Seminar: "Geometry&Physics, DFT" (http://events.theory.nipne.ro/gap/index.php/seminar) (http://www.nipne.ro/indico/categoryDisplay.py?categId=5)

Location: DFT seminar room

Date: Thursday, 30 July 2015, 12:00 noon

Title: Singular sets of Yang-Mills minimizers in dimension higher than 4

Speaker: Dr. Mircea Petrache (Université Pierre et Marie Curie - Paris 6)

Abstract: "The celebrated results of Uhlenbeck furnished the bases for a complete variational study of the Yang-Mills functional in dimension 4, leading to the definition of new differential invariants of 4-manifolds by Donaldson. In this setting the objects of study were Sobolev connections over smooth bundles. In the first part of the talk I will show why it is not sufficient for the variational study of the Yang-Mills functional in dimensions higher than 4.

In the second part of the talk I will describe a space of weak connections introduced in collaboration with Tristan Riviere, as well as our existence-andregularity theory for Yang-Mills minimizers in dimensions greater than 4.

Similar to the Federer-Fleming introduction of rectifiable currents in the 60's to study minimal surfaces and soap films, our space of weak connections selects precisely the class of connections where the direct minimization as in the calculus of variations can be applied to the Yang-Mills functional. At the same time its description is concrete enough to allow to recover for weak connections a sharp regularity result that says that the topological singular set of stationary Yang-Mills connections has codimension 5.

This gives a first step for connecting the variational theory to the algebraic geometry framework which motivated some conjectures of Tian. The optimality of the codimension 5 above is shown by an explicit example of a minimizer, and the proof of its minimality uses a new combinatorial technique together with a decomposition theorem for vector fields by Smirnov. I will save some time at the end to indicate how our singular Yang-Mills bundles can also play a role for the rigorous construction of the Yang-Mills-Gibbs measure from statistical physics in dimension higher than 2."