dS Space in Gauge/Gravity Duality

Lilia Anguelova
INRNE, Bulgarian Academy of Sciences

arXiv:1412.8422 [hep-th]; arXiv:1507.04053 [hep-th]

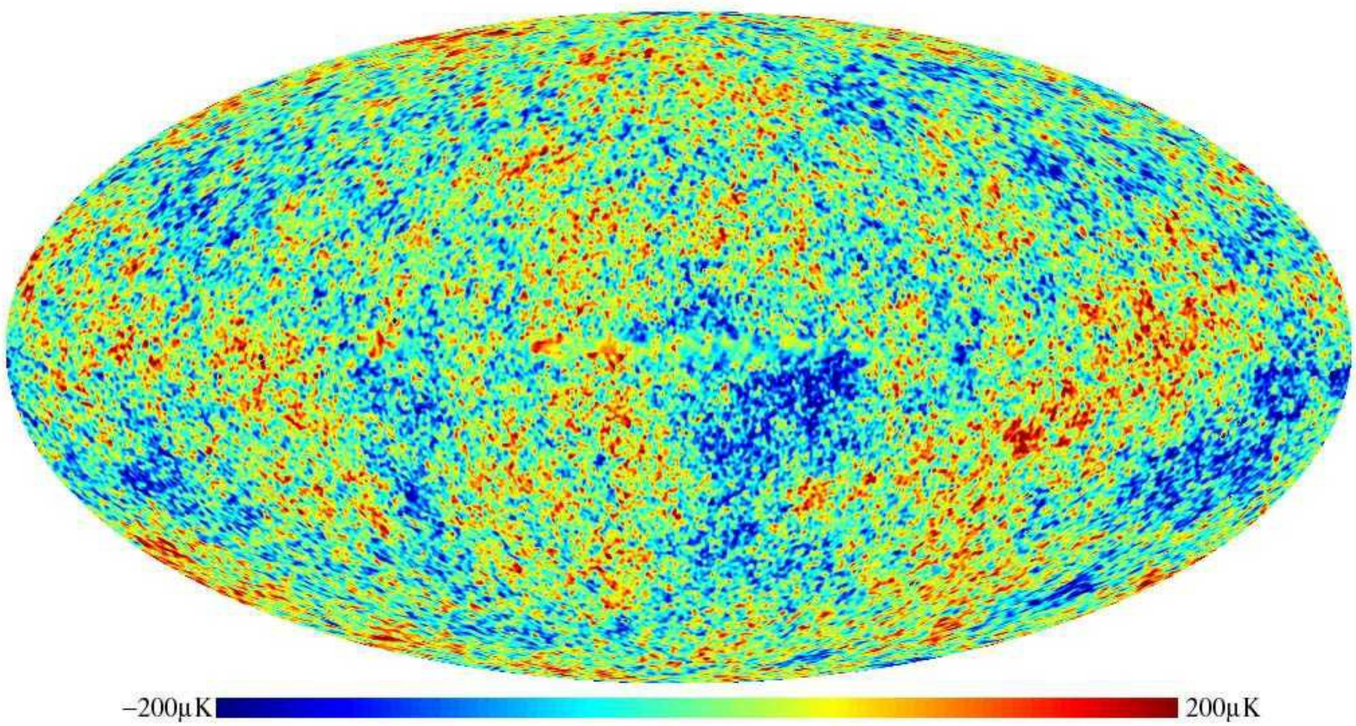
(with P. Suranyi, L.C.R. Wijewardhana)

Motivation

Cosmic Microwave Background (CMB) radiation:

WMAP (2003-2012) and Planck (2013) satellites:

Detailed map of CMB temperature fluctuations on the sky



$\overline{T} = 2.7\text{K}$

According to CMB data:

- On large scales:

  Universe is homogeneous and isotropic

- In Early Universe:

  Small perturbations that seed structure formation

  [(Clusters of) Galaxies]

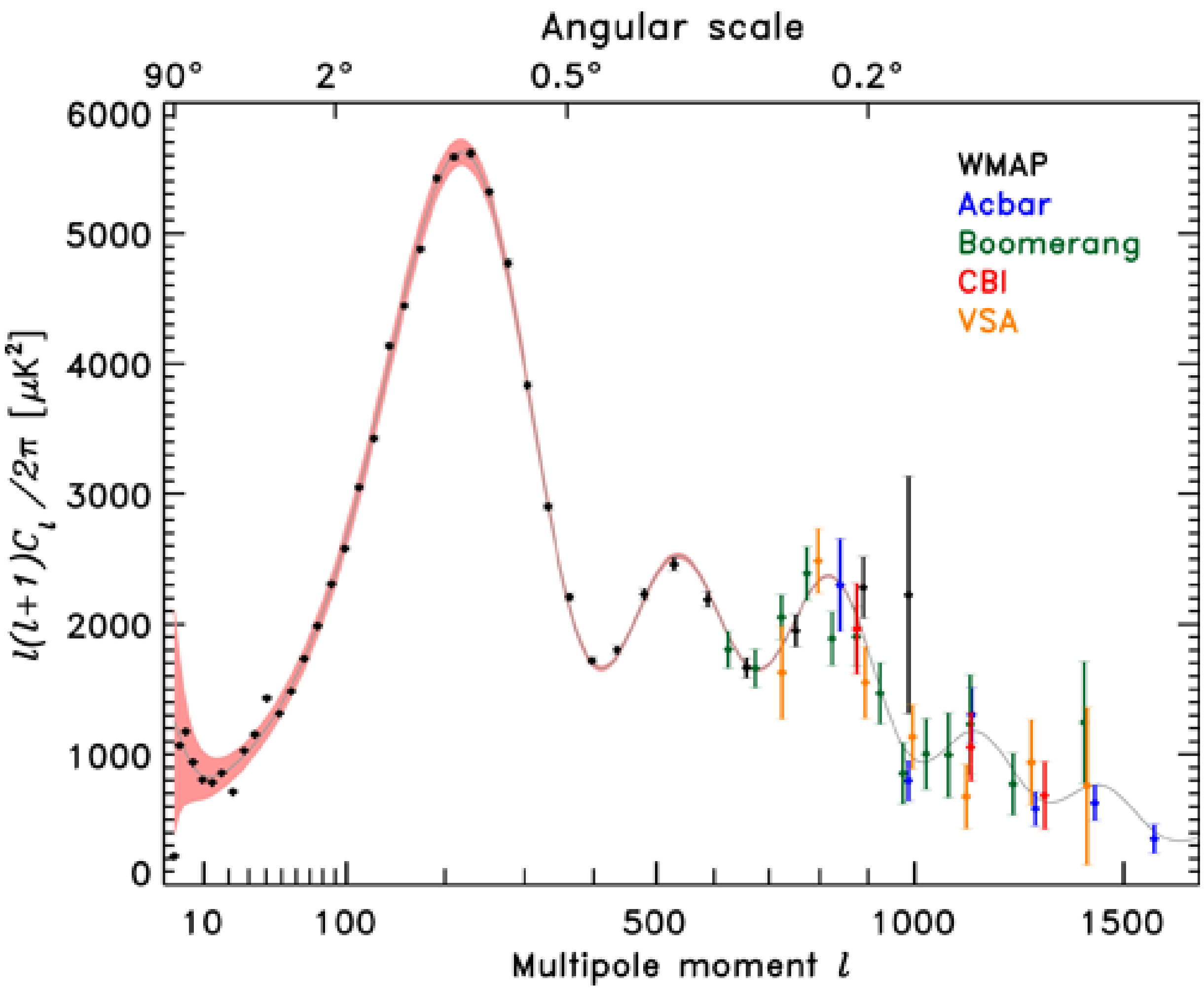
- Spectrum of temperature fluctuations:

  Expand: $$\frac{\delta T(\theta,\varphi)}{\overline{T}} = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi) \ ,$$

  Y_{lm} - standard spherical harmonics

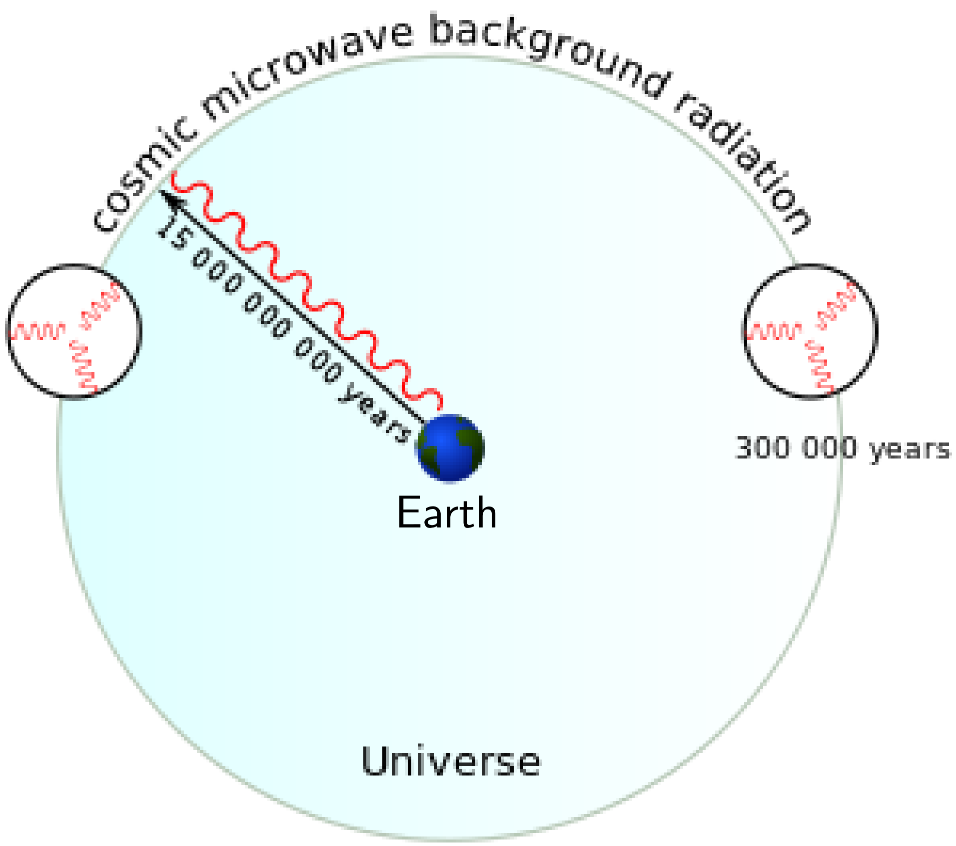
Rotationally invariant angular power spectrum:

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2$$



Cosmological Inflation:

Question: How can CMB be so uniform, if when it formed
Universe had many causally disconnected regions ?!?

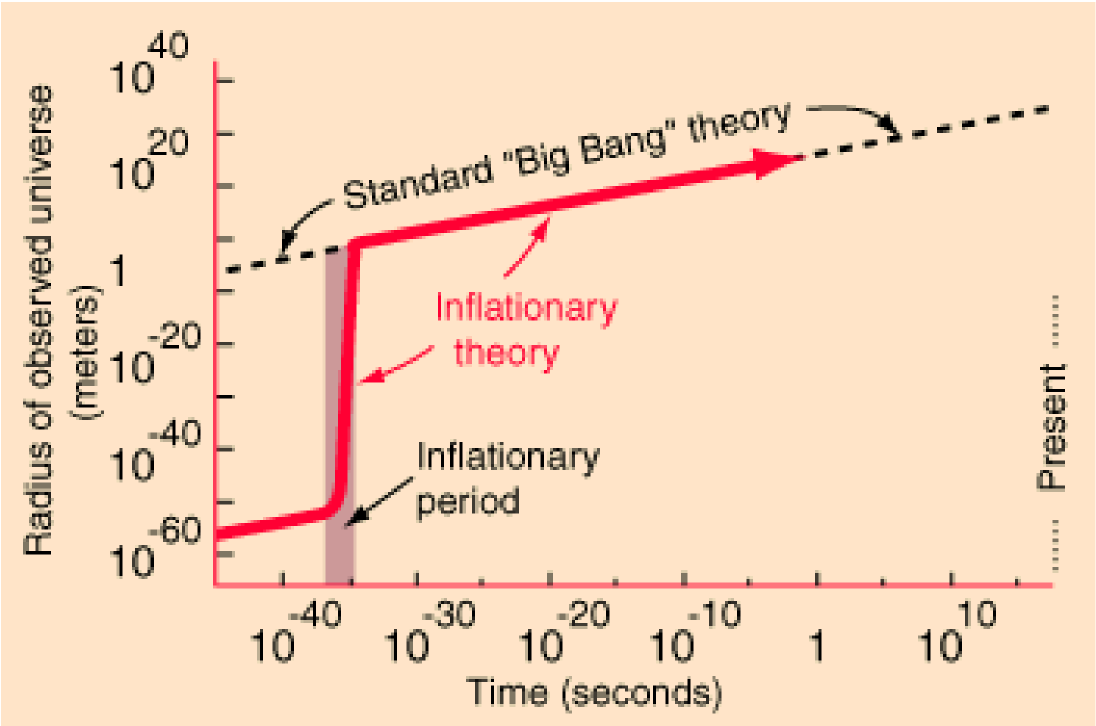


Cosmological Inflation:

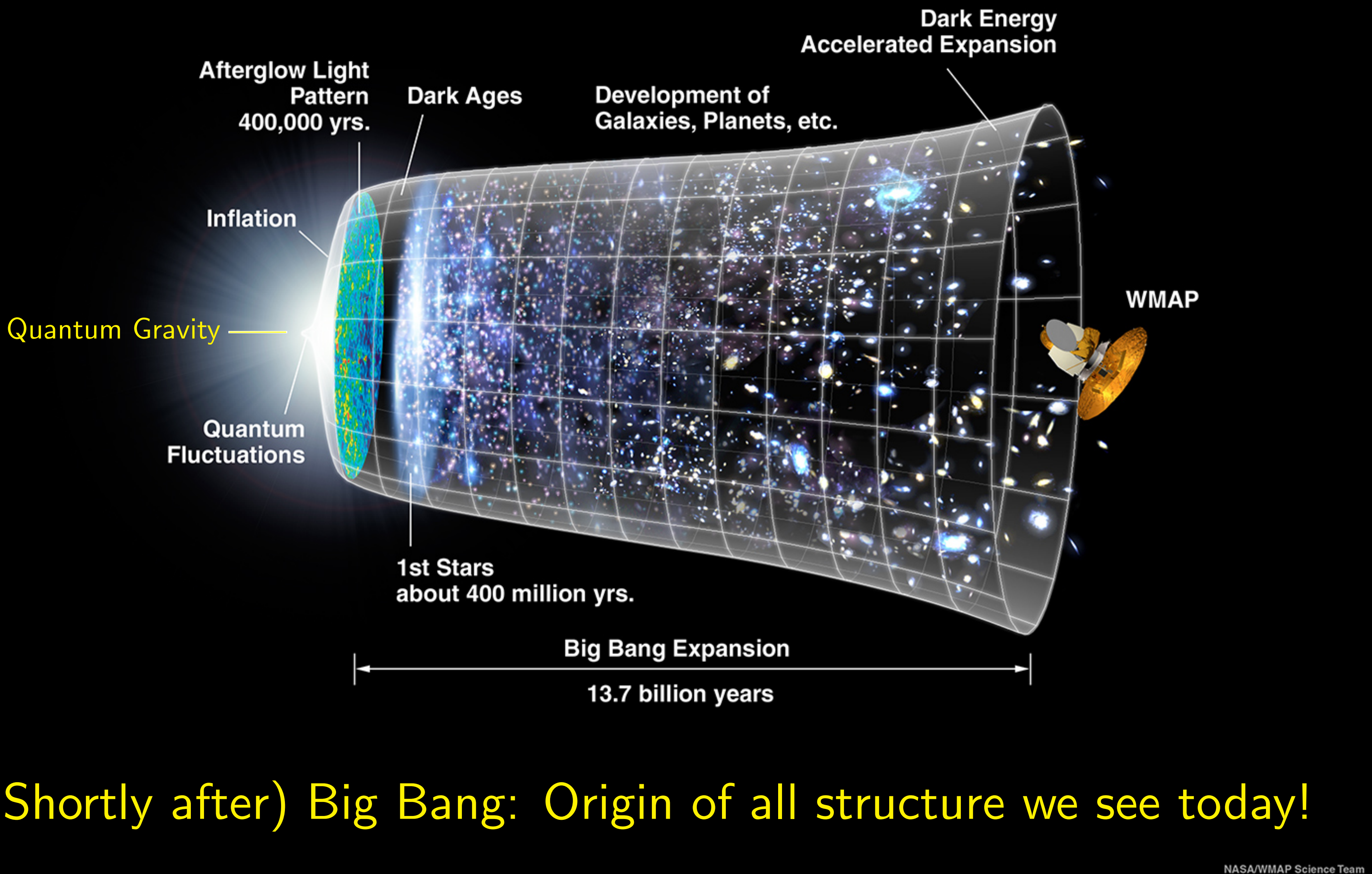
Answer: Period of very fast expansion of space in Early Universe

(faster than speed of light)

$\Rightarrow$ homogeneity and isotropy observed today



Inflation: Traces of Quantum Gravity ?



(Shortly after) Big Bang: Origin of all structure we see today!

Cosmological Inflation:

Standard description:

– expansion driven by the potential energy of a scalar field φ called inflaton

– weakly coupled Lagrangian for the inflaton within QFT framework:

$$S = \int d^4x \sqrt{-\det g} \left[\frac{R}{2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - V(\varphi) \right]$$

– preferably: small field models

($\Delta\varphi \ll M_P \quad \Rightarrow \quad$ EFT reliable)

BUT:

η problem:

Recall the slow roll conditions:

$$\varepsilon = \frac{V'(\varphi)}{V(\varphi)} \ll 1 \quad , \quad \eta = \frac{V''(\varphi)}{V(\varphi)} \ll 1$$

(consistency with observations $\Rightarrow$ slow roll inflation)

However: Quantum corrections drive inflaton mass ($m_\varphi^2 = V''$) to cutoff of effective theory (at least Hubble scale $H \approx \sqrt{V}$)

$\rightarrow$ $\Delta\eta \approx \mathcal{O}(1)$ or larger $\Rightarrow$ inflation ends prematurely

Hence need a symmetry... (ex.: axion monodromy inflation...)

Composite Inflation:

A possible different approach:

Inflaton - a composite state in a strongly coupled gauge theory

[F. Bezrukov, P. Channuie, J. Joergensen, F. Sannino, arXiv:1112.4054; inflaton - glueball]

Recently was argued that tensor-to-scalar ratio r can be large in such models [P. Channuie, K. Karwan, arXiv:1404.5879]

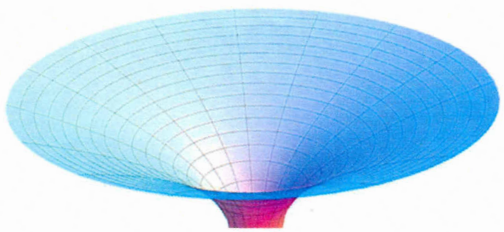
Our aim: Use Gauge/Gravity Duality (GGD) to study this class of inflationary models

(Recall: GGD - powerful nonperturbative method for studying strongly coupled gauge theories)

Gauge/Gravity Duality

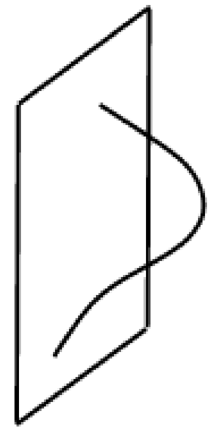
(AdS/CFT correspondence)

Two different perspectives on D-branes in string theory:



gravity background
[SUGRA solution]

$\longleftrightarrow$



open strings BCs
[gauge theory]

A stack of large number of D-branes:

Two sides of duality encode same degrees of freedom

[The two sides have equal partition functions!]

Gravity Backgrounds

Solutions of 10d SUGRA equations of motion

If a lower dimensional consistent truncation exists

$\Rightarrow$ we can, instead, study solutions of the lower dim. effective action for the relevant subset of fields

(Recall: Consistent truncation means that every solution of the lower dimensional action lifts to a solution of the full 10d action)

We will investigate a 5d consistent truncation of type IIB, established in [M. Berg, M. Haack, W. Muck, hep-th/0507285]

(This encompasses MN, KS solutions, but we will look for nonsusy ones.)

Consistent truncation:

IIB SUGRA:

– Bosonic fields: g_{MN}, Φ, C, H_3, F_3, F_5

– Ansatz for the consistent truncation:

$$ds^2_{10d} = e^{2p-q} ds^2_{5d} + e^{q+u}(\omega_1^2 + \omega_2^2) + e^{q-u}\left[(\tilde{\omega}_1 + v\omega_1)^2 + (\tilde{\omega}_2 - v\omega_2)^2\right]$$
$$+ e^{-6p-q}(\tilde{\omega}_3 + \omega_3)^2 \quad , \qquad ds^2_{5d} = g_{IJ}\, dx^I dx^J \quad ,$$

$$\tilde{\omega}_1 = \cos\psi d\tilde{\theta} + \sin\psi \sin\tilde{\theta} d\tilde{\varphi} \quad , \qquad \omega_1 = d\theta \quad ,$$
$$\tilde{\omega}_2 = -\sin\psi d\tilde{\theta} + \cos\psi \sin\tilde{\theta} d\tilde{\varphi} \quad , \qquad \omega_2 = \sin\theta d\varphi \quad ,$$
$$\tilde{\omega}_3 = d\psi + \cos\tilde{\theta} d\tilde{\varphi} \quad , \qquad \omega_3 = \cos\theta d\varphi$$

(Topology of internal 5d: $S^1 \times S^2 \times S^2$)

Ansatz continued:

$$\Phi = \phi(x^I) \quad , \quad C = 0 \quad , \quad H_3 = 0 \quad ,$$

$$\begin{aligned} F_3 = P\,[&-(\tilde{\omega}_1 + b d\theta) \wedge (\tilde{\omega}_2 - b \sin\theta d\varphi) \wedge (\tilde{\omega}_3 + \cos\theta d\varphi) \\ &+ (\partial_I b)\, dx^I \wedge (-d\theta \wedge \tilde{\omega}_1 + \sin\theta d\varphi \wedge \tilde{\omega}_2) \\ &+ (1 - b^2)(\sin\theta d\theta \wedge d\varphi \wedge \tilde{\omega}_3)\,, \end{aligned}$$

$$F_5 = \mathcal{F}_5 + \star\mathcal{F}_5 \;\; , \;\; \mathcal{F}_5 = Q\, vol_{5d} \;\; , \;\; P = const \;\; , \;\; Q = const$$

$\rightarrow$ 5d fields:

- metric: $g_{IJ}(x^I)$
- 6 scalars: $\phi(x^I)$, $p(x^I)$, $q(x^I)$, $u(x^I)$, $v(x^I)$, $b(x^I)$

[Note: Possible to extend considerations to $H_3 \neq 0$ ...]

5d action:

Let us denote $\{\varphi^i\} = \{\phi, p, q, u, v, b\}$:

$$S = \int d^5x \sqrt{-detg} \left[-\frac{R}{4} + \frac{1}{2} G_{ij}(\varphi) \partial_I \varphi^i \partial^I \varphi^j + V(\varphi) \right] ,$$

$G_{ij}(\varphi)$ - sigma model metric ,

$V(\varphi)$ - complicated potential

Equations of motion:

$$\begin{aligned} \nabla^2_{5d} \varphi^i + \mathcal{G}^i{}_{jk}\, g^{IJ} (\partial_I \varphi^j)(\partial_J \varphi^k) - V^i &= 0 , \\ -R_{IJ} + 2G_{ij}(\partial_I \varphi^i)(\partial_J \varphi^j) + \frac{4}{3} g_{IJ} V &= 0 , \end{aligned}$$

$\mathcal{G}^i{}_{jk}$ - Christoffel symbols for G_{ij} , $V^i = G^{ij} \partial_{\varphi^j} V$.

dS and Inflationary Solutions

Want to find a solution with the 5d metric:

$$ds_{5d}^2 = e^{2A(z)}\left[-dt^2 + a(t)^2 d\vec{x}^2\right] + dz^2$$

[K. Ghoroku, M. Ishihara, A. Nakamura, hep-th/0609152 : Used a 10d solution in IIB with such external 5d metric and $a(t) = e^{\sqrt{\frac{\Lambda}{3}}\,t}$ to study gauge theory in dS space. But the two scalars in that solution: $\phi(z)$, $C(z)$ $\Rightarrow$ not compatible with above consistent truncation.]

Hubble parameter: $\quad H(t) \equiv \dfrac{\dot{a}(t)}{a(t)} \quad \left(\Rightarrow \ \dot{H} = \dfrac{\ddot{a}}{a} - H^2\right)$

Note:

- dS space: $\quad H = const$
- Slow roll inflation: $\quad H = H(t)$, but $\dot{H}$ small

[More precisely: $\ddot{a} > 0 \ \Leftrightarrow \ \epsilon \equiv -\frac{\dot{H}}{H^2} < 1$; slow roll: $\epsilon \ll 1$]

Solving the coupled system of EoMs:

- Subtruncation of the consistent truncation:

Can consistently set $u \equiv 0\,,\ v \equiv 0\,,\ b \equiv 0$

[EoMs identically solved]

$\rightarrow$ Study class of solutions with only nontrivial scalars:

$$\phi(x^I)\ ,\ p(x^I)\ ,\ q(x^I)$$

- Look for quasi-de Sitter solutions (i.e. with $H \approx const$):

In gauge/gravity duality context: these scalar fields - glueballs

Discrete mass spectrum $\rightarrow$ inflaton mass dynamically fixed

$\Rightarrow$ No η problem!

BUT:

Number of EoMs for scalar fields and metric functions is with one more than number of unknown functions

$\rightarrow$ No solution?

Fortunately, we showed: [as long as $A'(z) \neq 0$]

One equation is dependent on the others!

Solutions with $H = const$:

- 3-parameter family with $q = -6\,p$ and $\phi = 0$

  [analytical solution]

- two 4-parameter families with $q = -\frac{3}{2}\,p$ and $\phi = 3\,p$

  [numerical solutions]

Solutions with time-dependent H:

[work in progress...]

Look for small time-dependent deviations from exact $H = const$ (i.e. pure dS) solution:

$\rightarrow$ Deform 5d metric ansatz and ansatz for scalar field(s)

Recall: $$ds_5^2 = e^{2A}\left[-dt^2 + e^{2\tilde{H}}d\vec{x}^2\right] + dz^2$$

- Expand in $\alpha \ll 1$ around the analytical solution:

$$\begin{aligned} \phi &= \alpha\,\phi_{(1)} + \alpha^3\phi_{(3)} + \mathcal{O}(\alpha^5) \\ A &= A_{(0)} + \alpha^2 A_{(2)} + \mathcal{O}(\alpha^4) \\ \tilde{H} &= H_{(0)} + \alpha^2 H_{(2)} + \mathcal{O}(\alpha^4) \end{aligned}$$

At leading order, find:

$$\phi_{(1)} = C_1 + C_2\, e^{-kt} \qquad \text{with} \qquad C_1, C_2, k = const$$

$\rightarrow$ Ultra-slow roll inflation [arXiv:gr-qc/0503017, W. Kinney]

Gives $n_s \approx 1$ (i.e. scale-invariant spectrum), but does not last for more than a few e-foldings

$\Rightarrow$ Can only be a transient phase, before usual slow roll

However, such a phase could explain low-l anomaly in CMB power spectrum

- To explore: t-dep. def. of numerical solutions $\rightarrow$ slow roll?...

Summary

Found so far:

- Three multi-parameter solutions in 5d consistent truncation of IIB supergravity

  [dS_4 space fibered over the fifth dimension]

- Ultra-slow roll glueball inflation model [t-dep. deformation]

Open issues:

- Slow roll Glueball Inflation?...
- Microscopic realization ?...
- Inflaton mass (mass-spectrum of fluctuations) ?...

Thank you!