Sven Köppel

The Generalized Uncertainty Principle and Extra Dimensions

Geometry and Physics seminar
DFT at IFIN-HH, Bucharest
September 25, 2015

FIAS Frankfurt Institute for Advanced Studies

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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The Generalized Uncertainty Principle and Extra Dimensions

Coworkers:

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PD Dr. Piero Nicolini

FIAS Frankfurt Institute for Advanced Studies

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Maximiliano Isi
Prof. Jonas Mureika

Caltech

LMU|LA Loyola Marymount University

Outline

Why do we need models for Quantum Black Holes?

Modifying Heisenberg's uncertainty principle

Features of a certain class of nonlocal gravity spacetimes

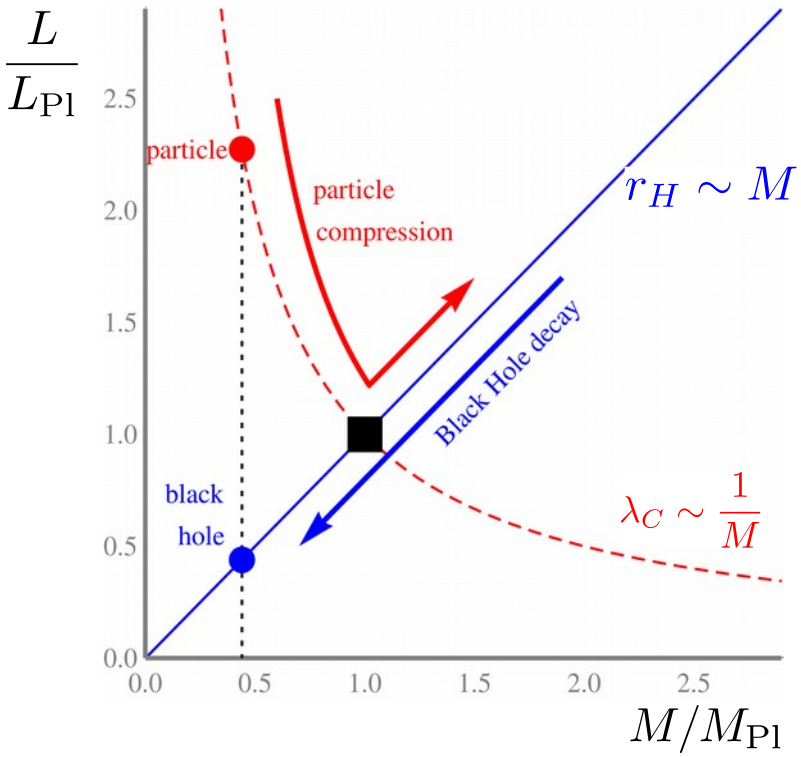
Extension to Large extra dimensions

Further extensions / comments about ...

… Analytic continuation to negative radii

Motivation: Why minimal length physics?

UV-Complete Gravity,
Minimal length



Aurilia, Spallucci 2002
Dvali, Gomez 2010
Carr 2014

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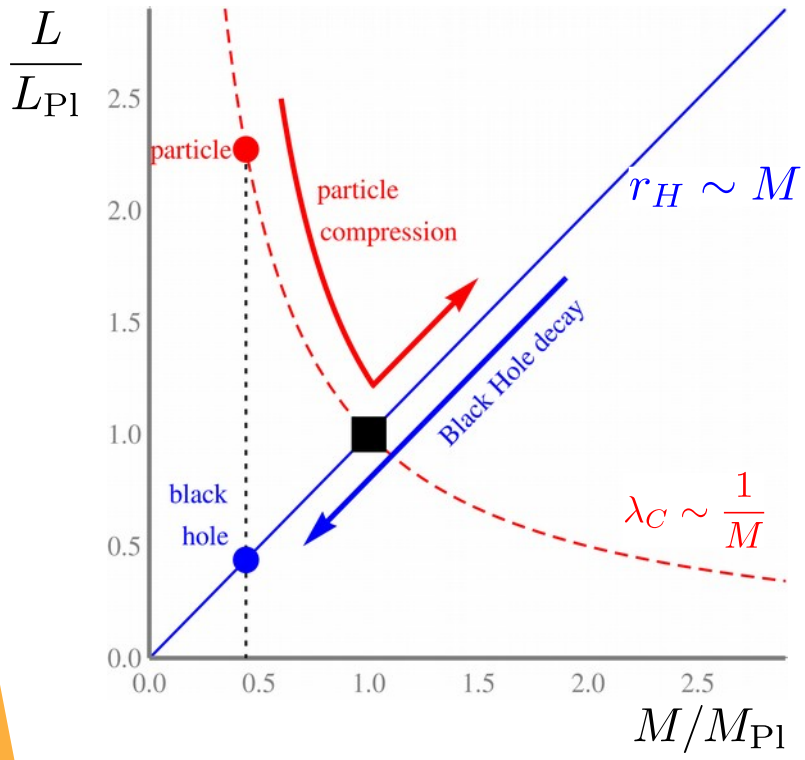

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The Planck scale

$$M_{\rm Pl} = \sqrt{\frac{c^3}{\hbar G}} \approx 10^{-8}\,{\rm kg}$$

$$L_{\rm Pl} = 1/M_{\rm Pl} \approx 10^{-35}\,{\rm m}$$

$$T_{\rm Pl} = L_{\rm Pl}/c \approx 10^{-44}\,{\rm s}$$

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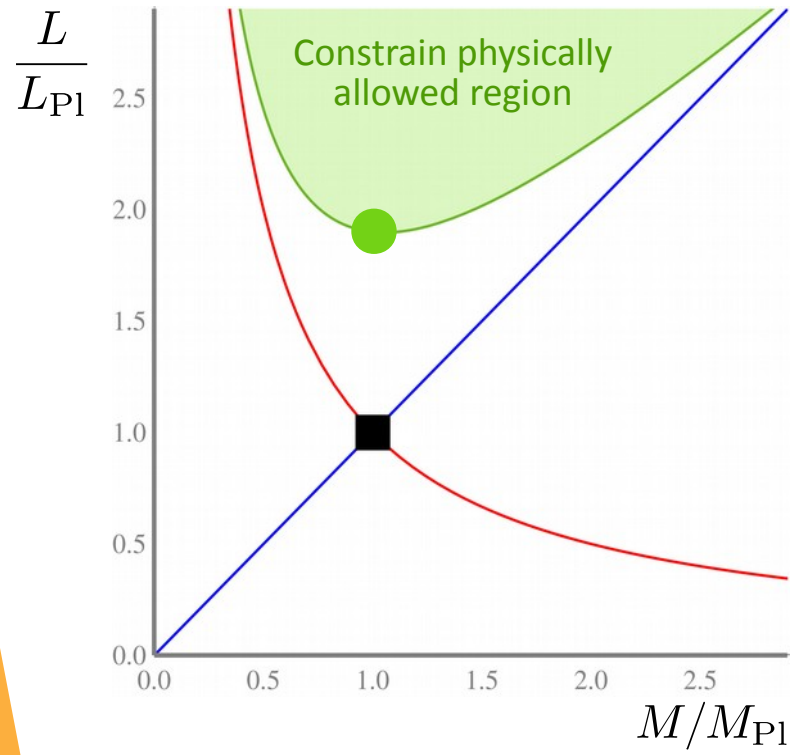

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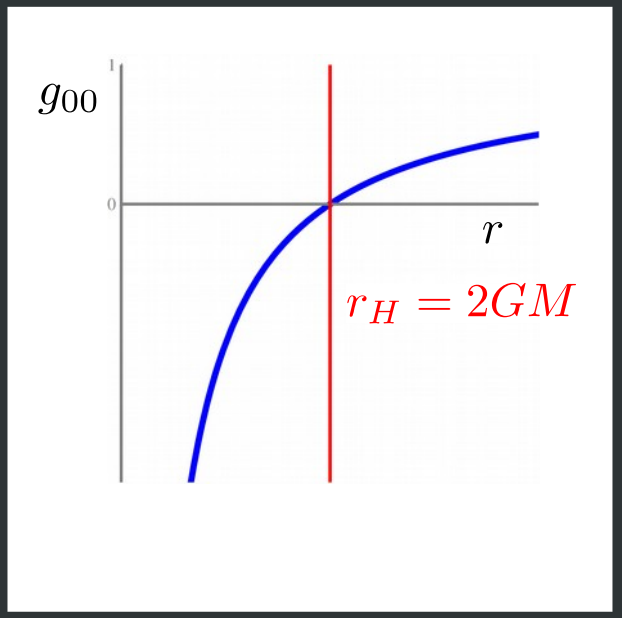
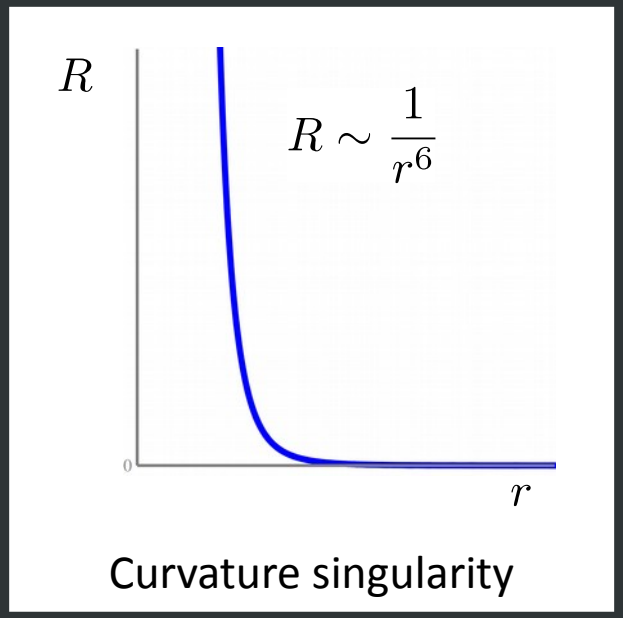
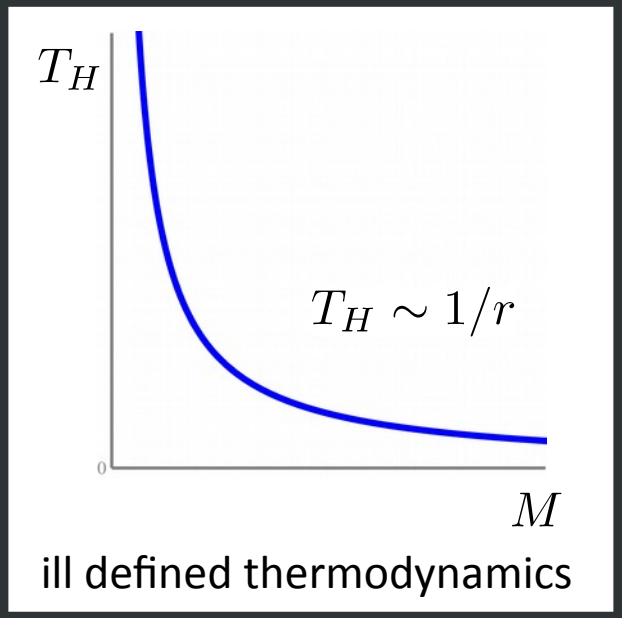
Schwarzschild BHs exhibit problems

The Schwarzschild black hole is not suitable to describe „smallest distances“:

$$\mathrm{d}s^2 = -\left(1 - \frac{2GM}{r}\right)\mathrm{d}t^2 + \frac{1}{1 - \frac{2GM}{r}}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2$$





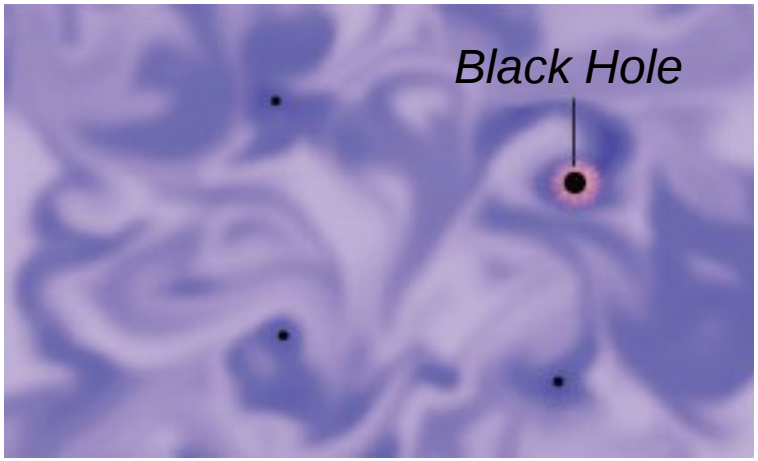


Quantum black holes in nature?

Carr 2014
Bleicher, Nicolini 2010, 2014

Quantum black holes in nature?

Primordial black holes



Evaporate ~ „now“

Carr 2014
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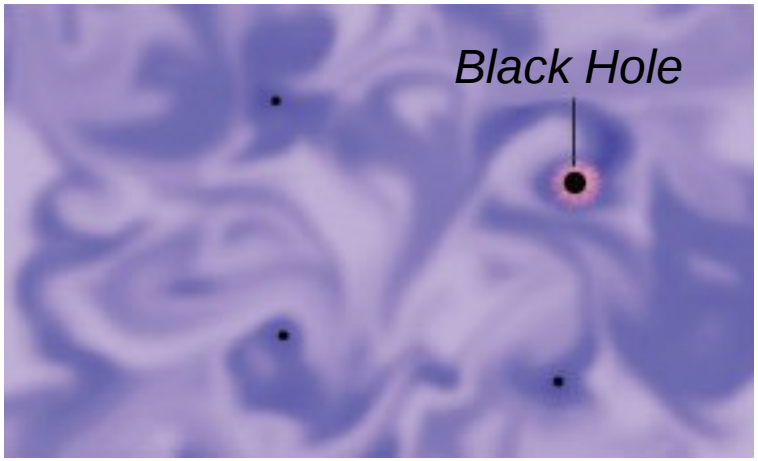
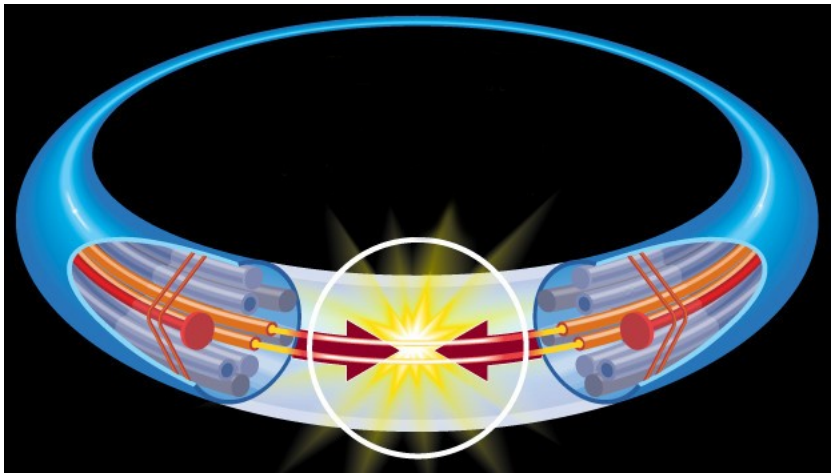
Primordial black holes



Evaporate ~ „now“

Particle accelerators

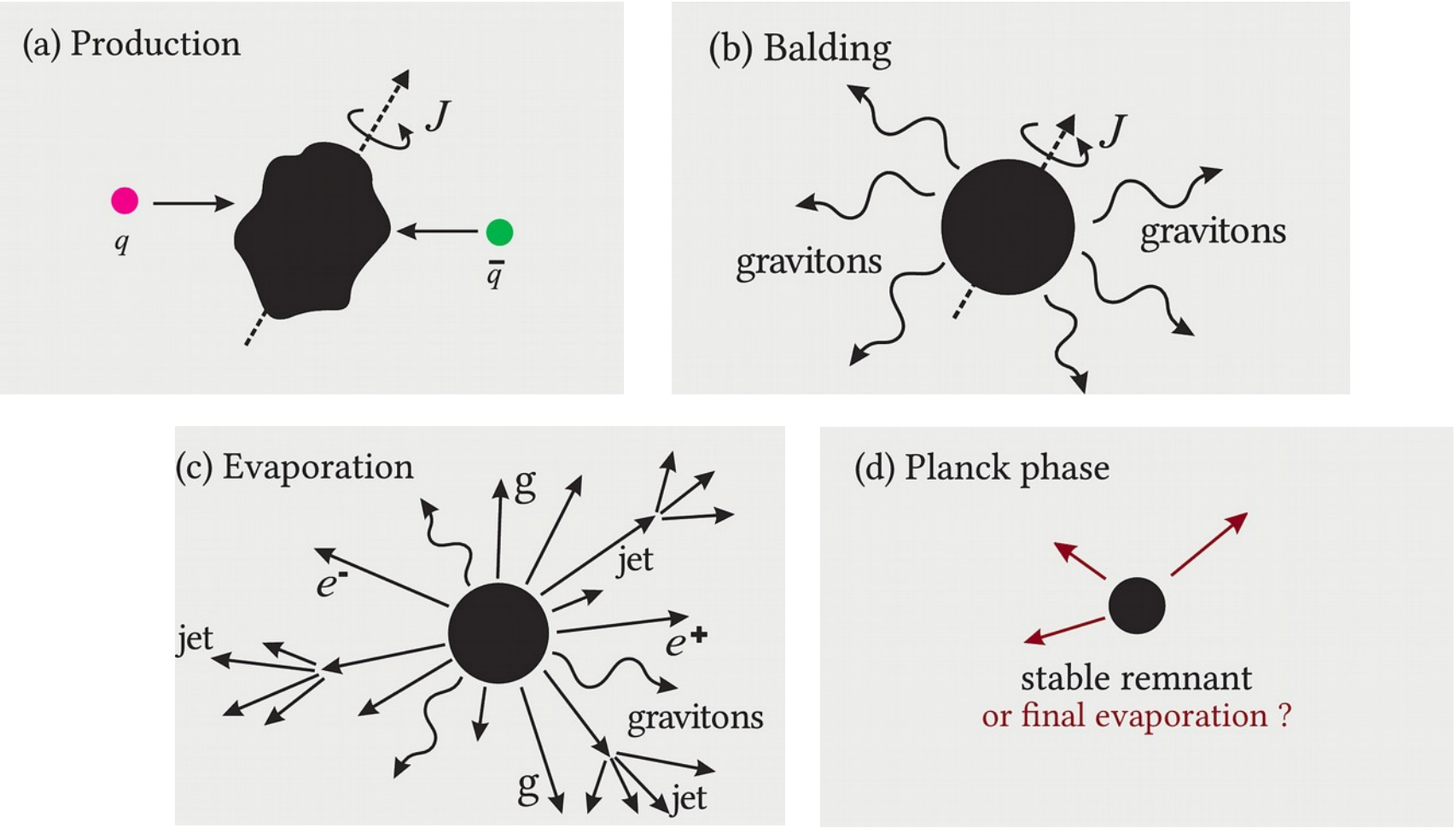


Immediately evaporates

10 TeV: Requires extra dimensions!

Carr 2014
Bleicher, Nicolini 2010, 2014

Black hole evaporation



Hossenfelder 2004
Hossenfelder LRR 2013

Remnants: Candidates for cold dark matter

Effective quantum gravity equations

Influenced by one of:

String theory
Loop quantum gravity
Asymptotically safe gravity
Noncommutative geometry

Nonlocal operator

Quantum matter

$$G_{\mu\nu} = -8\pi G \; \mathcal{A}^{-2}(\Box/\Lambda^2) T_{\mu\nu}$$

$$\int \mathrm{d}^3 x e^{ixp}$$

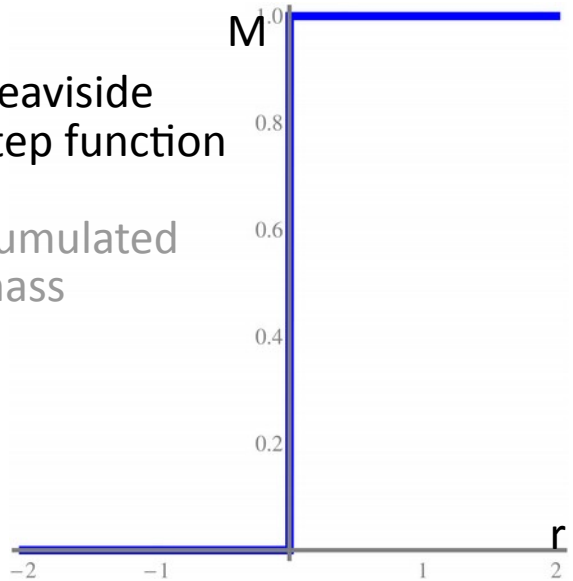
$$\mathcal{A}^{2}(\Box/\Lambda^2) G_{\mu\nu} = -8\pi G \; T_{\mu\nu}$$

Nonlocal spacetime

A quantum theory of gravity must be nonlocal

Heaviside
Step function

Cumulated
mass



$\mathcal{A}^{-2}$

„Smeared“
Mass

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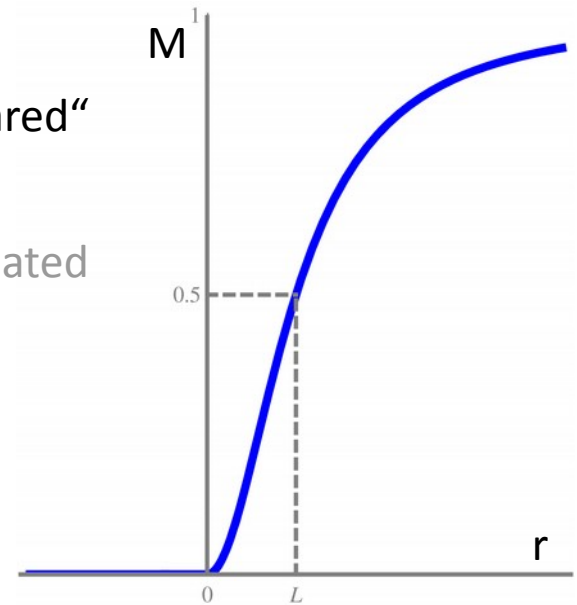
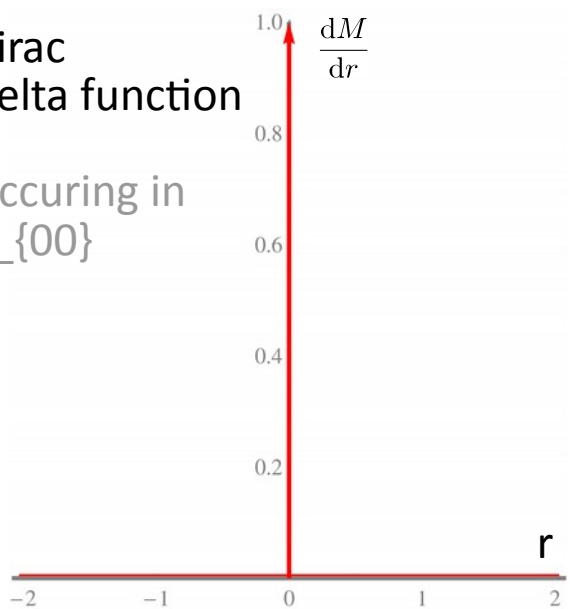


Dirac
Delta function

Occuring in
T_{00}



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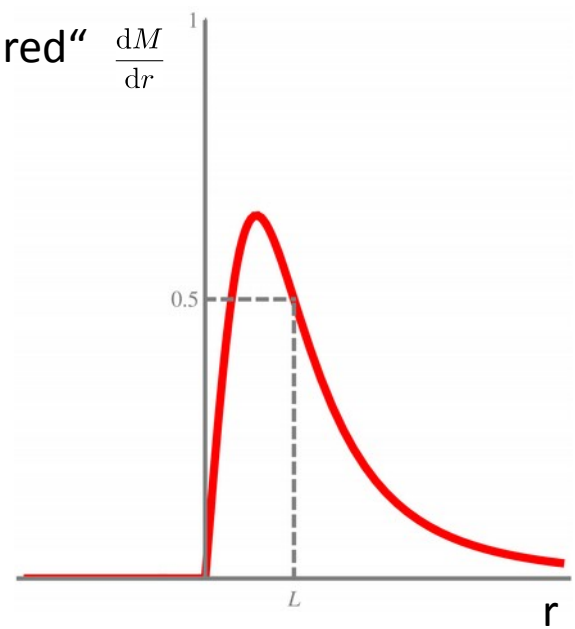

The Generalized Uncertainty Principle

as a guiding principle for the quantum corrections to GR

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+f(p)\right) \qquad f(p) = \beta p^2$$

Veneziano 1986
Amati, Veneziano, Ciafaloni 1995
Michele Maggiore 1995
Kempf, Mangano, Mann 1996

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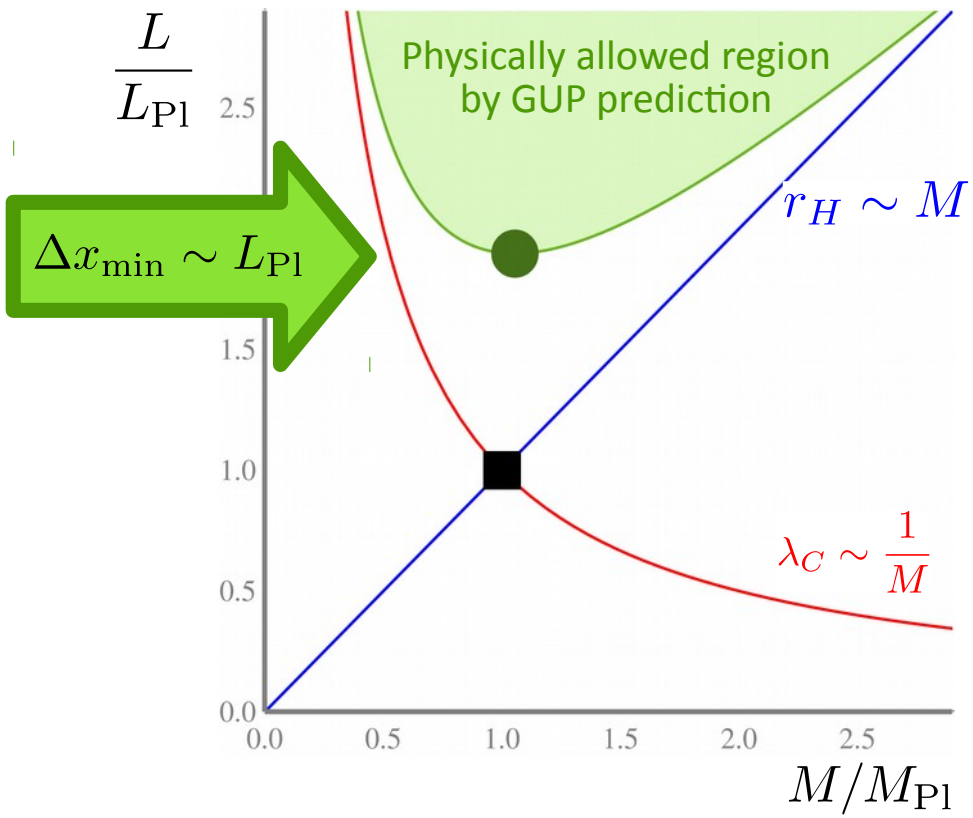

GUP: Approximation to Temperature: Hot remnants

$$\Delta x \Delta p \sim 1 + (L_{\mathrm{Pl}} \Delta p)^2$$

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$$\Delta p_{1,2} \sim \frac{\Delta x}{2 L_{\mathrm{Pl}}^2} \left(1 \pm \sqrt{1 - \frac{L_{\mathrm{Pl}}^2}{\Delta x^2}} \right)$$

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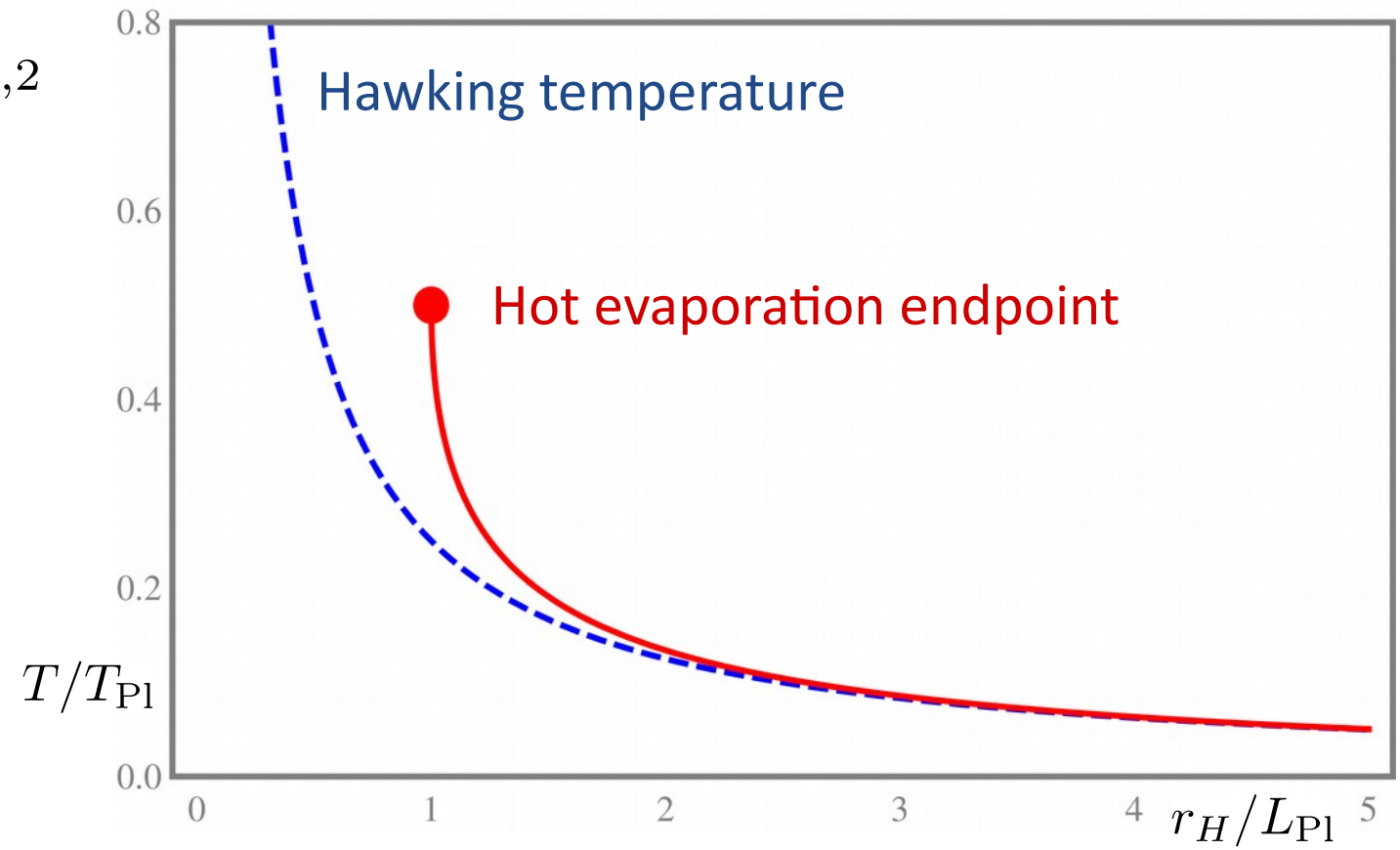
$$T \sim M \sim \Delta p_{1,2}$$



GUP: Exploiting a Hilbert space representation

Hilbert space representation

$$\int \frac{\mathrm{d}^3 p}{1 + f(p)} \, |p\rangle\langle p| = 1$$

Kempf, Mangano, Mann 1996

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Approximation in nonlocal Lagrangian gives nonlocal EFE

$$\mathcal{A}(\Box) G_{\mu\nu} = 8\pi G \; T_{\mu\nu}$$

Choose $\mathcal{A}(\Box) = \dfrac{1}{1 + \beta \Box}$

Krasnikov 1987, Modesto 2012, 2013 Moffat 2012

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What is the effect on a Schwarzschild BH?

$$T^0_0 = -M \delta^{(3)}(r)$$

Isi, Mureika, Nicolini 2013 Balazin, Nachbagauer 1994

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Modified Newton's potential

$$\mathrm{d}s^2 = -\left(1 - \frac{2G\mathcal{M}(r)}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2G\mathcal{M}(r)}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$\mathcal{M}(r) = -4\pi M \int_0^r \mathrm{d}r' \; r'^2 \; \mathcal{A}(\Box)\delta^3(r') \sim \int_0^r \mathrm{d}r' \; r'^2 \int \frac{\mathrm{d}^3 p}{1+\beta\vec{p}^{\,2}} e^{i\vec{x}\cdot\vec{p}}$$

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Analytical result:

$$\mathcal{M}(r) = M\left(1 - e^{-r/\sqrt{\beta}} - (r/\sqrt{\beta})e^{-r/\sqrt{\beta}}\right)$$

GUP inspired Black holes: Self completeness

$$\mathrm{d}s^2 = -\left(1 - \frac{2GM}{r}\gamma(2, r/\sqrt{\beta})\right)\mathrm{d}t^2 + \left(1 - \frac{2GM}{r}\gamma(2, r/\sqrt{\beta})\right)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

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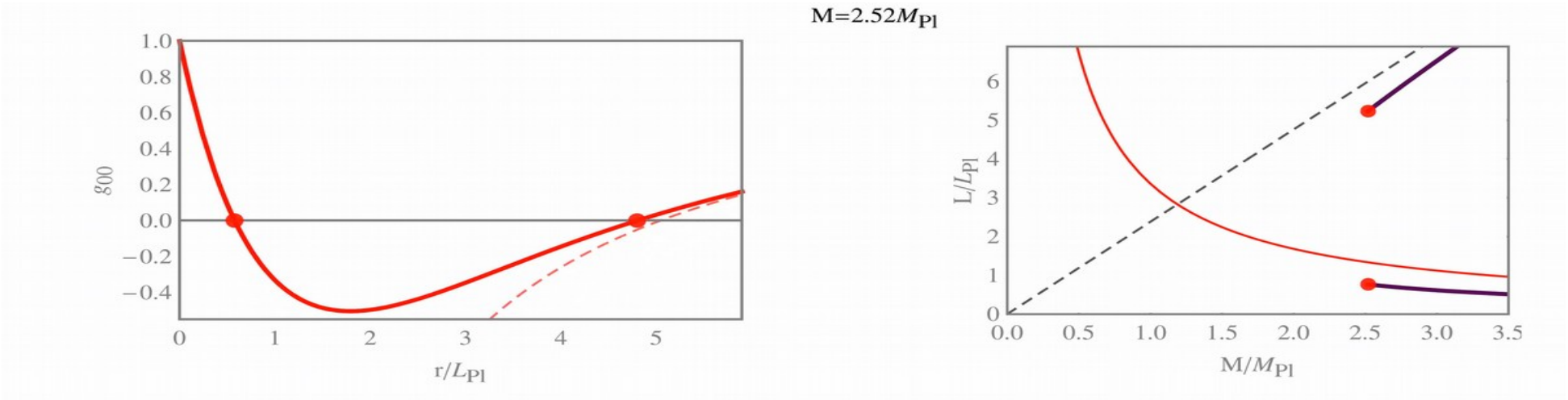

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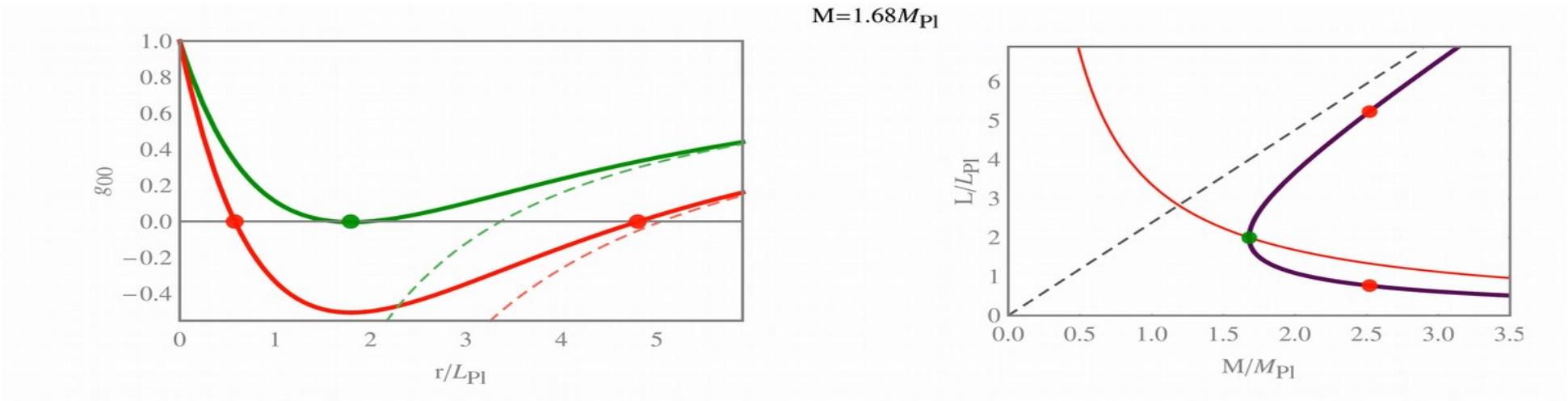

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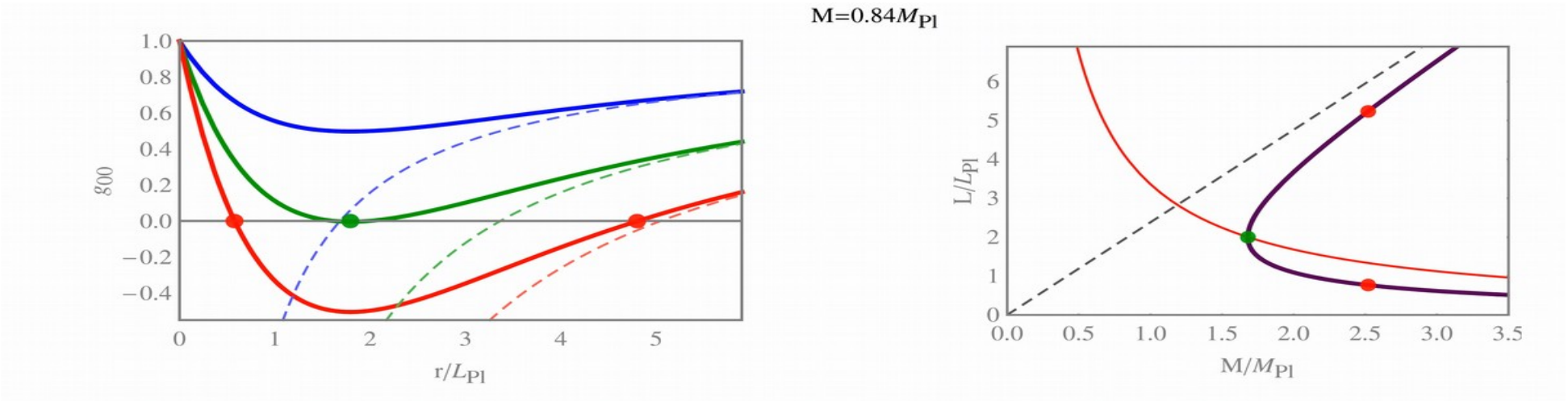

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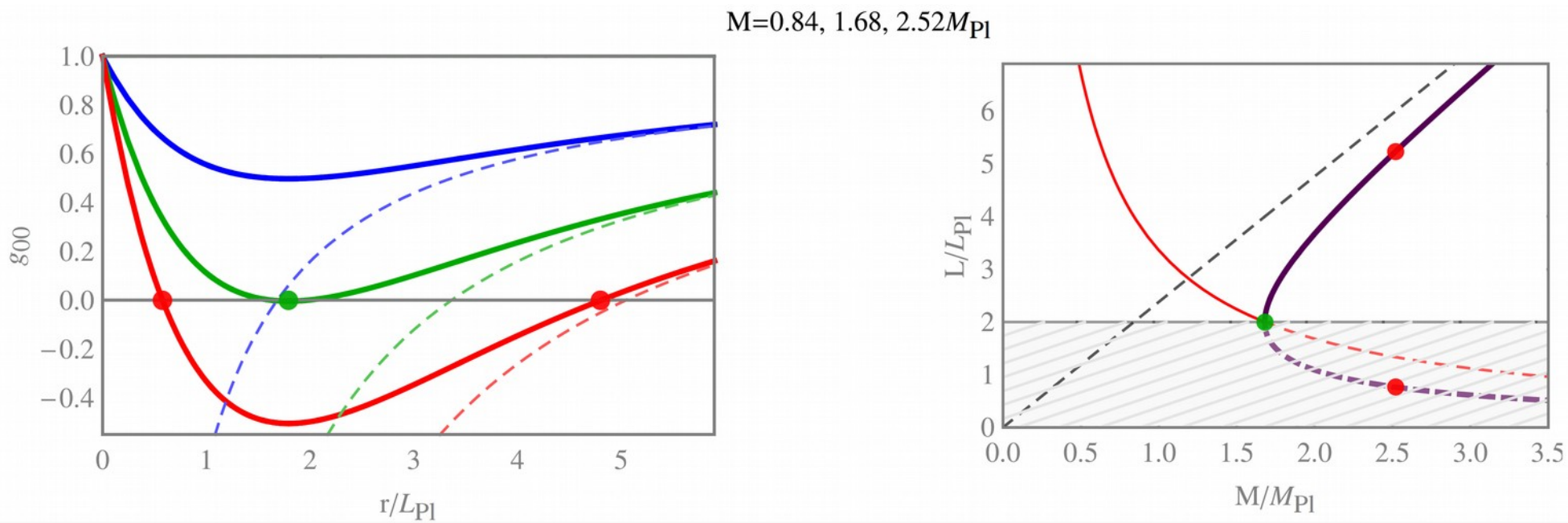
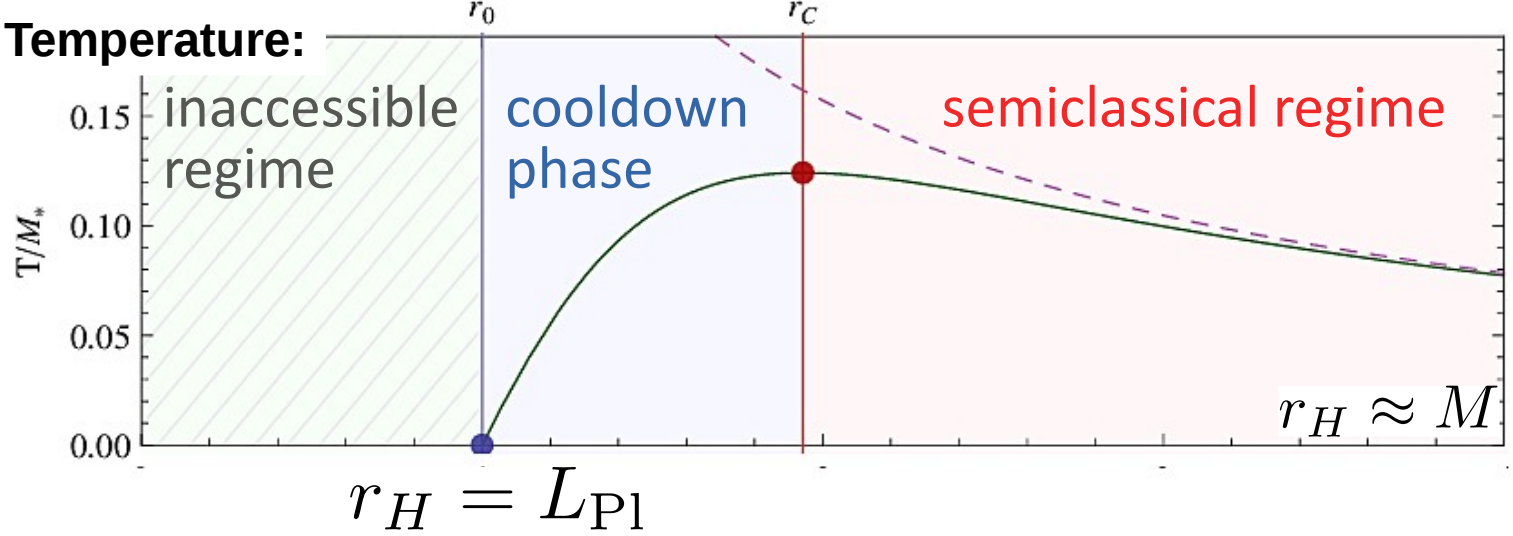


Self complete: Can relate free parameter to fundamental length

$$\beta = 1.534 L_{\mathrm{Pl}}$$

Temperature:



The black hole remnant: thermodynamics

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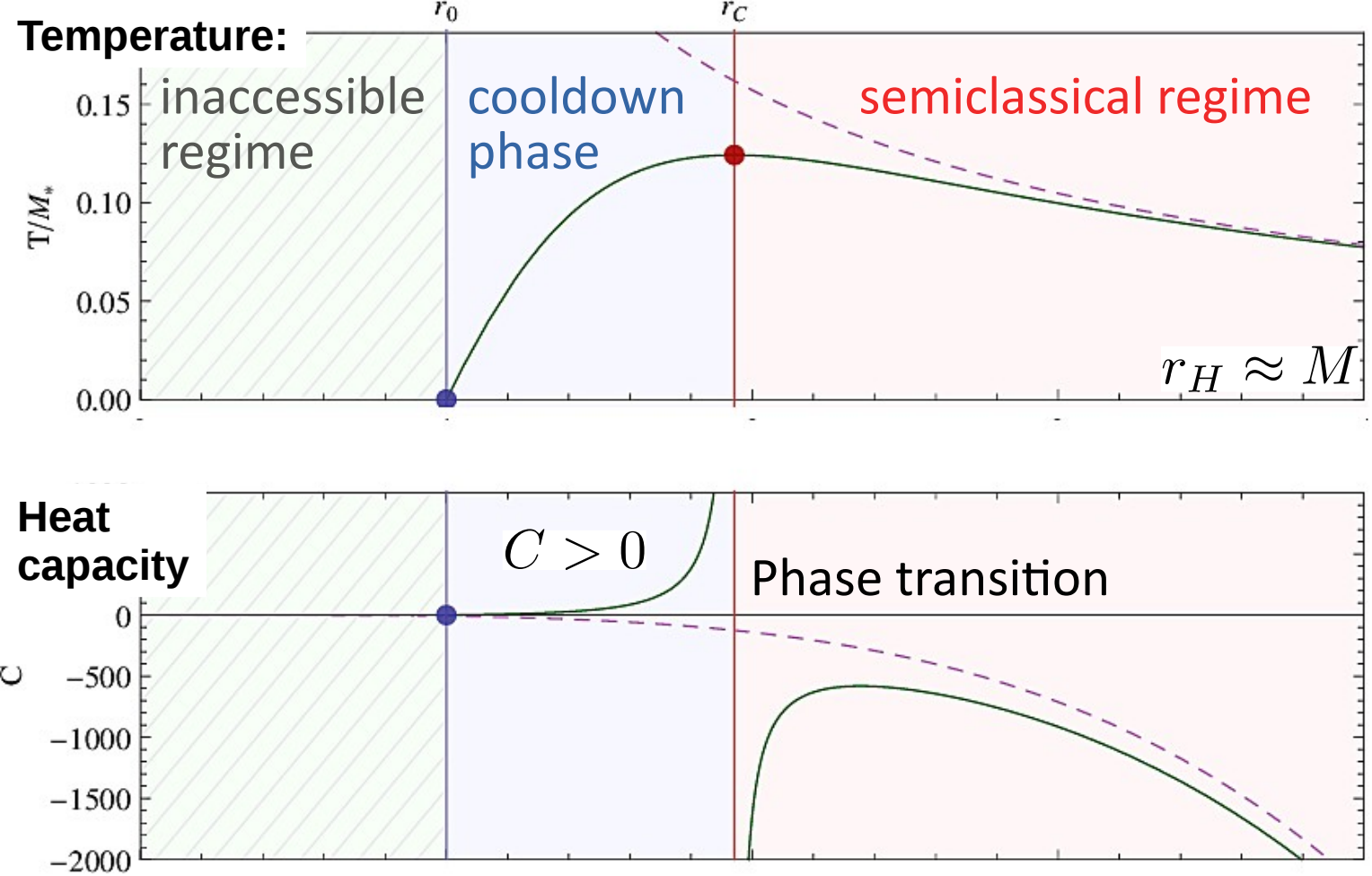

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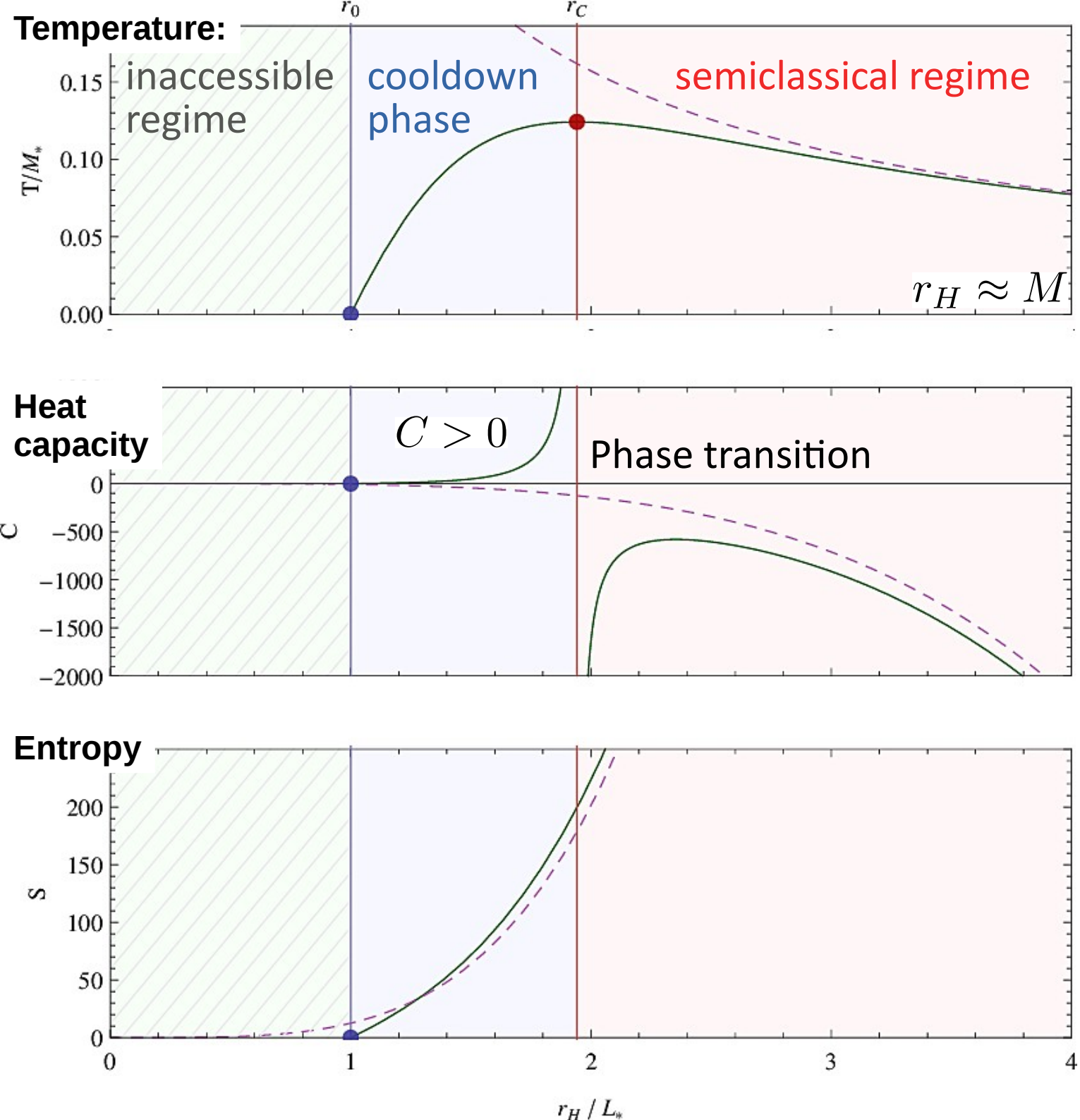


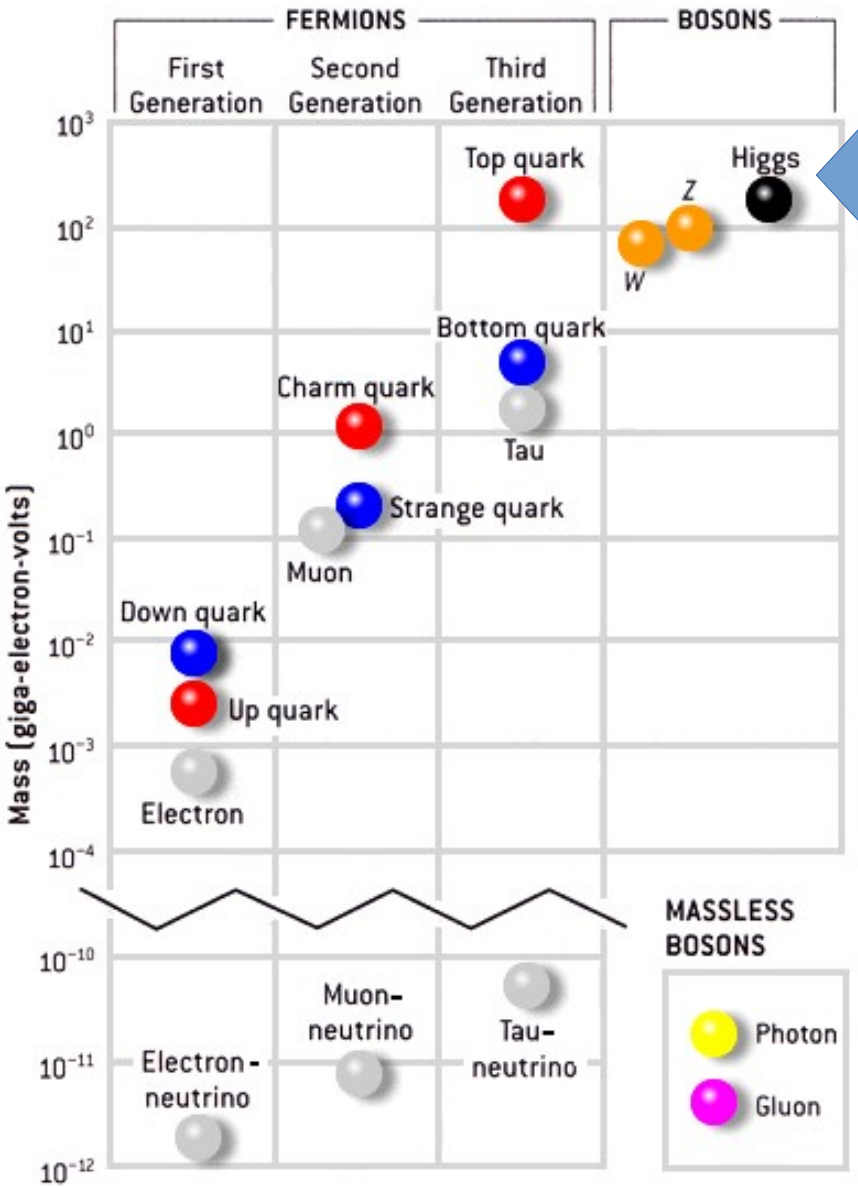
The remnant:
A cold stable evaporation endpoint

Extension of such models

Alternative theories of gravity:
Large extra dimensions
+ Terascale physics
+ Hawking emission on brane/bulk

or adding more complexity
+ Angular momentum
+ Electrical charge
+ AdS background

Adressing the weak hierarchy problem of the Standard Model



Elektroweak scale

$\Lambda_{\mathrm{EW}} \sim 123\ \mathrm{GeV}$

vs.

Planck scale

$M_{\mathrm{Pl}} \sim 10^{19}\ \mathrm{GeV}$

Large Extra Dimensions

ADD scenario

$$M_{\mathrm{Pl}}^2 = V_n M_*^{n+2}$$

Integrated huge Volume of extra dimensions

$$V_n = (2\pi R_c)^n$$

Large $\cong$ **µm** up to **mm**

Large Extra Dimensions

ADD scenario ← **A**rkani-Hamed **D**imopoulos **D**vali 1998

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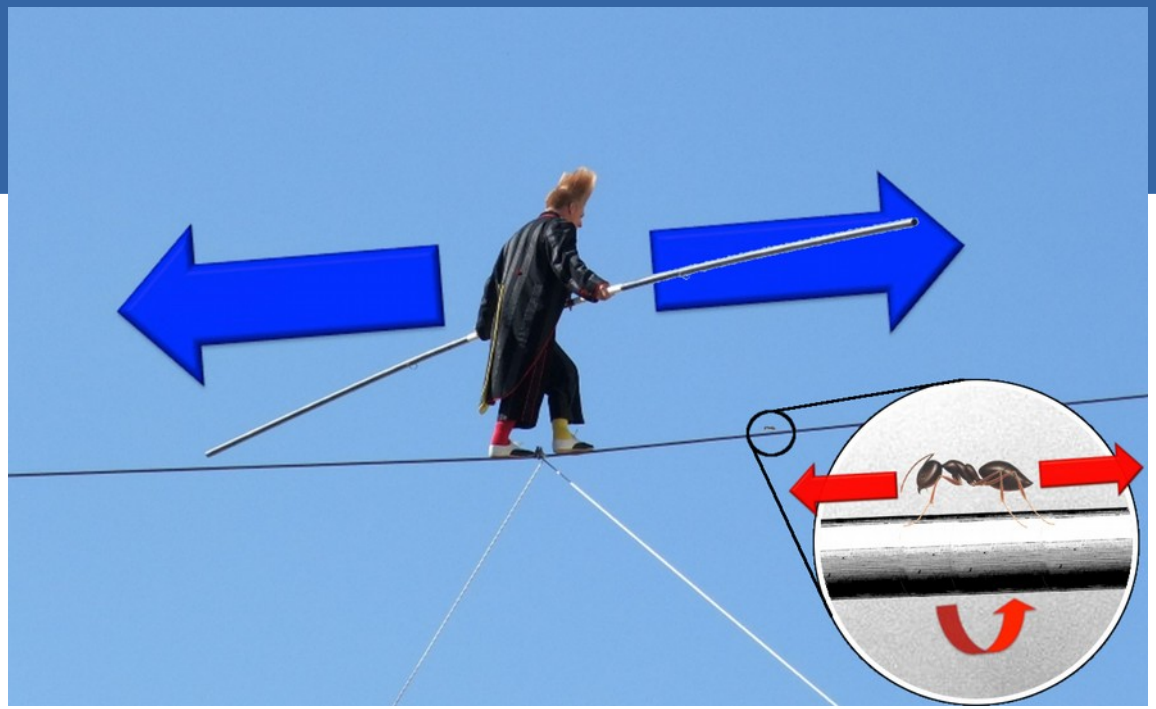

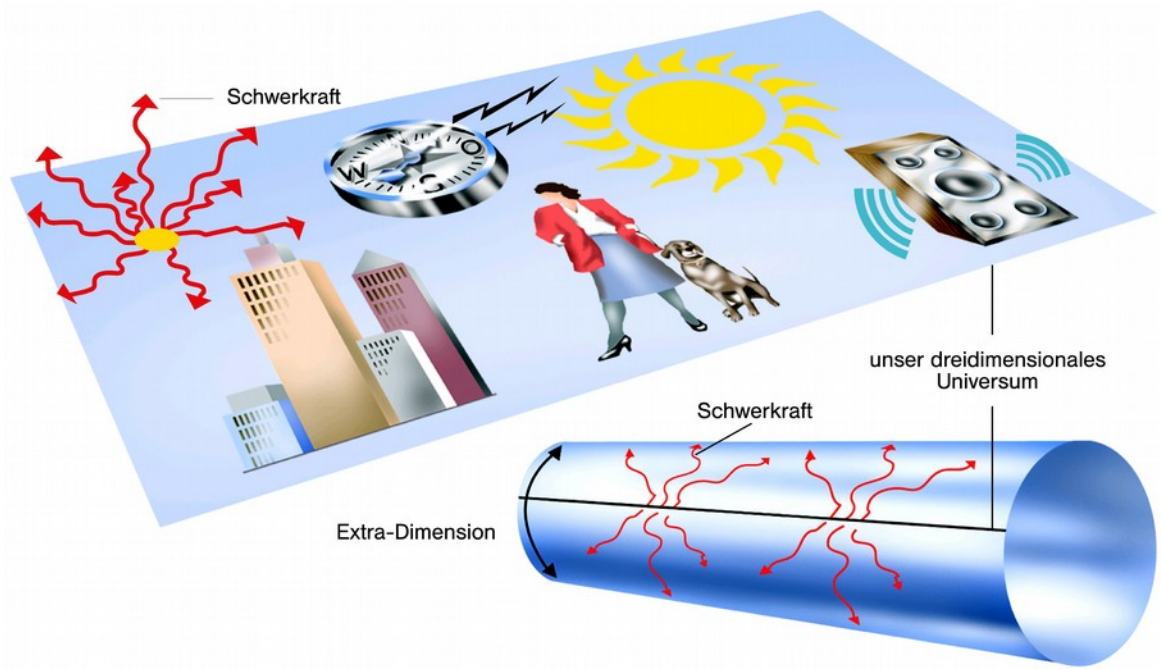

The Kempf-Mangano-Mann model in N+1 dimensions

Apply the momentum space regularization in *N* spatial dimensions (ADD model)

$$\int \frac{\mathrm{d}^N p}{1+\beta \vec{p}^2} \, |p\rangle\langle p| = 1$$

Kempf, Mangano, Mann 1996
Scardigli, Casadio 2003

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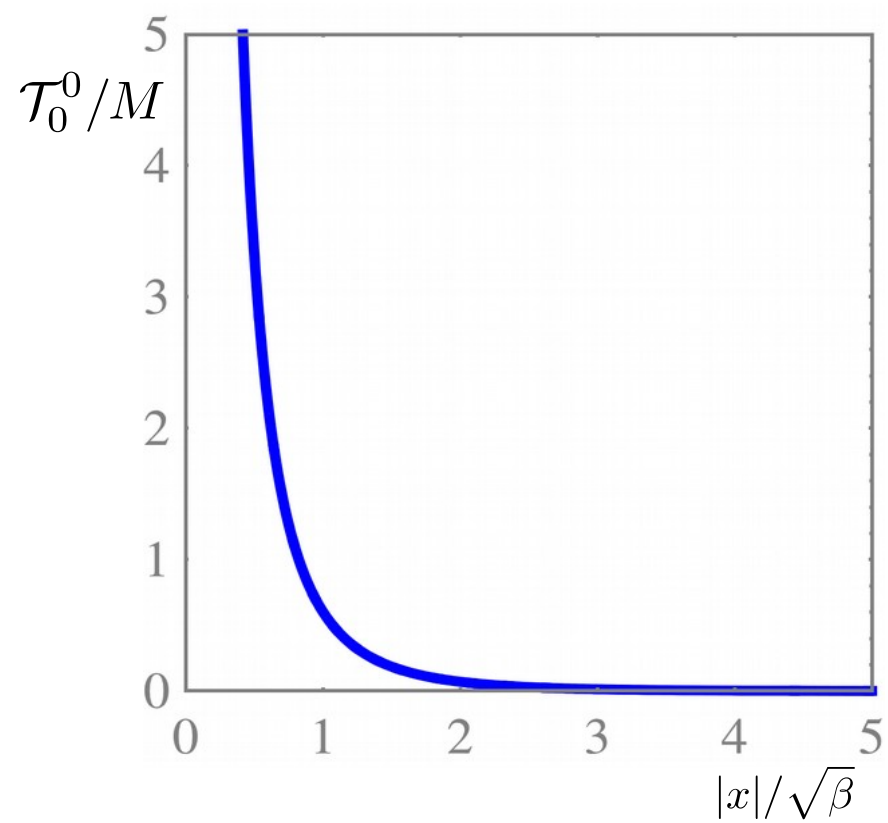
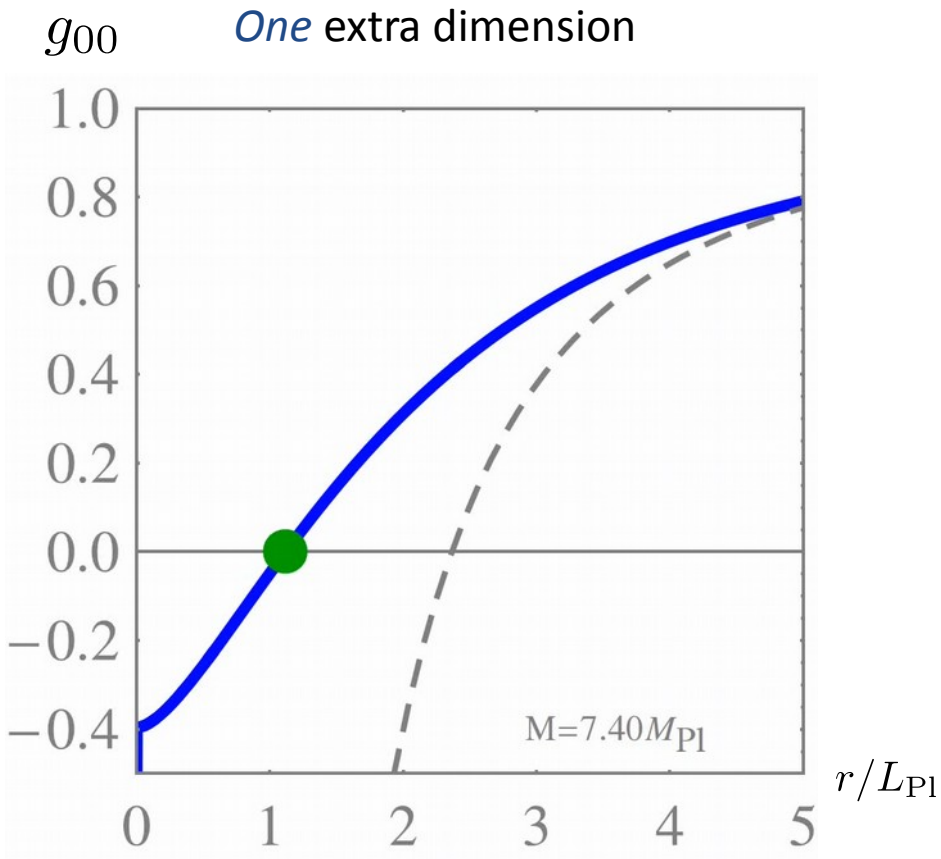
We can solve the nonlocal operator analytically in any number of extradimensions

$$\mathcal{T}_0^0 = \frac{M}{(2\pi)^N}\int \frac{\mathrm{d}^N p\; e^{i\vec{x}\cdot\vec{p}}}{1+\beta p^2}$$
$$\sim \frac{1}{|x|^{N/2-1}} K_{N/2-1}\left(|x|/\sqrt{\beta}\right)$$



GUP-inspired BH in higher dimensions

One extra dimension



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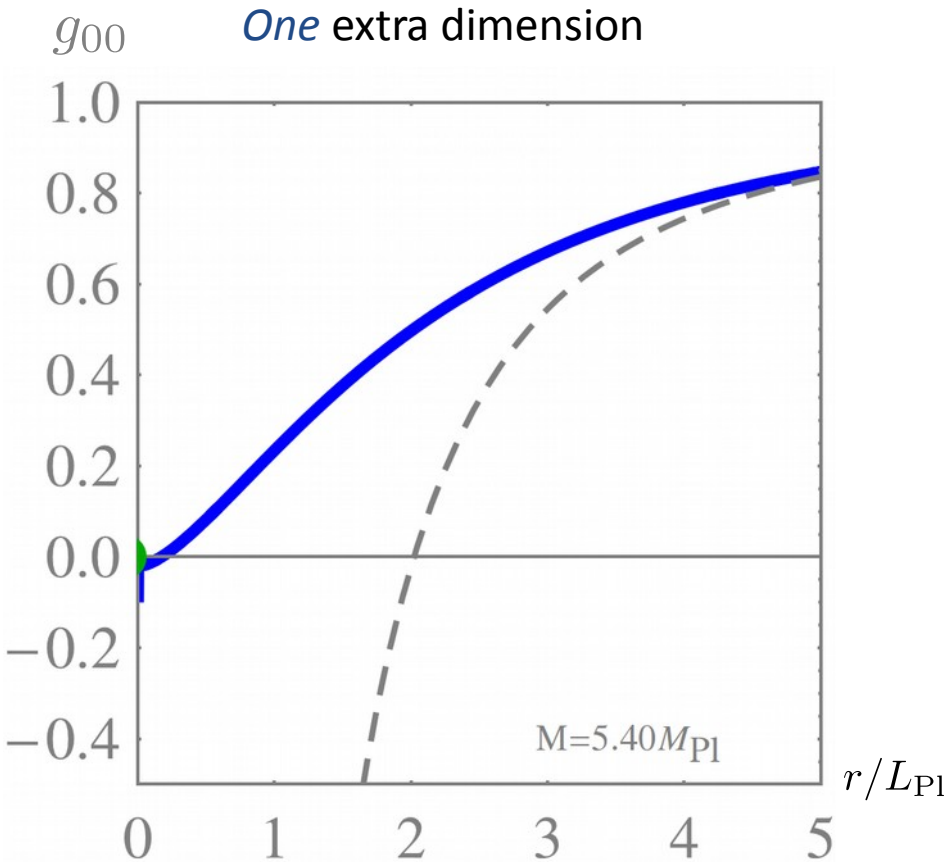
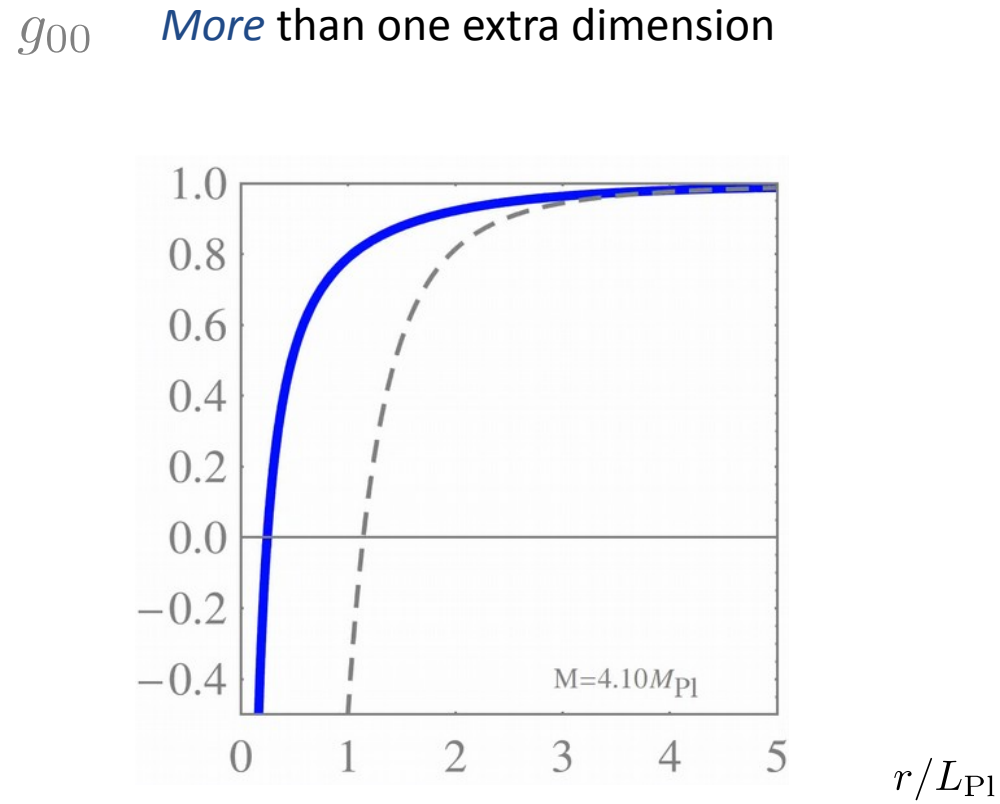
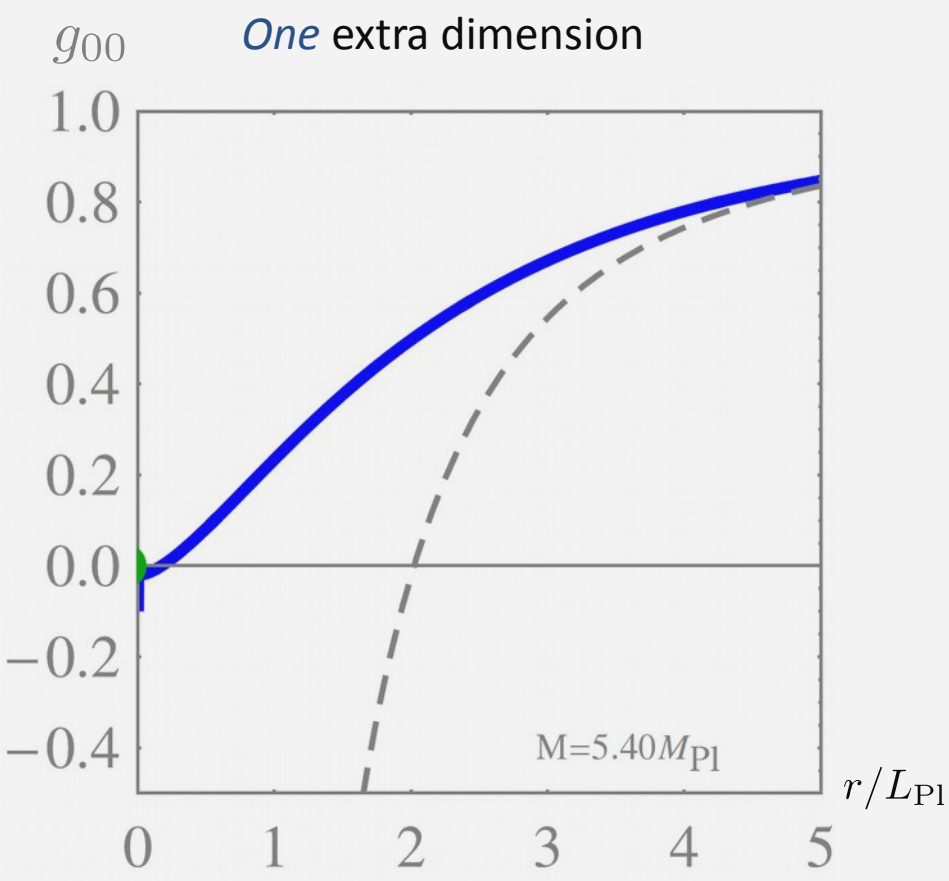
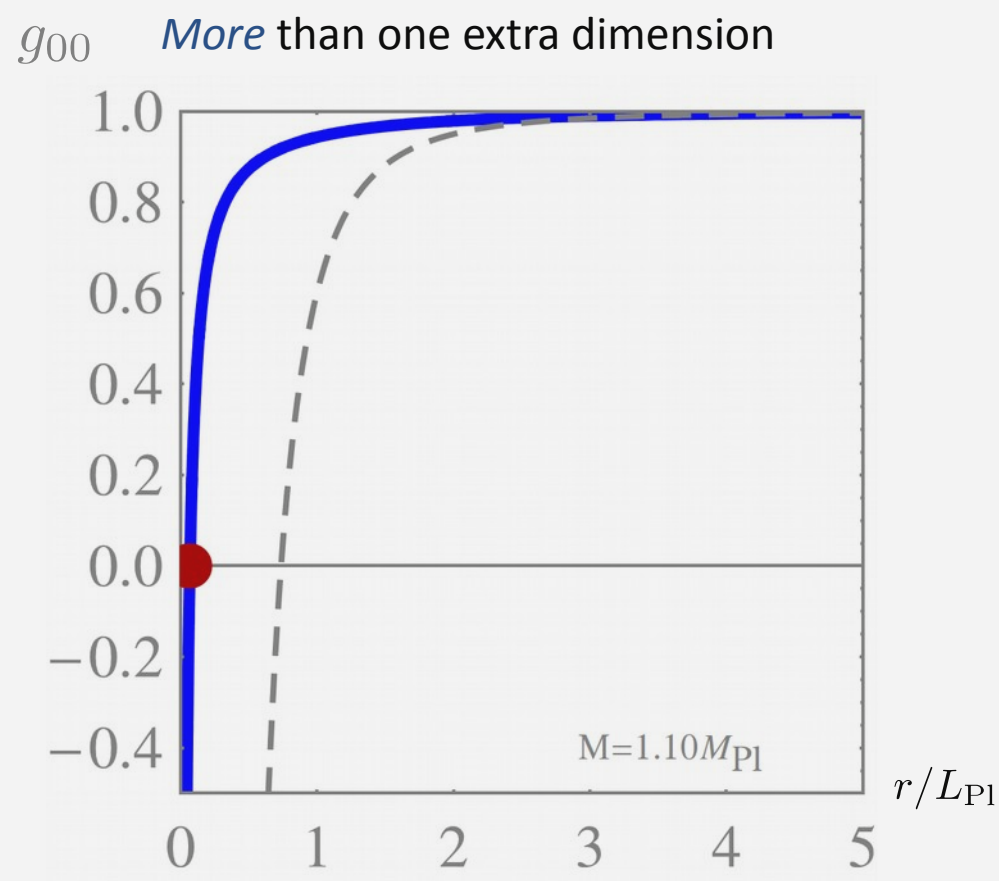


More than one extra dimension



GUP-inspired BH in higher dimensions





No more remnants in higher ($N > 4$) dimensions.

$$\mathcal{T}_0^0 \sim M \int \frac{\mathrm{d}^N p \; e^{i\vec{x}\cdot\vec{p}}}{1+\beta p^2} \sim M \int \mathrm{d}p \; e^{irp} \; p^{N-3}$$

Retracing a popular Gedankenexperiment

The Heisenberg microscope in *n* LXDs:

Adler 2010
Carr 2014

$$\Delta x \sim \Delta x_c + \Delta x_g$$

$$\Delta x_c \sim \lambda \sim 1/\Delta p$$

$$\Delta x_g \sim G_{(n)} \frac{M_{\text{eff}}}{r^{2+n}} \frac{r^2}{c^2} \sim G_{(n)} \frac{\Delta p}{r^n} \sim L_{(\text{n})}^{2+n} \Delta p^{1+n}$$

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This Gedankenexperiment motivates a modified GUP:

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+(\sqrt{\beta}p)^{2+n}\right) \qquad \mathcal{T}_0^0 \sim M \int \frac{\mathrm{d}^{3+n}p \; e^{i\vec{x}\cdot\vec{p}}}{1+(\sqrt{\beta}p)^{2+n}} \sim M \int \mathrm{d}^N p \; e^{i\vec{x}\cdot\vec{p}}$$

Constant integrand: Sounds reasonable

We can solve this spacetime analytically in any dimension.

Modified GUP inspired Black Holes

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+(\sqrt{\beta}p)^{2+n}\right)$$

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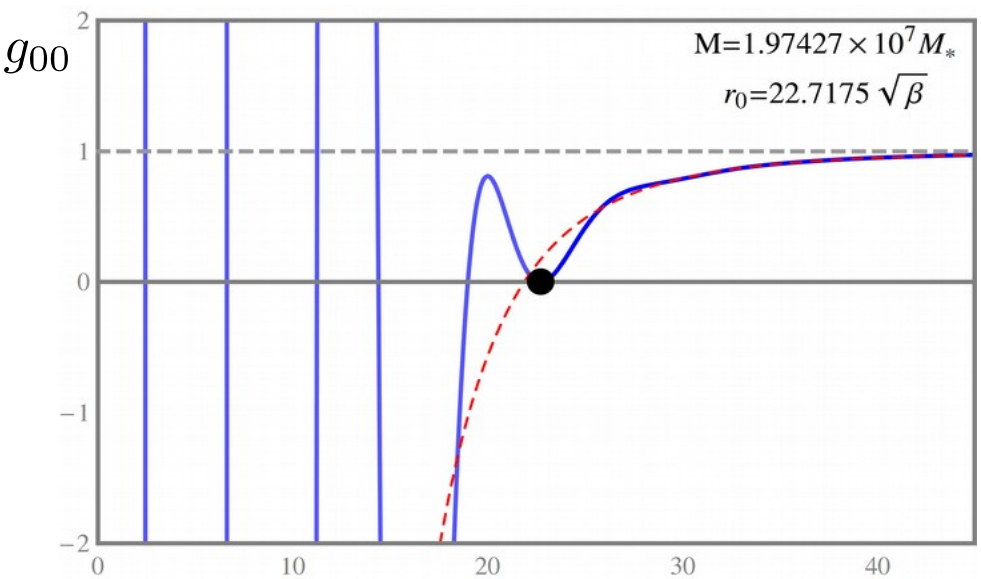
Gravitational potential with 4 extra dimensions:



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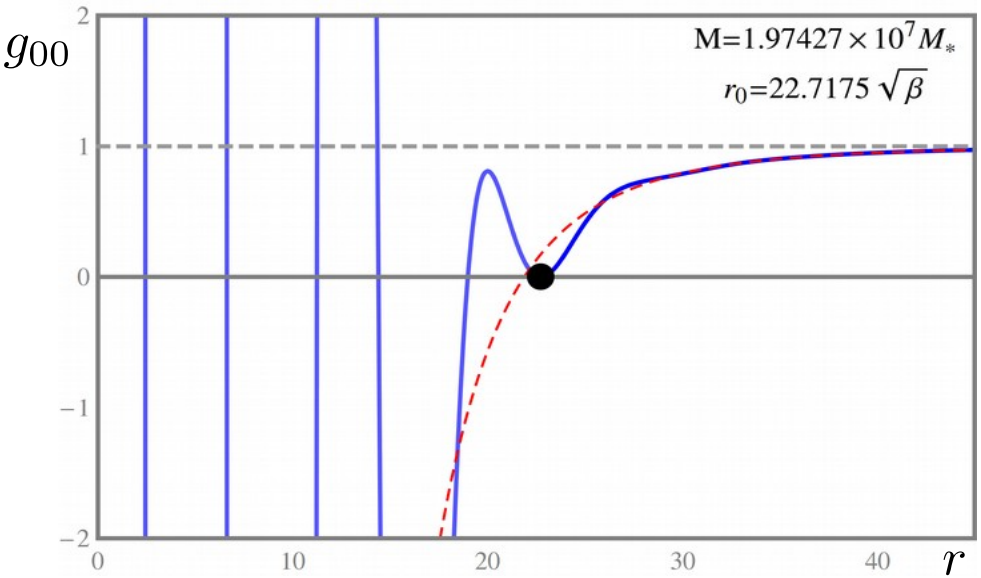
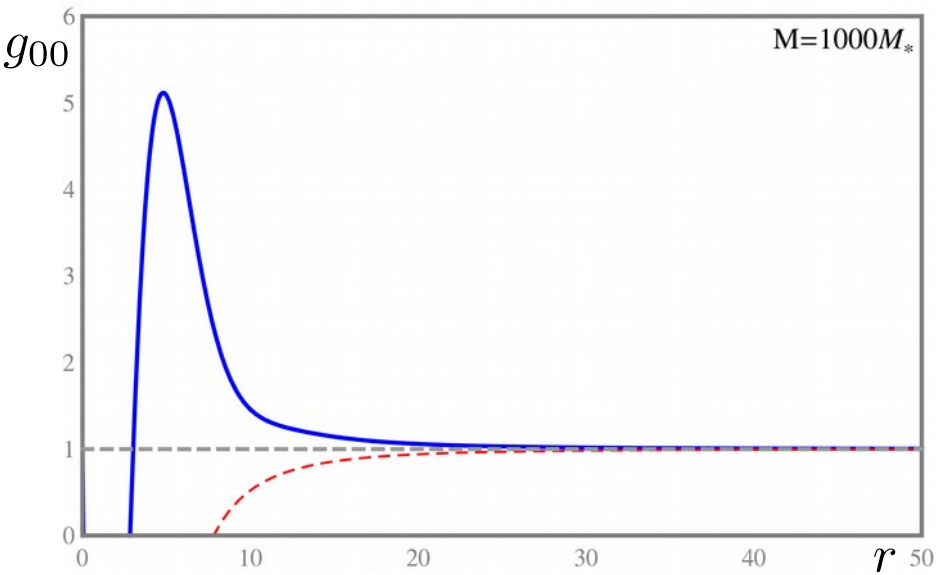


Gravitational potential with 1 extra dimension:



Pathologies!

GUP in extra dimensions: Ways out

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- Improve GUP. *Example:*

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+\left(\sqrt{\beta}p\right)^{(2+n)/(1+n)}\right)$$

Maziashvili 2013

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- Propose nonsingular model.
  Model engineering

Hayward 2006

Analytical continuation

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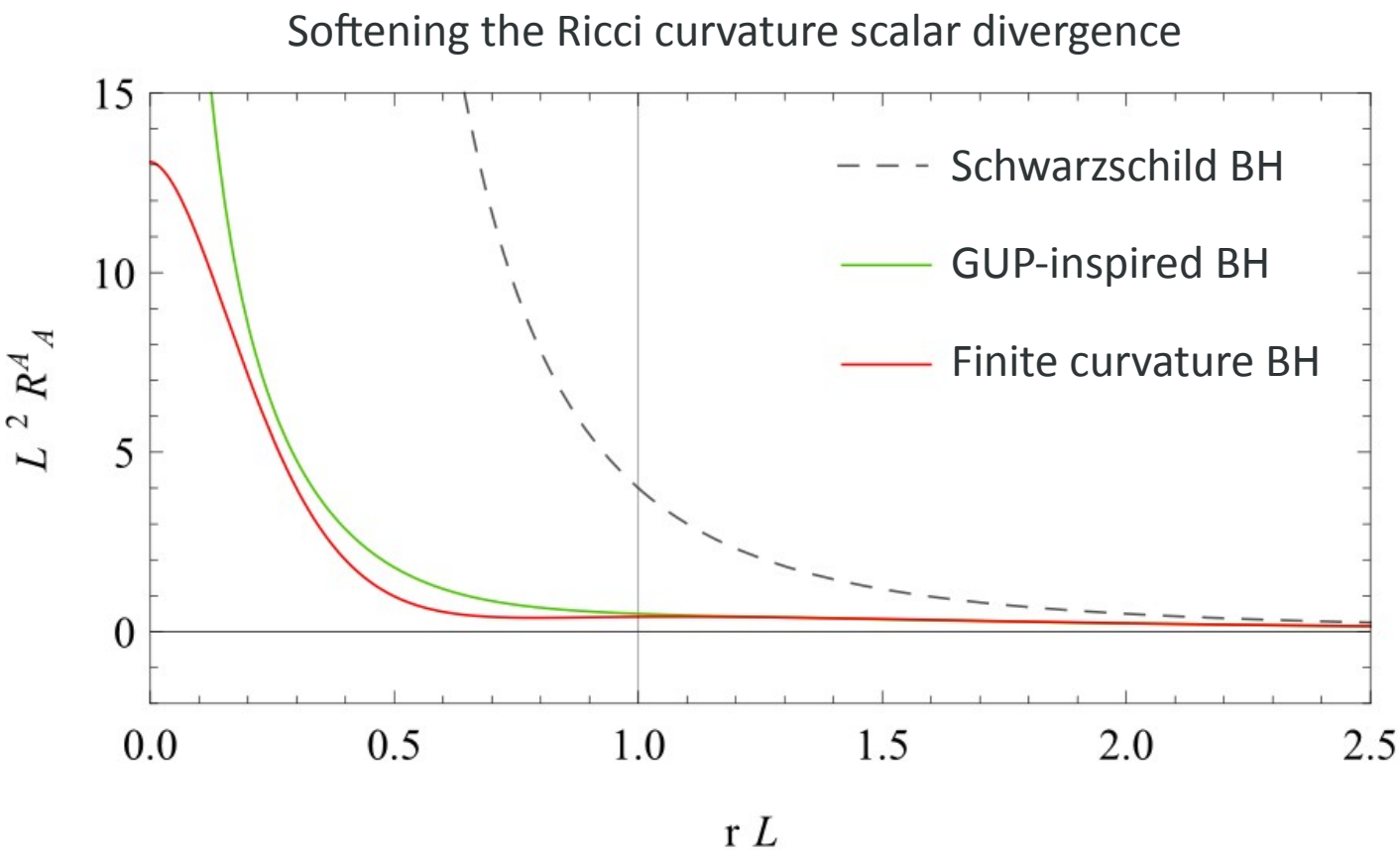
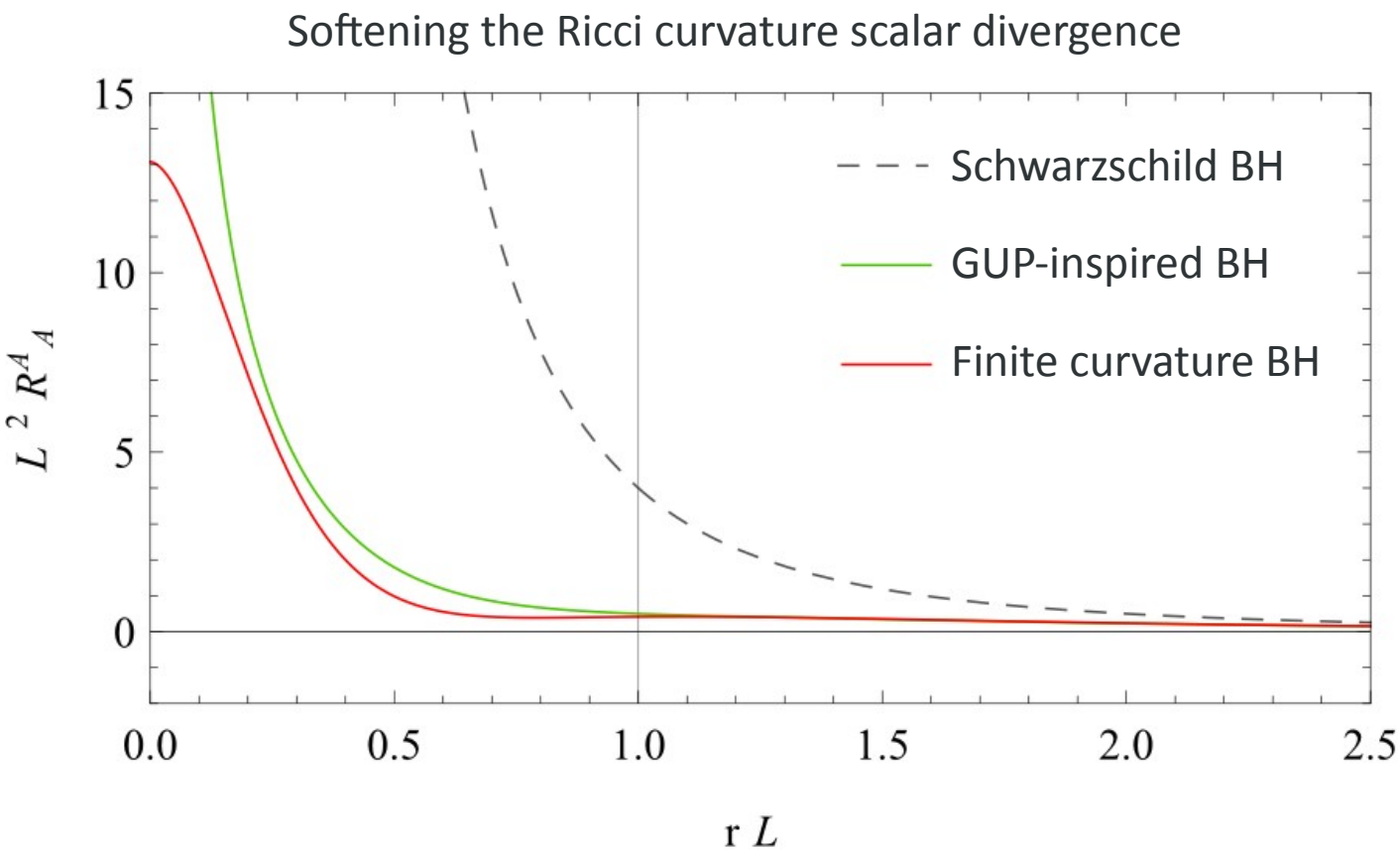


Analytical continuation



Analytical continuation:

$$\Theta = 0, |z| = r \quad \Rightarrow \quad g_{00} = 1 - \frac{2M(|z|)}{|z|}$$

Sven Köppel

The Generalized Uncertainty Principle and Extra Dimensions

FIAS Frankfurt Institute for Advanced Studies
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Thank you for your attention!

Ongoing

Köppel, Isi, Knipfer, Mureika, Nicolini
Self-Completeness and the GUP in extradimensions
Köppel, Knipfer, Dirkes, Frassino, Nicolini, Bleicher
GUP and BHs – a pedagogical review

References

Isi, Mureika, Nicolini
Self-Completeness and the GUP [JHEP 2013]