Seminar: "Geometry&Physics@DFT"

Location : DFT Seminar Room
(Seminar Homepage) (Indico Page)

Date: Monday, October 12, 2015, 12:00 noon

Title: **Complete integrability of nonlocal nonlinear Schrödinger equation**

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Abstract: We start with the generic AKNS system

$$L\psi \equiv i\frac{d\psi}{dx} + (q(x,t) - \lambda\sigma_3)\,\psi(x,t,\lambda) = 0, \qquad q(x,t) = \begin{pmatrix} 0 & q_+ \\ q_- & 0 \end{pmatrix}, \quad (1)$$

whose potential $q(x,t)$ belongs to the class of smooth functions vanishing fast enough for $x \to \pm\infty$. By generic here we mean that the complex-valued functions $q_+(x,t)$ and $q_-(x,t)$ are independent. Using L as a Lax operator we can integrate a system of two equations for $q_+(x,t)$ and $q_-(x,t)$ generalizing the famous NLS equation (GNLS). After the reduction $q_+(x,t) = q_-^*(x,t)$, this system reduces to the NLS equation; applying different 'nonlocal' reduction $q_+(x,t) = \epsilon q_-^*(-x,t) = u(x,t)$, $\epsilon^2 = 1$ we obtain the nonlocal NLS [1]:

$$i\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + \epsilon u^2(x,t)u^*(-x,t) = 0. \quad (2)$$

which also finds physical applications.

We prove that the 'squared solutions' of (1) form complete set of functions thus generalizing the results of [2,3], see also [4]. Then, using the expansions of $q(x,t)$ and $\sigma_3 q_t(x,t)$ over the 'squared solutions' we extend the interpretation of the inverse scattering method as a generalized Fourier transform also to the nonlinear evolution equations related to L. Next, following [3] we introduce a symplectic basis, which also satisfies the completeness relation and denote by $\delta\eta(\lambda)$ and $\delta\kappa(\lambda,t)$ the expansion coefficients of $\sigma_3\delta q_t$ over it. If we consider the special class of variations $\sigma_3\delta q(x) \simeq \sigma_3 q_t\delta t$ then the expansion coefficients $\delta\eta(\lambda) \simeq \eta_t\delta t$ and $\delta\kappa(\lambda,t) \simeq \kappa_t\delta t$ provide us with the action-angle variables for the generalized NLS system and for the nonlocal NLS (2).

References

[1] M. Ablowitz and Z. Musslimani, Integrable Nonlocal Nonlinear Schrödinger Equation, Phys. Rev. Lett. **110** (2013) 064105(5).

[2] D.J. Kaup, *Closure of the Squared Zakharov-Shabat Eigenstates*, J. Math. Analysis and Applications **54**, No. 3, 849-864 (1976).

[3] V. S. Gerdjikov, E. Kh. Khristov. *On the evolution equations solvable with the inverse scattering problem. II. Hamiltonian structures and Backlund transformations.* Bulgarian J. Phys. **7,** No.2, 119–133, (1980) (In Russian).

[4] V. S. Gerdjikov, G. Vilasi, A. B. Yanovski. *Integrable Hamiltonian Hierarchies. Spectral and Geometric Methods* Lecture Notes in Physics **748**, Springer Verlag, Berlin, Heidelberg, New York (2008). ISBN: 978-3-540-77054-1.