Dynamics of tachyon fields on (non-)Archimedean spaces -- towards origin of inflation

Geometry and Physics Seminar

Department of Theoretical Physic, (IFIN-HH)

November 13, 2015 Bucharest, Romania

Goran S. Djordjević

SEEMET-MTP Office & Department of Physics,

Faculty of Science and Mathematics

University of Niš, Serbia

In cooperation with:

D. Dimitrijevic, M. Milosevic, M. Stojanovic, N. Bilic, Lj Nesic and D. Vulcanov

Cosmology and Tachyonic Inflation on Non-Archimedean Spaces

- Introduction and motivation

- Adelic Quantum Theory
- Minisuperspace quantum cosmology as quantum mechanics over minisuperspace
- p-Adic Inflation
- Tachyons
- Tachyons - From Field Theory to Classical Analogue –
  DBI and Sen approach
- Classical Canonical Transformation and Quantization
- Reverse Engineering and (Non)Minimal Coupling,
- Tachyon inflation in an AdS braneworld

- Conclusion and perspectives

1. Introduction and Motivation

- The main task of quantum cosmology is to describe the evolution of the universe in a very early stage.
- Since quantum cosmology is related to the Planck scale phenomena it is logical to consider various geometries (in particular nonarchimedean, noncommutative …)
- Supernova Ia observations show that the expansion of the Universe is accelerating, contrary to FRW cosmological models.
- Besides, cosmic microwave background (CMB) radiation data are suggesting that the expansion of our Universe seems to be in an accelerated state which is referred to as the “dark energy” effect.
- A need for understanding these new and rather surprising facts, including (cold) “dark matter”, has motivated numerous authors to reconsider different inflation scenarios.
- Despite some evident problems such as a non-sufficiently long period of inflation, tachyon-driven scenarios remain highly interesting for study.

2. Adelic Quantum Theory

- Reasons to use *p*-adic numbers and adeles in quantum physics:
- The field of rational numbers **Q**, which contains all observational and experimental numerical data, is a dense subfield not only in **R** but also in the fields of *p*-adic numbers $\mathbf{Q}_p$.
- There is an analysis within and over $\mathbf{Q}_p$ like that one related to **R.**
- General mathematical methods and fundamental physical laws should be invariant *[I.V. Volovich, (1987), Vladimirov, Volovich, Zelenov (1994)]* under an interchange of the number fields **R** and $\mathbf{Q}_p$.
- There is a quantum gravity uncertainty ($\Delta x \geq l_0 = \sqrt{\hbar G / c^3}$), when measures distances around the Planck length, which restricts priority of Archimedean geometry based on the real numbers and gives rise to employment of non-Archimedean geometry.
- It seems to be quite reasonable to extend standard Feynman's path integral method to non-Archimedean spaces.

Adelic Quantum Theory

- ***p*-ADIC FUNCTIONS AND INTEGRATION**
- There are primary two kinds of analyses on $\mathbf{Q}_p : \mathbf{Q}_p \to \mathbf{Q}_p$ (class.) and $\mathbf{Q}_p \to \mathbf{C}$ (quant.).
- Usual complex valued functions of *p*-adic variable, which are employed in mathematical physics, are:
- an additive character $\chi_p(x) = \exp 2\pi i \{x\}_p,$
- fractional part $\{x\}_p$
- locally constant functions with compact support

$$\Omega(|x|_p) = \begin{cases} 1 & |x|_p \leq 1 \\ 0 & |x|_p > 1 \end{cases}$$

- The number theoretic function $\lambda_p(x)$

Adelic Quantum Theory

- There is well defined Haar measure and integration. Important integrals are

$$\int_{Q_p} \chi_p(ayx)dx = \delta_p(ay) = |a|_p^{-1} \delta_p(y), a \neq 0$$

$$\int_{Q_p} \chi_p(\alpha x^2 + \beta x)dx = \lambda_p(\alpha)|2\alpha|_p^{-1/2} \chi_p\left(-\frac{\beta^2}{4\alpha}\right), \alpha \neq 0$$

- Real analogues of integrals

$$\int_{Q_\infty} \chi_\infty(ayx)dx = \delta_\infty(ay) = |a|_\infty^{-1} \delta_\infty(y), a \neq 0$$

$$\int_{Q_\infty} \chi_\infty(\alpha x^2 + \beta x)dx = \lambda_\infty(\alpha)|2\alpha|_\infty^{-1/2} \chi_\infty\left(-\frac{\beta^2}{4\alpha}\right), \alpha \neq 0$$

$$Q_\infty \equiv R, \quad \chi_\infty(x) = \exp(-2\pi i x)$$

Adelic Quantum Theory

- **Dynamics of *p*-adic quantum model**
- *p*-adic quantum mechanics is given by a triple

$$(L_2(Q_p), W_p(z_p), U_p(t_p))$$

- Adelic evolution operator is defined by

$$U(t)\psi(x) = \int_A K_t(x,y)\psi(y)dy = \prod_{v=\infty,2,3,\dots,p,\dots} \int_{Q_v} K_t^v(x_v,y_v)\psi^{(v)}(y_v)dy_v$$

- The eigenvalue problem

$$U(t)\psi_\alpha(x) = \chi(E_\alpha t)\psi_\alpha(x)$$

Adelic Quantum Theory

- The main problem in our approach is computation of *p*-adic transition amplitude in Feynman's PI method

$$K_p(x'',t'';x',t') = \int_{(x',t')}^{(x'',t'')} \chi_p\left(-\frac{1}{h}\int_{t'}^{t''} L(\dot{q},q,t)\right) Dq$$

- Exact general expression ($\bar{S}$ - classical action)

$$K_p(x'',t'';x',t') = \lambda_p\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x'\partial x''}\right) \times \left|\frac{1}{h}\frac{\partial^2 \bar{S}}{\partial x'\partial x''}\right|_p^{1/2} \chi_p\left(-\bar{S}(x'',t'';x',t')\right)$$

$$K_p(x'',y'',z'',t'';x',y',z',t') = \lambda_p\left(\det\begin{pmatrix} -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial x'\partial x''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial x'\partial y''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial x'\partial z''} \\ -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial y'\partial x''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial y'\partial y''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial y'\partial z''} \\ -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial z'\partial x''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial z'\partial y''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial z'\partial z''} \end{pmatrix}\right)$$

$$\times \left|\det\begin{pmatrix} -\frac{\partial^2 \bar{S}}{\partial x'\partial x''} & -\frac{\partial^2 \bar{S}}{\partial x'\partial y''} & -\frac{\partial^2 \bar{S}}{\partial x'\partial z''} \\ -\frac{\partial^2 \bar{S}}{\partial y'\partial x''} & -\frac{\partial^2 \bar{S}}{\partial y'\partial y''} & -\frac{\partial^2 \bar{S}}{\partial y'\partial z''} \\ -\frac{\partial^2 \bar{S}}{\partial z'\partial x''} & -\frac{\partial^2 \bar{S}}{\partial z'\partial y''} & -\frac{\partial^2 \bar{S}}{\partial z'\partial z''} \end{pmatrix}\right|_p^{1/2} \chi_p\left(-\bar{S}(x'',y'',z'',t'';x',y',z't')\right)$$

Adelic Quantum Theory

- Adelic quantum mechanics *[Dragovich (1994), G. Dj. and Dragovich (1997, 2000), G. Dj, Dragovich and Lj. Nesic (1999)]*.
- Adelic quantum mechanics: $(L_2(A), W(z), U(t))$
  - adelic Hilbert space, $L_2(A)$
  - Weyl quantization of complex-valued functions on adelic classical phase space, $W(z)$
  - unitary representation of an adelic evolution operator, $U(t)$
- The form of adelic wave function

$$\psi = \psi_\infty(x_\infty) \cdot \prod_{p \in M} \psi_p(x_p) \cdot \prod_{p \notin M} \Omega(|x|_p)$$

Adelic Quantum Theory

- **Exactly soluble p-adic and adelic quantum mechanical models:**
  - a free particle and harmonic oscillator *[VVZ, Dragovich]*
  - a particle in a constant field *[G. Dj, Dragovich]*
  - a free relativistic particle *[G. Dj, Dragovich, Nesic]*
  - a harmonic oscillator with time-dependent frequency *[G. Dj, Dragovich]*

- **Resume of AQM:** AQM takes in account ordinary as well as *p*-adic effects and may me regarded as a starting point for construction of more complete quantum cosmology and string/M theory. In the low energy limit AQM effectively becomes the ordinary one.

3. Minisuperspace quantum cosmology as quantum mechanics over minisuperspace

(ADELIC) QUANTUM COSMOLOGY

- The main task of AQC is to describe the very early stage in the evolution of the Universe.
- **At this stage, the Universe was in a quantum state, which should be described by a wave function (complex valued and depends on some real parameters).**
- However, QC is related to Planck scale phenomena - it is natural to reconsider its foundations.
- We maintain here the standard point of view that the wave function takes complex values, but we treat its arguments in a more complete way!
- We regard space-time coordinates, gravitational and matter fields to be adelic, i.e. they have real as well as *p*-adic properties simultaneously.
- There is no Schroedinger and Wheeler-De Witt equation for cosmological models.
- Feynman's path integral method was exploited and minisuperspace cosmological models are investigated as a model of adelic quantum mechanics *[Dragovich (1995), G Dj, Dragovich, Nesic and Volovich (2002), G.Dj and Nesic (2005, 2008)…]*.

Minisuperspace quantum cosmology as quantum mechanics over minisuperspace

(ADELIC) QUANTUM COSMOLOGY

- Adelic minisuperspace quantum cosmology is an application of adelic quantum mechanics to the cosmological models.
- Path integral approach to standard quantum cosmology

$$< h''_{ij}, \phi'', \Sigma'' \,|\, h'_{ij}, \phi', \Sigma' >_{\infty} = \int D(g_{\mu\nu})_{\infty} D(\phi)_{\infty} \chi_{\infty}(-S_{\infty}[g_{\mu\nu}, \phi])$$

$$< h''_{ij}, \phi'', \Sigma'' \,|\, h'_{ij}, \phi', \Sigma' >_{p} = \int D(g_{\mu\nu})_{p} D(\phi)_{p} \chi_{p}(-S_{p}[g_{\mu\nu}, \phi])$$

4. *p*-Adic Inflation

- *p*-Adic string theory was defined *[Volovich, Freund, Olson (1987); Witten at al (1987,1988)]* replacing integrals over ***R*** (in the expressions for various amplitudes in ordinary bosonic open string theory) by integrals over Q_p , with appropriate measure, and standard norms by the *p*-adic one.

- This leads to an exact action in *d* dimensions, , .

$$S = \frac{m_s^4}{g_p^2}\int d^4x\left(-\frac{1}{2}\phi e^{-\frac{-\partial_t^2+\nabla^2}{m_p^2}}\phi + \frac{1}{p+1}\phi^{p+1}\right), \qquad \frac{1}{g_p^2} \equiv \frac{1}{g_s^2}\frac{p^2}{p-1} \qquad m_p^2 \equiv \frac{2m_s^2}{\ln p}.$$

- The dimensionless scalar field describes the open string tachyon.
- m_s is the string mass scale and
- g_s is the open string coupling constant
- Note, that the theory has sense for any integer and make sense in the limit $p \to 1$

p-Adic Inflation

- The corresponding equation of motion:

$$e^{-\frac{-\partial_t^2+\nabla^2}{m_p^2}}\varphi=\varphi^p$$

- In the limit (the limit of local field theory) the above eq. of motion becomes a local one

$$(-\partial_t^2+\nabla^2)\varphi=2\varphi\ln\varphi$$

- Very different limit $p>>1$ leads to the models where nonlocal structures is playing an important role in the dynamics
- Even for extremely steep potential p-adic scalar field (tachyon) rolls slowly! This behavior relies on the nonlocal nature of the theory: the effect of higher derivative terms is to slow down rolling of tachyons.
- Approximate solutions for the scalar field and the quasi-de-Sitter expansion of the universe, in which starts near the unstable maximum $\phi=1$ and rolls slowly to the minimum $\phi=0$.

5. Tachyons

- A. Somerfeld - first discussed about possibility of particles to be faster than light (100 years ago).
- G. Feinberg - called them tachyons: Greek word, means fast, swift (almost 50 years ago).
- According to Special Relativity: $m^2 < 0, \quad v = \dfrac{p}{\sqrt{p^2 + m^2}}.$
- From a more modern perspective the idea of faster-than-light propagation is abandoned and the term "tachyon" is recycled to refer to a quantum field with $m^2 = V'' < 0.$

Tachyons

- Field Theory
- Standard Lagrangian (real scalar field):

$$L(\varphi, \partial_\mu \varphi) = T - V = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi_0) - V'(\varphi_0)\varphi - \frac{1}{2}V''(\varphi_0)\varphi^2 - \ldots$$

- Extremum (min or max of the potential): $V'(\varphi_0) = 0$
- Mass term: $V''(\varphi_0) = m^2$
- Clearly V'' can be negative (about a maximum of the potential). Fluctuations about such a point will be unstable: tachyons are associated with the presence of instability.

$$L(\phi, \partial_\mu \phi) = L_{kin} - V = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 + const$$

6. Tachyons-From Field Theory to Classical Analogue – DBI and Sen approach

- String Theory
- A. Sen – proposed (effective) tachyon field action (for the *Dp*-brane in string theory):

$$S = -\int d^{n+1}x V(T)\sqrt{1+\eta^{ij}\partial_i T \partial_j T}$$

$$\eta_{00} = -1$$

$$\eta_{\mu\nu} = \delta_{\mu\nu} \quad \mu,\nu = 1,\dots,n$$

- $T(x)$ - tachyon field
- $V(T)$ - tachyon potential
- Non-standard Lagrangian and DBI Action!

Tachyons-From Field Theory to Classical Analogue – DBI and Sen approach

- Equation of motion (EoM):

$$\ddot{T}(t) - \frac{1}{V(T)}\frac{dV}{dT}\dot{T}^2(t) = -\frac{1}{V(T)}\frac{dV}{dT}$$

- Can we transform EoM of a class of non-standard Lagrangians in the form which corresponds to Lagrangian of a canonical form, even quadratic one? Some classical canonical transformation (CCT)?

7. Classical Canonical Transformation and Quantization

- CCT: $T, P \quad \mapsto \quad \tilde{T}, \tilde{P}$
- Generating function: $G(\tilde{T}, P) = -PF(\tilde{T})$

$$T = -\frac{\partial G}{\partial P} = F(\tilde{T}) \qquad\qquad \tilde{P} = -\frac{\partial G}{\partial \tilde{T}} \Rightarrow P = \left(\frac{dF(\tilde{T})}{d\tilde{T}}\right)^{-1} \tilde{P}$$

- EoM transforms to

$$\ddot{\tilde{T}} + \left(\frac{\dfrac{d^2 F(\tilde{T})}{d\tilde{T}^2}}{\dfrac{dF(\tilde{T})}{d\tilde{T}}} - \frac{dF(\tilde{T})}{d\tilde{T}} \frac{d \ln V(F)}{dF} \right) \dot{\tilde{T}}^2 + \frac{1}{\dfrac{F(\tilde{T})}{d\tilde{T}}} \frac{d \ln V(F)}{dF} = 0$$

Classical Canonical Transformation and Quantization

- Choice: $F^{-1}(T) = \int_{T_0}^{T} \frac{dX}{V(X)}$
- EoM reduces to:

$$\ddot{\tilde{T}} + \frac{1}{F'} \frac{d \ln V(F)}{dF} = 0$$

- This EoM can be obtained from the standard type Lagrangians $\mathcal{L} = L_{kin} - V$

Classical Canonical Transformation and Quantization

- Example: $V(T)=\dfrac{1}{\cosh(\beta T)}$

$$F^{-1}(T)=\int^{T}\frac{dx}{V(x)}=\frac{1}{\beta}\sinh(\beta T)$$

- Generating function: $G(\tilde{T},P)=-PF(\tilde{T})=-\dfrac{P}{\beta}\operatorname{arcsinh}(\beta T)$
- EoM: $\ddot{\tilde{T}}(t)-\beta^2\tilde{T}(t)=0$
- This EoM can be obtained from the standard-type (quadratic)Lagrangian

$$\mathcal{L}_{quad}(\tilde{T},\dot{\tilde{T}})=\frac{1}{2}\dot{\tilde{T}}^2+\frac{1}{2}\beta^2\tilde{T}^2$$

Classical Canonical Transformation and Quantization

- Action (quadratic): $$S_{cl}=\int_0^{\tau}\mathcal{L}_{quad}dt=\frac{\beta}{2}\left(\left(\tilde{T}_1^2+\tilde{T}_2^2\right)\coth(\beta\tau)-\frac{2\tilde{T}_1\tilde{T}_2}{\sinh(\beta\tau)}\right)$$
- Quantization; Transition amplitude, $v=\infty,2,3,\ldots p,\ldots$

$$\mathcal{K}_v(\tilde{T}_2,\tau;\tilde{T}_1,0)=\lambda_v\left(\frac{1}{2\tau}\right)\left|-\frac{1}{\tau}\right|_v^{1/2}\chi_v\left(-S_{cl}(\tilde{T}_2,\tau;\tilde{T}_1,0)\right)$$

- The necessary condition for the existence of a p-adic (adelic) quantum model is the existence of a p-adic quantum-mechanical ground (vacuum) state in the form of a characteristic Ω -function; we get expression which defines constraints on parameters of the theory

$$\int_{|\tilde{T}_1|_p\le 1}\mathcal{K}_p(\tilde{T}_2,\tau;\tilde{T}_1,0)d\tilde{T}_1=\Omega(|\tilde{T}_2|_p)$$

Classical Canonical Transformation and Quantization

- Using *p*-Adic Gauss integral

$$\int_{|y|_p \le 1} \chi_p(ay^2 + by)dy = \begin{cases} \Omega(|b|_p), & |a|_p \le 1 \\ \dfrac{\lambda_p(a)}{|a|_p^{1/2}} \chi_p(-\dfrac{b^2}{4a})\Omega(|\dfrac{b}{a}|_p), & |a|_p > 1 \end{cases}$$

- we get (for the case of inverse power-law potential) $V \sim \tilde{T}^{-n}, \quad n = 1$

$$\frac{\lambda_p(\frac{1}{2\tau})}{|\tau|_p^{1/2}} \chi_p\left(-\frac{1}{2\tau}\tilde{T}_2^2 - \frac{1}{2}k\tau\tilde{T}_2 + \frac{1}{24}k^2\tau^3\right) \times I_{Gauss} = \Omega(|\tilde{T}_2|_p)$$

Classical Canonical Transformation and Quantization

- Case 1 $|\tau|_p > 1$ impossible to fulfill
- Case 2 $|\tau|_p = 1$

$$\chi_p\Big(-\frac{1}{2\tau}\tilde{T}_2^2 - \frac{1}{2}k\tau\tilde{T}_2 + \frac{1}{24}k^2\tau^3\Big)\Omega(|\frac{\tilde{T}_2}{\tau} - \frac{1}{2}k\tau|_p) = \Omega(|\tilde{T}_2|_p)$$

- Case 3 $|\tau|_p < 1$

$$\chi_p\Big(-k\tau\tilde{T}_2 + \frac{1}{6}k^2\tau^3\Big)\Omega(|-2\tilde{T}_2 + k\tau^2|_p) = \Omega(|\tilde{T}_2|_p)$$

8. Review of Reverse Engineering Method - “REM”

Ar.	$R(t)/R_0$	$\phi(t)-\phi_0$	$V(\phi)$	$4\pi B^2$
1.	$exp(\omega t)$	$\pm\frac{B}{\omega}exp(-\omega t)$	$\frac{3\omega^2}{8\pi}+\omega^2(\phi-\phi_0)^2$	$\frac{k}{R_0^2}$
2.	$sinh(\omega t)$	$\pm\frac{B}{\omega}log\frac{e^{\omega t}-1}{e^{\omega t}+1}$	$\frac{3\omega^2}{8\pi}+B^2\left[sinh\left(\frac{2\omega}{B}(\phi-\phi_0)\right)\right]^2$	$\frac{k}{R_0^2}+\omega^2$
3.	$cosh(\omega t)$	$\pm\frac{2B}{\omega}tan^{-1}(e^{\omega t})$	$\frac{3\omega^2}{8\pi}+B^2\left[sin\left(\frac{2\omega}{B}(\phi-\phi_0)\right)\right]^2$	$\frac{k}{R_0^2}-\omega^2$
4.	t^n	$\pm Blog\ t$	$\frac{B^2}{2}(3n-1)\exp\left(\pm\frac{2}{B}(\phi-\phi_0)\right)$	n
5.	t	$\pm Blog\ t$	$B^2\exp(2\frac{\phi-\phi_0}{B})$	$1+\frac{k}{R_0^2}$

Table 1.

where we denoted with an *"0"* index all values at the initial actual time.

Cosmology with non-minimally coupled scalar field

We shall now introduce the most general scalar field as a source for the cosmological gravitational field, using a Lagrangian as :

$$L = \sqrt{-g}\left[\frac{1}{16\pi}R - \frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi) - \frac{1}{2}\xi R\phi^2\right]$$

where ζ is the numerical factor that describes the type of coupling between the scalar field and the gravity.

Cosmology with non-minimally coupled scalar field

- Although we can proceed with the reverse method directly with the Friedmann eqs. it is rather complicated due to the existence of nonminimal coupling. We appealed to the numerical and graphical facilities of a Maple platform in the Einstein frame with a minimal coupling!.

- For sake of completeness we can compute the Einstein equations for the FRW metric.

Cosmology with non-minimally coupled scalar field

- After some manipulations we have:

$$3H(t)^2 + 3\frac{k}{R(t)^2} = \left[\frac{1}{2}\dot{\varphi}(t)^2 - V(t) + 3\zeta\dot{H}(t)(\varphi(t)^2)\right]$$

$$3H(t)^2 + 3\dot{H}(t) = \left[-\dot{\varphi}(t)^2 + V(t) - \frac{3}{2}\zeta H(t)(\dot{\varphi}(t)^2)\right]$$

$$\ddot{\phi}(t) = \frac{\partial V}{\partial \phi} - 6\zeta\frac{k}{R(t)^2} - 6\zeta H(t)\phi(t)$$

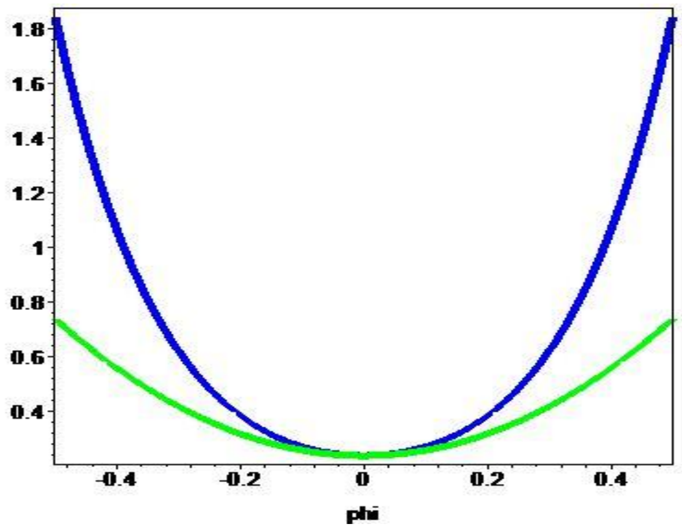
$$-12\zeta H(t)^2\phi(t) - 3H(t)\dot{\phi}(t)$$

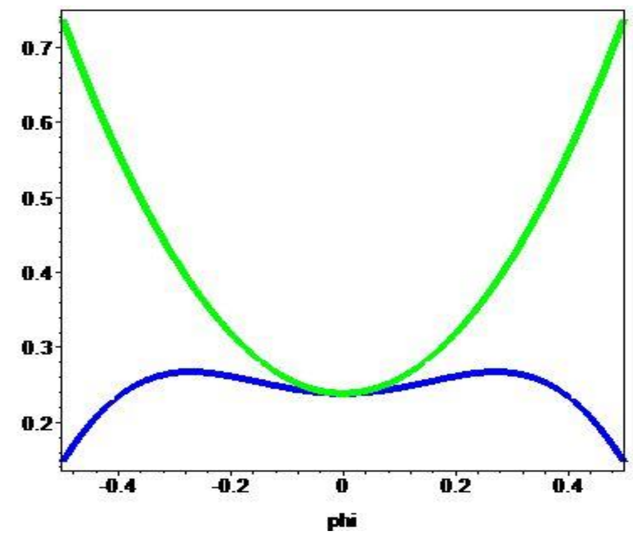
Where $8\pi G = 1,\ c = 1$

These are the new Friedman equations !!!

Some numerical results

- The exponential expansion and the corresponding potential depends on the field



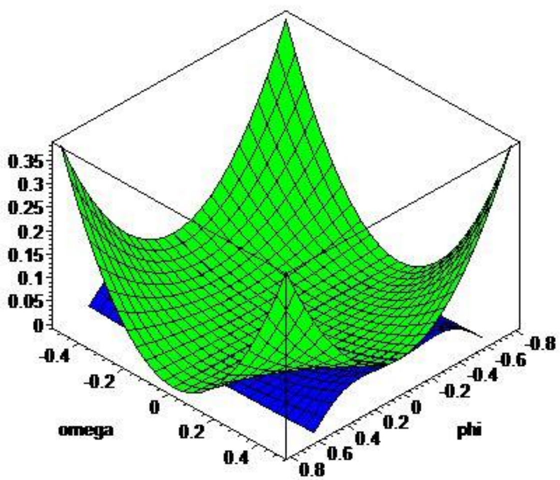


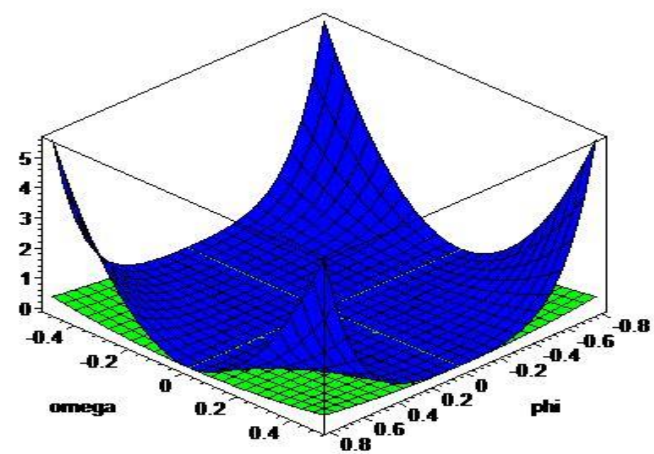
The potential in terms of the scalar field for $\omega = 1$, $\xi = 0$ (with green line in both panels) and for $\xi = -0.1$ (left panel) and $\xi = 0.1$ (right panel) with blue line

ω

Some numerical results

- The exponential expansion and the corresponding potential depends on the field and “omega factor”





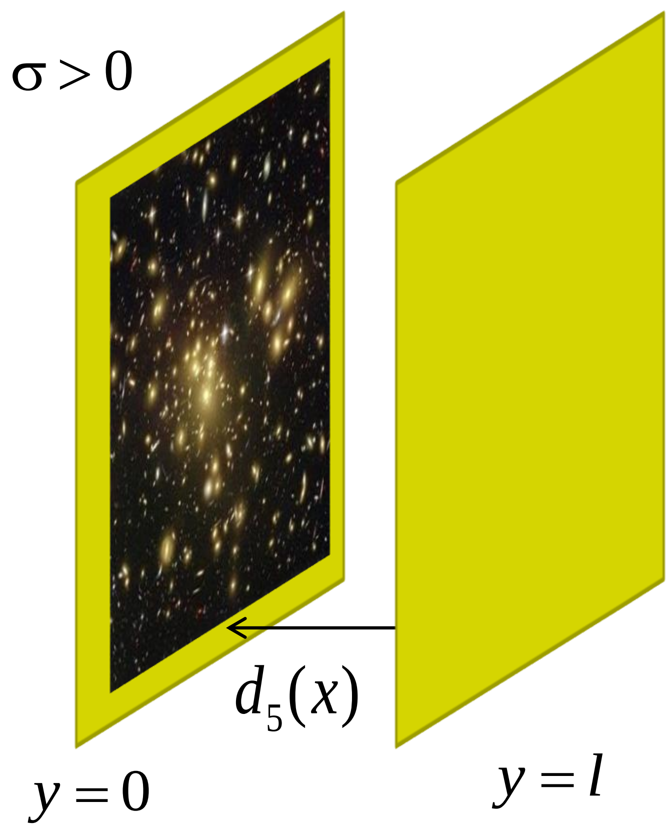
The potential in terms of the scalar field and ω , for $\xi = 0$ (the green surface in both panels) and for $\xi = -0.1$ (left panel) and $\xi = 0.1$ (right panel) the blue surfaces

9. Tachyon inflation in an AdS braneworld

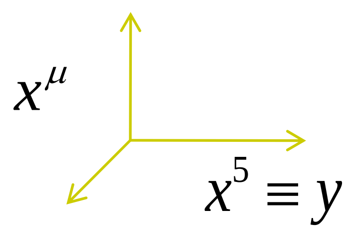
- Inflation is driven by the tachyon field originating in string theory
- A simple model of this kind is based on the second Randall-Sundrum (RSII) model
- The model was originally proposed as a possible mechanism for localizing gravity on the 3+1 universe embedded in a 4+1 dimensional space-time without compactification of the extra dimension.
- Originally proposed as a possible mechanism for localizing gravity on the 3+1 universe embeded in a 4+1 dimensional spacetime without compactication of the extra dimension.
- The RSII model is a 4+1 dimensional Anti de Sitter (AdS_5) universe containing two 3-branes with opposite tensions separated in the fifth dimension: observers reside on the positive tension brane and the negative tension brane is pushed off to infinity.
- The Planck mass scale is determined by the curvature of the AdS spacetime rather than by the size of the fifth dimension.
- The fluctuation of the interbrane distance along the extra dimension implies the existence of the radion.
- Radion - a massless scalar field that causes a distortion of the bulk geometry.

RS II Model

- We discuss a dynamics of 3-brane moving in the spacetime of the extended RSII model which includes the back-reaction of the radion field



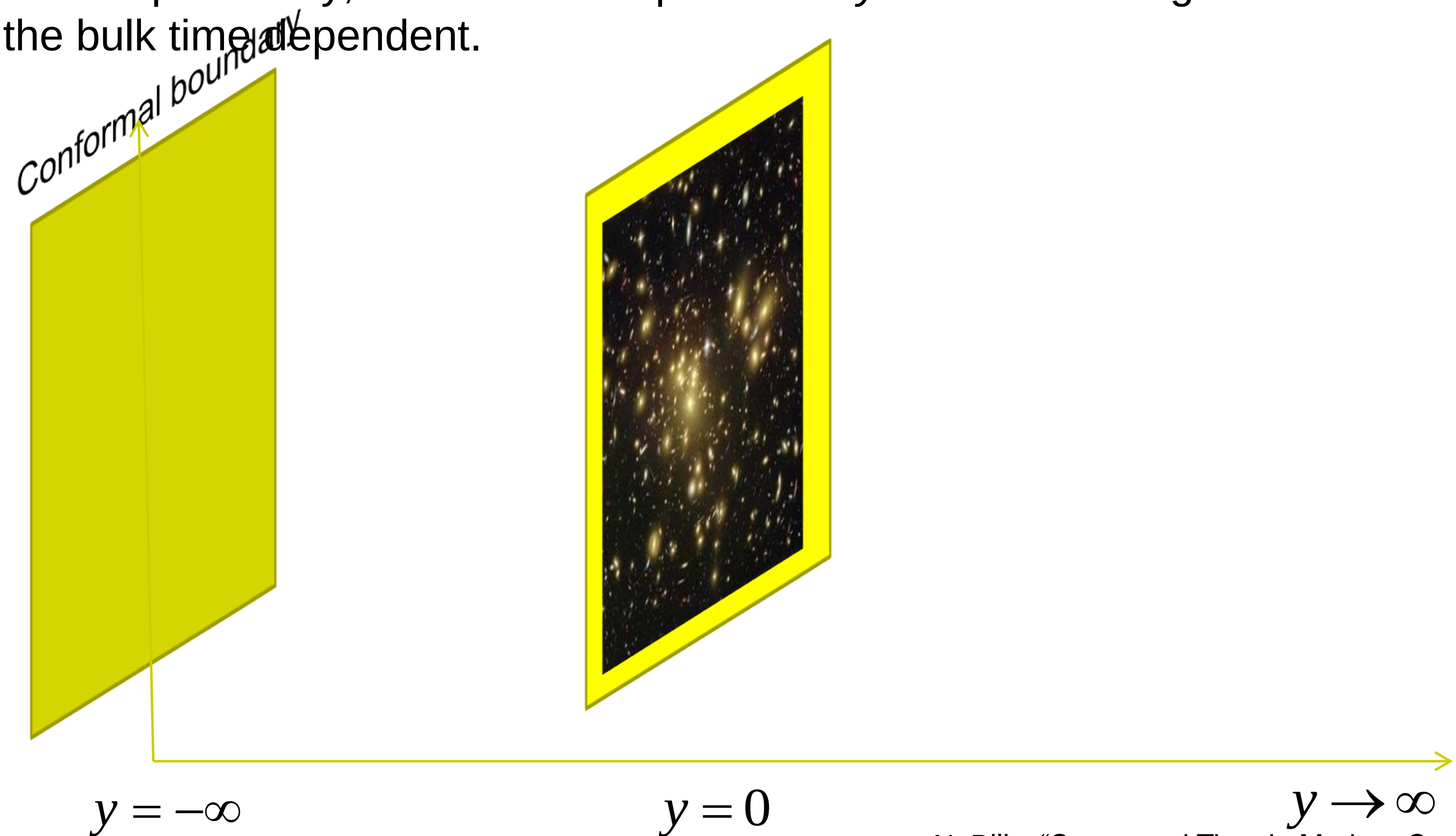
σ - the dynamical brane tension



$y \to \infty$

RS II Model

- Cosmology on the brane is obtained by allowing the brane to move in the bulk. Equivalently, the brane is kept fixed at *y=0* while making the metric in the bulk time dependent.



RS II Model

- The brane Lagrangian, after integrating out the fifth coordinate, is:

$$S=\int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G}+\frac{1}{2}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}\right)+S_{\text{br}}$$

$$S_{\text{br}}=-\int d^4x\sqrt{-g}\,\frac{\sigma}{k^4\Theta^4}(1+k^2\Theta^2\eta)^2\sqrt{1-\frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{(1+k^2\Theta^2\eta)^3}}$$

- Where Φ is the radion field, Θ is the tachyon field, k is the inverse of the AdS curvature radius $k=1/\ell$ and η is

$$\eta=\sinh^2\left(\sqrt{\frac{4\pi G}{3}}\Phi\right)$$

- In the absence of radion, the combined brane-radion Lagrangian is

$$L = \frac{1}{2} g^{\mu\nu} \Phi_{,\mu} \Phi_{,\nu} - \frac{\lambda \psi^2}{\Theta^4} \sqrt{1 - \frac{g^{\mu\nu} \Theta_{,\mu} \Theta_{,\nu}}{\psi^3}}$$

- Where

$$\lambda = \frac{\sigma}{k^4} \qquad \psi = 1 + k^2 \Theta^2 \eta$$

- The treatment of our the system in a cosmological context is conveniently performed in the Hamiltonian formalism.
- The Hamiltonian density is

$$H = \frac{1}{2} \Pi_\Phi^2 + \frac{\lambda \psi^2}{\Theta^4} \sqrt{1 + \Pi_\Theta^2 \Theta^8 / (\lambda^2 \psi)}$$

Hamilton's equations

$$\dot{\phi} = \pi_\phi$$

$$\dot{\theta} = \frac{\theta^4 \psi \pi_\theta}{\sqrt{1+\theta^8 \pi_\theta^2 / \psi}}$$

$$\dot{\pi}_\phi = -3h\pi_\phi - \frac{\psi}{2\theta^2} \frac{4+3\theta^8 \pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}} \eta'$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{\psi}{\theta^5} \frac{4-3\theta^{10}\eta\pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}$$

- Friedman equation

$$\frac{\dot{a}}{a} \equiv h = \sqrt{\frac{\kappa^2}{3}\bar{\rho}}$$

- Where

$$\psi = 1 + \theta^2 \eta$$

$$\eta = \sinh^2\left(\sqrt{\frac{\kappa^2}{6}}\phi\right)$$

$$\eta' = \frac{d\eta}{d\phi} = \sqrt{\frac{\kappa^2}{6}} \sinh\left(\sqrt{\frac{2\kappa^2}{3}}\phi\right)$$

$$\bar{\rho} = \left(\frac{1}{2}\pi_\phi^2 + \frac{\psi^2}{\theta^4}\sqrt{1+\frac{\theta^8\pi_\theta^2}{\psi}}\right)$$

- Besides, we rescaled the time as

$$t = \tau / k$$

- and express the system in terms of dimensionless quantities

$$h = H/k, \quad \phi = \Phi/(k\sqrt{\lambda}),$$

$$\pi_\phi = \Pi_\Phi/(k^2\sqrt{\lambda})),$$

$$\theta = k\Theta, \quad \pi_\theta = \Pi_\Theta/(k^4\lambda)$$

Conditions for inflation and slow-roll parameters

- Slow-roll parameters are defined as

$$\epsilon_0 \equiv \frac{H_*}{H} \qquad H_*\text{-Hubble rate at an arbitrarily chosen time}$$

$$\epsilon_i \equiv \frac{d \ln |\epsilon_{i-1}|}{H dt}, \qquad i \geq 1$$

- The first two parameters:

$$\epsilon_1 = -\frac{1}{H^2}\frac{dH}{dt}$$

$$\epsilon_2 = 2\epsilon_1 + \frac{1}{H}\left(\frac{dH}{dt}\right)^{-1}\frac{d^2H}{dt^2}$$

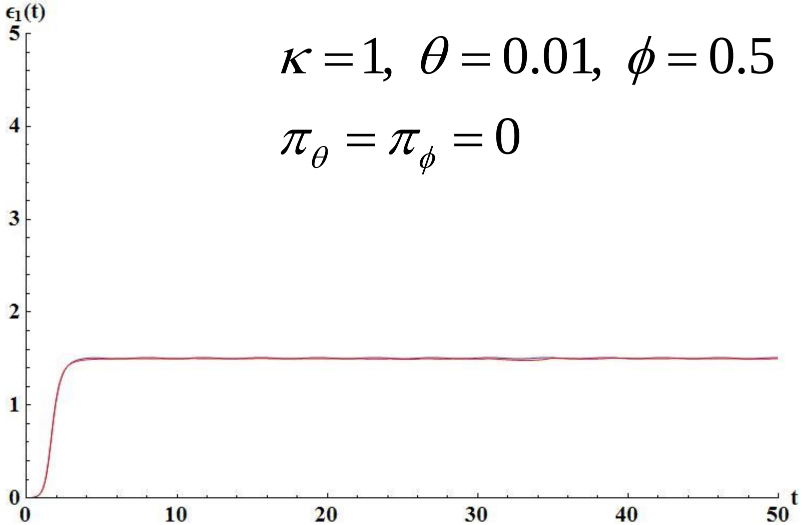
- In the absence of the radion

$$\epsilon_1 = \frac{3}{2}\left(\frac{d\Theta}{dt}\right)^2$$

$$\epsilon_2 = \frac{3}{H\epsilon_1}\frac{d^2\Theta}{dt^2}$$

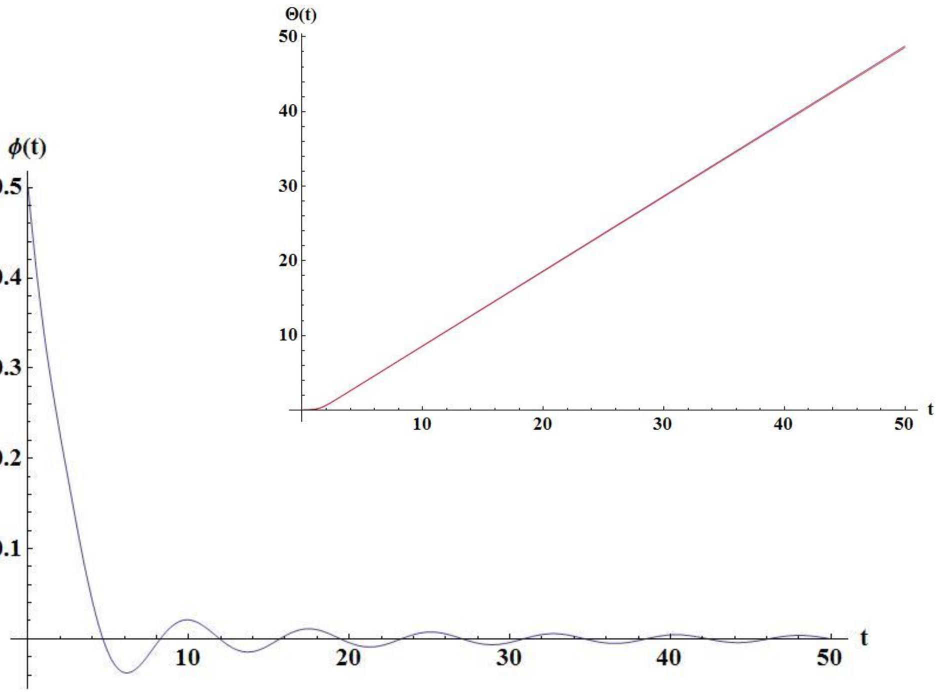
Results

- We analyze the effects of radion to tachyon field by comparing results for the RS II model to the results obtained for the inverse quartic potential (absence of radion)
- The system of equations is solved numerically and the slow roll parameters are calculated

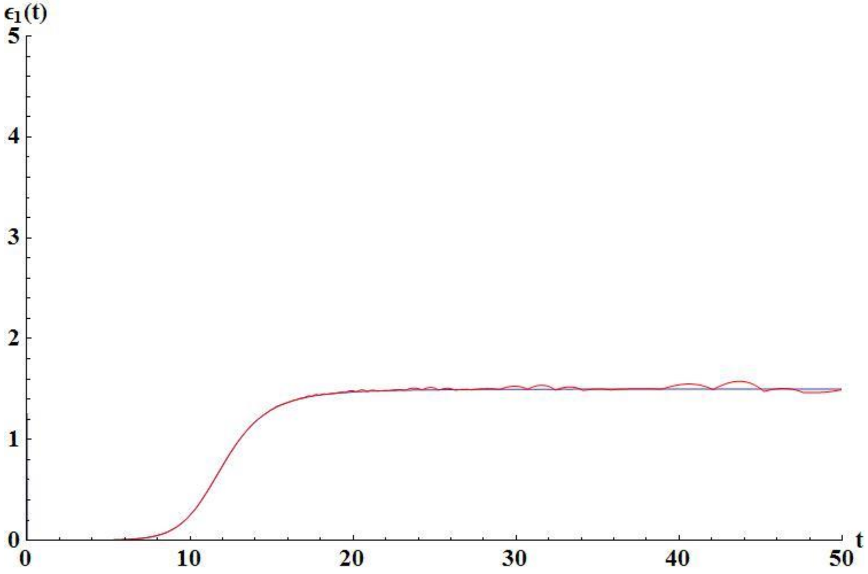


$$\kappa = 1,\ \theta = 0.01,\ \phi = 0.5$$

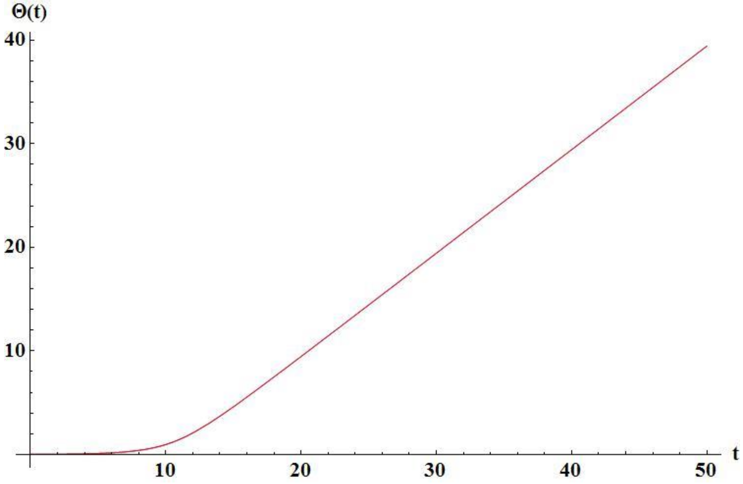
$$\pi_\theta = \pi_\phi = 0$$

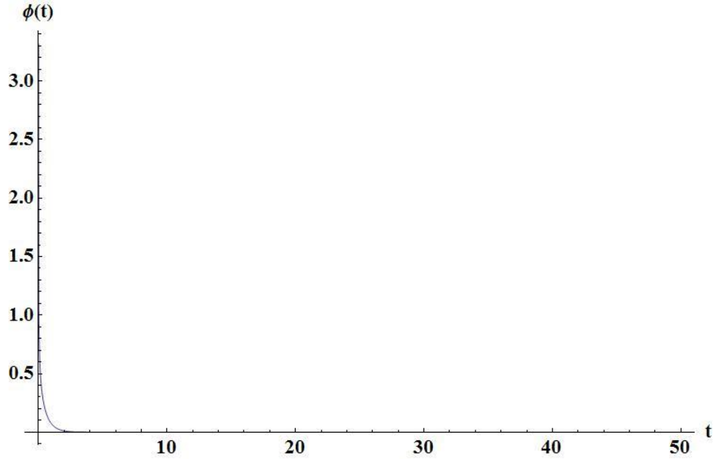


Results

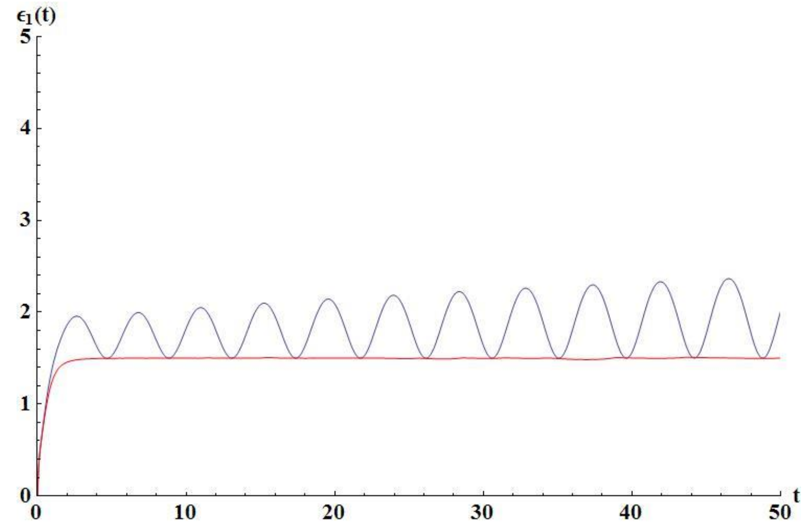


$$\kappa = 5,\ \theta = 0.01,\ \phi = 1.0$$
$$\pi_\theta = \pi_\phi = 0$$



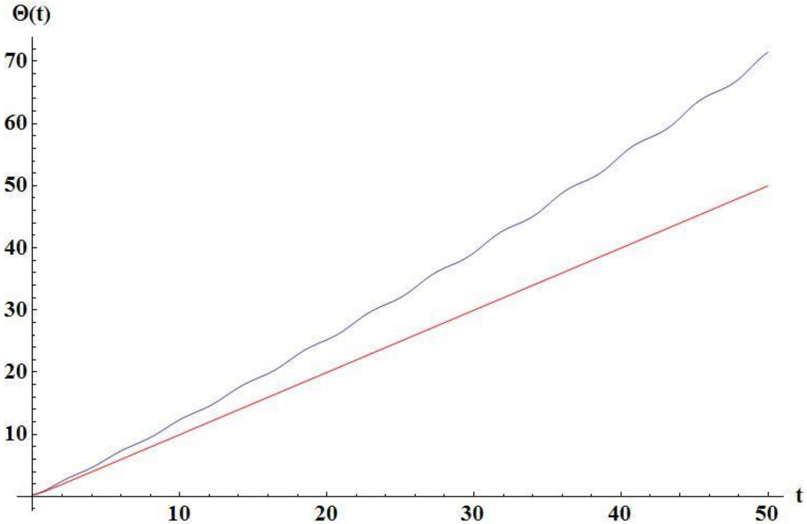


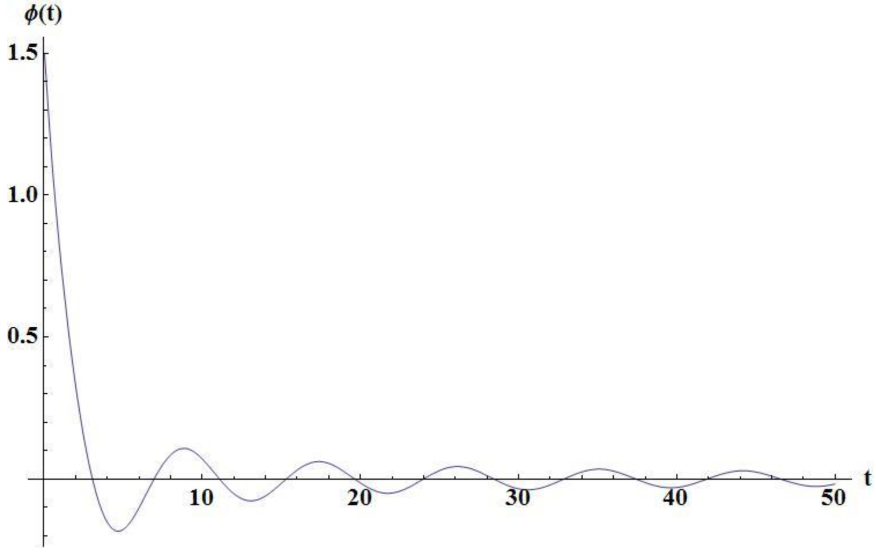
Results



$$\kappa = 1.2,\ \theta = 0.3,\ \phi = 1.5$$

$$\pi_\theta = \pi_\phi = 0$$





Conclusion and perspectives

- Sen’s proposal and similar conjectures have attracted important interests.
- Our understanding of tachyon matter, especially its quantum aspects is still quite pure.
- Perturbative solutions for classical particles analogous to the tachyons offer many possibilities in quantum mechanics, quantum and string field theory and cosmology on archimedean and nonarchimedean spaces.
- It was shown *[Barnaby, Biswas and Cline (2007)]* that the theory of *p*-adic inflation can compatible with CMB observations.
- Quantization of tachyons in homogenous case is possible and done in the “locally equivalence” limit
- Quantum tachyons could allow us to consider even more realistic inflationary models including quantum fluctuation.
- Reverse Engineering Method-REM remains a valuable auxiliary tool for investigation on tachyonic–universe evolution for nontrivial models.
- Randal-Sundrum model allows numerical consideration for tachyon like systems with promising preliminary results.

Some references

- G. S. Djordjevic, B. Dragovich, Mod. Phys. Lett. A 12(20), 1455 (1997).
- G. S. Djordjevic, B. Dragovich, L. Nesic, Inf. Dim. Analys, Quant. Prob. and Rel. Topics 06(02), 179-195 (2003).
- I.Ya.Aref'eva, B.Dragovich, P.Frampton and I.V.Volovich, Int. J. Mod.
- Phys. A6, 4341 (1991).
- G. S. Djordjevic, B. Dragovich, L. Nesic, I. V. Volovich, Int. Jour. Mod. Phys. A 17(10), 1413-1433 (2002).
- A. Sen, Journal of High Energy Physics 2002(04), 048{048 (2002).
- D. D. Dimitrijevic, G. S. Djordjevic, L. Nesic, Fortschritte der Physik 56(4-5), 412-417 (2008).
- G.S. Djordjevic and Lj. Nesic, Tachyon-like Mechanism in Quantum Cosmology and Inflation, in Modern trends in Strings, Cosmology and Particles, Monographs Series: Publications of the Astronomical Observatory of Belgrade, No 88, 75-93 (2010).
- N. Bilic G. B. Tupper and R. D. Viollier, Phys. Rev. D 80, 023515 (2009).
- D.N. Vulcanov and G. S. Djordjevic, Rom. Journ. Phys., Vol. 57, No. 5–6, 1011-1016 (2012).
- D. D. Dimitrijevic, G. S. Djordjevic and M. Milosevic, Romanian Reports in Physics, vol. 57 no. 4 (2015).
- G. S. Djordjevic, D. D. Dimitrijevic and M. Milosevic, Romanian Journal of Physics, vol. 61, no. 1-2 (2016).

Acknowledgement

The Work is supported by the Serbian Ministry of Education and Science Projects No. 176021 and 174020.

The financial support under the Abdus Salam International Centre for Theoretical Physics (ICTP) – Southeastern European Network in Mathematical and Theoretical Physics (SEENET-MTP) Project PRJ-09 "Cosmology and Strings" is kindly acknowledged.

Multumesc/хвала!