) Model with nonlocal term $\sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}$

$$S = \frac{1}{16\pi G}\int_M \Big(R - 2\Lambda + \sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}\Big)\sqrt{-g}\, d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=1}^{\infty} f_n \Box^n$

- Cosmological solution: $a(t) = A\, e^{\frac{\Lambda}{6}t^2}$
  $(p = -6\gamma,\ n = 0,\ R_0 = -12\gamma = -2\Lambda,\ m = \frac{1}{2})$
  $\Box\sqrt{R-2\Lambda} = -\Lambda\sqrt{R-2\Lambda}$
  $\mathcal{F}(-\Lambda) = -1, \quad \mathcal{F}'(-\Lambda) = 0, \quad \Lambda \neq 0.$
- Cosmological solution: $a(t) = A\, t^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$
  $(p = -6\gamma,\ n = \frac{2}{3},\ R_0 = -28\gamma = -2\Lambda,\ m = \frac{1}{2})$
  $\Box\sqrt{R-2\Lambda} = -\frac{3}{7}\Lambda\sqrt{R-2\Lambda}$
  $\mathcal{F}(-\frac{3}{7}\Lambda) = -1, \quad \mathcal{F}'(-\frac{3}{7}\Lambda) = 0, \quad \Lambda \neq 0.$

4. Cosmological solutions: 1) Model with nonlocal term $\sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}$

- Cosmological solution: $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}$

  $\Lambda > 0, \quad k = \pm 1, \quad R(t) = \frac{6k}{A^2} e^{-2\sqrt{\frac{\Lambda}{6}}t} + 2\Lambda,$
  $\Box\sqrt{R-2\Lambda} = \frac{\Lambda}{3}\sqrt{R-2\Lambda},$
  $\mathcal{F}\left(\frac{\Lambda}{3}\right) = -1, \quad \mathcal{F}'\left(\frac{\Lambda}{3}\right) = 0.$

4. Cosmological solutions: 1) Model with nonlocal term $\sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}$

- Cosmological solutions for $R = 4\Lambda > 0$
  Equations of motion are satisfied without conditions on function $\mathcal{F}(\Box)$, because $\Box\sqrt{R-2\Lambda} = 0$.

  (i) Solution $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{3}}\, t}$

  $k = 0$. One has $H(t) = \pm\sqrt{\frac{\Lambda}{3}}$.

  (ii) Solution $a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}}\, t$

  $k = +1$. Now $H(t) = \sqrt{\frac{\Lambda}{3}} \tanh \sqrt{\frac{\Lambda}{3}}\, t$.

  (iii) Solution $a(t) = \sqrt{\frac{3}{\Lambda}} \left| \sinh \sqrt{\frac{\Lambda}{3}}\, t \right|$

  $k = -1$. Here $H(t) = \sqrt{\frac{\Lambda}{3}} \coth \sqrt{\frac{\Lambda}{3}}\, t$.

4. Cosmological solutions: 1) Model with nonlocal term $\sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}$

- Cosmological solutions with $R \to 0,\ k = 0$
  The Minkowski space can be obtained from the previous de Sitter case $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{3}}\, t},\ \ k = 0$ in the limit $\Lambda \to 0$.
- Cosmological solutions for $R = 4\Lambda < 0$
  The corresponding solution has the form
  $a(t) = \sqrt{-\frac{3}{\Lambda}} \left|\cos \sqrt{-\frac{\Lambda}{3}}\, t\right|$, where Λ is negative cosmological constant. In this case
  $H(t) = -\sqrt{-\frac{\Lambda}{3}} \tan \sqrt{-\frac{\Lambda}{3}}\, t$, and $k = -1$.

4. Cosmological solutions: 2) Model with nonlocal term $(R-2\Lambda)\,\mathcal{F}(\Box)\,(R-2\Lambda)$

$$S = \frac{1}{16\pi G}\int_M \Big(R - 2\Lambda + (R-2\Lambda)\,\mathcal{F}(\Box)\,(R-2\Lambda)\Big)\sqrt{-g}\; d^4x$$

- Cosmological solution: $a(t) = A\, e^{\frac{\Lambda}{2}t^2}$
  $\Box(R-2\Lambda) = -6\Lambda(R-2\Lambda)$
  $\mathcal{F}(-\frac{3}{2}\Lambda) = -\frac{1}{8\Lambda}, \quad \mathcal{F}'(-\frac{3}{2}\Lambda) = 0, \quad \Lambda \neq 0.$

4. Cosmological solutions: 3) Model with nonlocal term $(R-2\Lambda)\,\mathcal{F}(\Box)\,(R-2\Lambda)$

$$S = \frac{1}{16\pi G}\int_M \Big(R - 2\Lambda + (R-2\Lambda)\,\mathcal{F}(\Box)\,(R-2\Lambda)\Big)\sqrt{-g}\; d^4x$$

- Cosmological solution: $a(t) = A\,\sqrt{t}e^{\frac{\Lambda}{8}t^2}$
  $\Box(R-2\Lambda) = -\frac{3}{2}\Lambda(R-2\Lambda)$

4. Cosmological solutions: 4) Model with nonlocal term $R\,\mathcal{F}(\Box)\,R$

$$S = \frac{1}{16\pi G}\int_M \Big(R - 2\Lambda + R\,\mathcal{F}(\Box)\,R\Big)\sqrt{-g}\,d^4x$$

- $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$
- Linear Ansatz: $\Box R = rR + s$
- $a(t) = a_0 \cosh\left(\sqrt{\frac{\Lambda}{3}}\,t\right)$ for $k = 0$.
  (Biswas, Koivisto, Mazumdar and Siegel)
- $a(t) = a_0 e^{\frac{1}{2}\sqrt{\frac{\Lambda}{3}}t^2}$ for $k = 0$.
  (Koshelev and Vernov)
- $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$ for $k = 0, \pm 1$.
  (Dimitrijevic, B.D., Grujic, Rakic)

5. Concluding Remarks: Discussion

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda} \Big) \sqrt{-g}\, d^4x$$

- Nonsingular bounce solutions. $a(t) = A\, e^{\frac{\Lambda}{6} t^2}$
- An imitation of dark matter and dark energy. $a(t) = A\, t^{\frac{2}{3}} e^{\frac{\Lambda}{14} t^2}$
- Late time acceleration: $\frac{\ddot{a}(t)}{a(t)} = -\frac{2}{3} t^{-2} + \frac{\Lambda}{3} + \frac{\Lambda^2}{49} t^2$

5. Concluding Remarks: Conclusion

- We have presented a few nonlocal gravity models with some cosmological solutions.
- In particular nonlocal gravity model

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda}\Big) \sqrt{-g}\, d^4 x$$

  attracts further investigations.
- In future? Add matter to the action and investigate phenomenological aspects of this nonlocal gravity model.

Some relevant references

Based on joint work with I. Dimitrijevic, A. Koshelev. Z. Rakic and J. Stankovic

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