de Sitter in String Theory

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OUTLINE

Why are we interested in de Sitter space?

de Sitter from String Theory compactifications

No-GoTheorems

- Direct Product Space
- Type IIB Supergravity with Branes and Planes

Lift to M-theory

WHY DO WE CARE ABOUT DE SITTER SPACE?

ΛCDM(cosmologicalconstant+cold dark matter)is a success.

CMB cosmology: WMAP, Planck, etc,

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Inflation!: the paradigm of early universe cosmology ($f_{NL}^{loc} \sim 0$, n_s <1,...).BICEP2

What about Anti de Sitter spaces?

AdS Spaces are easy to obtain from String Theory

One needs to start with D3 and D5 branes in NS 3-form flux

Replace D5 branes with RR 3-form flux

Superpotential W written as a product of RR and NS fluxes

String Theory compactified on Calabi-Yau manifolds with NS and RR fluxes give rise to AdS spaces

De Sitter obtained by KKLT 0301240

$D3$ + non-perturbative effects$\Rightarrow$ non-susy AdS vacuum

$\bar{D}3 \Rightarrow$ positive energy contibution to 'uplift' to de Sitter solution

Bena, Grana, Kuperstein, Massai:
try to solve to full supergravity EOM in Klebanov-Strassler
0912.3519, 1102.2403, 1106.6165, 1205.1798, 1206.6369, 1212.4828

Find no solutions free from unphysical singularities (with no resolution by brane polarization a la Polchinski-Strassler)

DeSitterinStringTheory

Many proposals! Let's look at two of the more well studied

(1) $D3, \bar{D}3$ (KKLT)
Objection: Grana, Bena, et al. 1205.1798

(2) α' corrections (0611332)
Objection: Sethi, Quigley, Green, Martinec in Heterotic (1110.0545)

α' CORRECTIONS

In Type IIB (Becker, Becker, Haack, Louis 0204254): correction to the Kähler potential [1]: ${\alpha'}^3 R^4 \Rightarrow \delta\mathcal{K} \propto \chi$:

$$V = e^{\mathcal{K}} \left(|DW|^2 - 3|W|^2 \right)$$

de Sitter constructed: Westphal 0611332, many papers since

Green, Martinec, Quigley, Sethi: 1110.0545.

Leading correction in Heterotic R^2 does not satisfy Strong Energy Condition ($R_{00} > 0$)

$\Rightarrow$ no de Sitter!

NO-GO THEOREMS

Make some demands:

1. Poincare invariance in the 3+1
2. finite Newton's constant
3. Large Internal Space (no string-scale cycles, Einstein equations apply)
4. $R_4 > 0$, where $R_4 \equiv$ is the Ricci scalar of the 4d space after Kaluza Klein reduction from
   $10\text{d} \rightarrow 4d$

SIMPLEST CASE: DIRECT PRODUCT SPACE (NO WARPING)

Einstein equation:

$$R_{MN} = \frac{\mathcal{K}_D}{2}\left(T_{MN} - \frac{1}{D-2}g_{MN}T\right),$$

Assume spacetime is $\mathcal{M}_4 \times \mathcal{M}_6$. The Einstein equation becomes

$$R_4 = \frac{\mathcal{K}_{10}}{4}\left[T^{\mu}_{\mu} - T^{m}_{m}\right].$$

Simplest Case: direct product space (no warping)

Try fluxes (Gibbons: 0301117, Maldacena-Nunez: 0007018)

$$\mathcal{L}_F = -\sqrt{-G_D} F_{a_1 \ldots a_q} F^{a_1 \ldots a_q},$$

which gives a stress tensor

$$T^F_{MN} = -g_{MN} F^2 + 2q F_{M a_2 .. a_q} F_N^{\ a_2 \ldots a_q}.$$

Can this ever lead to $R_4 > 0$?

Try fluxes (Gibbons, Maldacena-Nunez)

$$\mathcal{L}^F_{\rm int} = -\sqrt{-G_D} F_{a_1 \ldots a_q} F^{a_1 \ldots a_q},$$

which gives a stress tensor

$$T^F_{MN} = -g_{MN} F^2 + 2q F_{M a_2 .. a_q} F_N^{\ a_2 \ldots a_q}.$$

Can this ever lead to $R_4 > 0$? This requires

$$\frac{(1-q)}{2q} F^2 > -F_{\mu a_2 .. a_q} F^{\mu a_2 .. a_q}.$$

Try fluxes (Gibbons, Maldacena-Nunez):

$$\frac{(1-q)}{2q}F^2 > -F_{\mu a_2..a_q}F^{\mu a_2..a_q}.$$

Case (1): all legs of flux are along $\mathcal{M}_6$

$F_{\mu...}F^{\mu\cdots} = 0 \;\;,\;\; F^2 > 0 \Rightarrow$ **Not satisfied!**

Case (2): flux fills $\mathcal{M}_4$ and has additional legs along $\mathcal{M}_6$

Easy to show: condition only satisfied for $q > 9$. **But there are no 10-form fluxes in string theory!**

Branes and Anti-Branes

The action for a Dp-brane in Einstein frame [3]:

$$S_{Dp} = -\int d^{p+1}\sigma \; T_p \; e^{\frac{\phi(p+1)}{4}} \; \sqrt{-\det(g_{ab} + \widetilde{F}_{ab})} + \mu_p \int \left(C \wedge e^{\widetilde{F}} \right)_{p+1} .$$

Here $\widetilde{F} = F_{ab} + B_{ab}$, F_{ab} is the gauge field on the brane, and g_{ab}, B_{ab} are the pullbacks of the metric and Kalb-Ramond two-form.

Note $T_p > 0$ for both brane and antibrane: **It is the sign of μ_p determines whether we have a brane or an anti-brane.**

Branes and Anti-Branes

It is the sign of μ_p determines whether we have a brane or an anti-brane.

And Chern-Simons terms do not contribute to the stress-energy tensor!

$$T_{mn}^{(CS)} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{CS}}{\delta g^{mn}} = 0$$

So Einstein equations can't tell the difference between a brane and antibrane.

Bianchi Identity cares about charge: compensates with fluxes, covered by Gibbons, Maldacena Nunez.

Branes and Anti-Branes

Work out the stress tensor (where $T_p > 0$):

$$T^{\mu}_{\mu\,(Dp,\bar{D}p)} = -4T_P N$$

$$T^{m}_{m\,(Dp,^{-}} = -(p-3)T_P N$$

where

$$N \equiv \exp\left[\frac{\phi(p+1)}{4}\right] \frac{\sqrt{-\det(g_{ab}+\widetilde{F}_{ab})}}{\sqrt{\det(-g_{10})}} \delta^{9-p}(x-\bar{x})$$

Example: $\bar{D}3$ gives $(T^{\mu}_{\mu} - T^{m}_{m}) = -4T_p N < 0 \Rightarrow R_4 < 0$

Easy to check: Need p=9, so D9 branes in string theory wrapped on CY 3-folds

No other way to get $R_{\ 4} > 0$

Branes and Anti-Branes

What about a combination of branes, antibranes, and fluxes?
Consider IIB supergravity with fluxes and branes [4],

$$R_4(x^\mu) = -\frac{G_3 \cdot \bar{G}_3}{12\ \mathrm{Im}\tau} + \frac{\widetilde{F}_{\mu abcd}\widetilde{F}_\mu^{\ abcd}}{4 \cdot 4!} + \frac{\kappa_{10}^2}{2}\left(T_\mu^{\mu\ \mathrm{loc}} - T_m^{m\ \mathrm{loc}}\right)$$

T_{MN} for the brane is localized: $T_{MN} \sim \delta(x - \bar{x})$. How to handle this?

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T_{MN} for the brane is localized: $T_{MN} \sim \delta(x - \bar{x})$. How to handle this?

Option (1): 'smear' the branes ($\delta(x) \to \Gamma(x)$), integrate over internal space. $\to$ no dS

Option (2): treat the branes as localized. R_4 is independent of x^m, so can be calculated at any x^m $\to$ branes only contribute through bulk fluxes. $\to$ no dS!

Warped Product Space

Generalize the metric:

$$ds^2 = e^{2A}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + e^{-2A}\tilde{g}_{mn}dx^m dx^n,$$

Calculate the Ricci Tensor:

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - \tilde{g}_{\mu\nu}e^{4A}\widetilde{\nabla}^2 A$$

where $\widetilde{\nabla}^2$ is the Laplacian on $\mathcal{M}_6$.

Include Negative Tension Objects: Op-planes

Op-planes:

$$S_{Op} = -\int d^{p+1}\sigma \; T_{Op} e^{\frac{\phi(p+1)}{4}} \sqrt{-\det f_{ab}} + \mu_{Op} \int C_{p+1},$$

1. Negative tension! $T_{Op} < 0$
2. Non-dynamical
3. Carry no gauge fields

CAN YOU GET dS?

Repeat the procedure, integrate over internal space:

$$\tilde{R}_4 = \frac{1}{\tilde{V}_6}\int d^6x\,\sqrt{\tilde{g}_6}\mathcal{I} + \frac{1}{\tilde{V}_6}\int d^6x\,\sqrt{\tilde{g}_6}\frac{\kappa_{10}^2}{2}e^{2A}\sum\left[T^\mu_\mu - T^m_m\right]_{Dp,Op}$$

where we have defined $\mathcal{I}_{\text{global}}$ and $\tilde{V}_6$ as

$$\mathcal{I} \equiv -\frac{e^{2A}G_3\cdot\bar{G}_3}{12\,\text{Im}\tau} + \frac{e^{2A}\hat{F}_{\mu abcd}\hat{F}^{\mu abcd}}{4\cdot 4!} - e^{-6A}\partial_m e^{4A}\partial^m e^{4A} \leq 0,$$

$$\tilde{V}_6 \equiv \int d^6x\sqrt{\tilde{g}_6} > 0.$$

Warping, fluxes, branes and anti-branes don't help! What about the Op-planes?

Can you get dS?

$$\frac{1}{\tilde{V}_6}\int d^6x\,\sqrt{\tilde{g}_6}\,\frac{\kappa_{10}^2}{2}e^{2A}\sum\left[T^{\mu}_{\mu}-T^{m}_{m}\right]_{Dp,Op}$$

Op-planes cannot be smeared: they are inherently localized!
How to deal with these?

Naive approach: ignore orientifold points, and incorporate backreaction via bulk fluxes and branes, $\rightarrow$ no dS . But this doesn't feel very honest....

Go to M-theory!

Some Insight From M-theory

Orientifold planes become geometry in M-theory:

O6 in IIA $\rightarrow$ (smooth) Atiyah-Hitchin manifold in M-theory

O8 in IIA $\rightarrow$ Hořava-Witten Wall in M-theory

Go to M-theory

Consider11dim.supergravity action with M2 branes and curvature corrections

$$S = S_{bulk} + S_{brane} + S_{corr},$$

$$S_{bulk} = \frac{1}{2\kappa^2} \int \mathrm{d}^{11}x \sqrt{-g} \left[R - \frac{1}{48} G^2 \right] - \frac{1}{12\kappa^2} \int C \wedge G \wedge G,$$

$$S_{brane} = - \frac{T_2}{2} \int \mathrm{d}^3\sigma \sqrt{-\gamma} \, [\, \gamma^{\mu\nu} \partial_\mu X^M \partial_\nu X^N g_{MN} - 1 + \frac{1}{3!} \epsilon^{\mu\nu\rho} \partial_\mu X^M \partial_\nu X^N \partial_\rho X^P C_{MNP} \,],$$

Go to M-theory

Consider the M-theory lift of a IIB
de Sitter solution. The IIB metric (in conformal time):

$$ds^2_{dS_4} \sim \frac{1}{t^2}\eta_{\mu\nu}dx^\mu dx^\nu$$

$$\begin{aligned} ds^2_{IIB} &= e^{-2A}g_{\mu\nu} + e^{2A}g_{mn}dy^m dy^n \\ &= \frac{1}{\Lambda(t)\sqrt{h}}(-dt^2 + \eta^{ij}dz_i dz_j + dx_3^2) + \sqrt{h}\tilde{g}_{mn}dy^m dy^n \end{aligned}$$

where $\Lambda(t) = \tilde{\Lambda}t^2$ is de Sitter. The corresponding M-theory metric:

$$\begin{aligned} ds^2 &= \frac{1}{(\Lambda(t)\sqrt{h})^{4/3}}(-dt^2 + \eta^{ij}dz_i dz_j) + h^{1/3}\left[\frac{\tilde{g}_{mn}dy^m dy^n}{(\Lambda(t))^{1/3}} + (\Lambda(t))^{2/3}|dz|^2\right] \\ &\equiv e^{2A(y,t)}(-dt^2 + \eta^{ij}dz_i dz_j) + e^{2B(y,t)}\tilde{g}_{mn}dy^m dy^n + e^{2C(y,t)}|dz|^2 \end{aligned}$$

WHAT ABOUT α' CORRECTIONS?

M-theory makes computations a lot simpler, so let's try and make things a bit more sophisticated.

1. In type IIB: come from sigma model, d-instantons, graviton scattering
2. no-go theorem for lowest-order corrections in Heterotic, explicit constructions in IIB

At lowest order: a Chern-Simons term ($R \equiv R_{MNPQ}$)

$$C \wedge X_8 \quad , \quad X_8 \sim \mathrm{Tr}R^2 - \mathrm{Tr}R^4$$

and an R^4 term:

$$\left(\frac{1}{8}\epsilon_{10}\epsilon_{10} - t_8 t_8\right) R^4,$$

General: possible infinite set of R^n, and $R^n G^m$ terms.

WHAT ABOUT α' CORRECTIONS?

How to study the curvature corrections? Could have a very complicated form on a CY manifold...

$$T^{MN}_{corr} = \frac{-2}{\sqrt{-g}} \frac{\delta \hat{S}_{corr}}{\delta g_{MN}}\Big|_{g,C} \equiv \sum_i [\Lambda(t)]^{\alpha_i + 1/3} \mathcal{C}^{MN,\, i}$$

Consider a general stress-energy tensor built out of curvatures:

A set of terms with time-dependence from $g_{\mu\nu}$ parametrized $\Lambda(t)$ and the g_{mn} dependence in coefficients C^{MN}.

dS SOLUTIONS?

Work out equations of motion to find consistency condition [5]:
(without curvature corrections)

$$\frac{1}{12}\int d^8x\sqrt{\tilde{g}}\,\widetilde{G}_{mnpa}\widetilde{G}^{mnpa} + 12\Lambda\int d^8x\sqrt{\tilde{g}}\,h^2 + 2\kappa^2 T_2(n_3+\bar{n}_3) = 0$$

All terms are positive definite $\Rightarrow$ No way to get de Sitter!

What does this mean?

Type IIB supergravity with fluxes, Dp-branes, anti Dp-branes, Op-planes, and by extension any linear combination thereof, does not lead to de Sitter space in the 3+1 non-compact directions.

[5] n_3, $\bar{n}_3$ are the number of $M2$ and anti-$M2$ branes, which correspond to space-filling $D3$ and anti D3 in IIB

Can α' corrections save us?

Include curvature corrections:

$$\frac{1}{12}\int d^8x\sqrt{\tilde{g}}\,\widetilde{G}_{mnpa}\widetilde{G}^{mnpa} + 12\Lambda\int d^8x\sqrt{\tilde{g}}\,h^2 + 2\kappa^2 T_2(n_3+\bar{n}_3)$$
$$+\quad \int d^8x\sqrt{\tilde{g}}h^{4/3}\left(\frac{1}{2}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}^{a,\,i}_{a} + \frac{1}{4}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}^{m,\,i}_{m} - \frac{2}{3}\sum_{\{\alpha_i\}=0}\tilde{\mathcal{C}}^{\mu,\,i}_{\mu}\right) = 0$$

de Sitter **is possible** only if the quantum corrections sum to a negative definite quantity:

$$\int d^8x\sqrt{\tilde{g}}h^{4/3}\,(\text{Quantum Corrections}) < 0$$

Conclusions:

(1) No dS in IIB with branes and or planes.

(2)Quantum corrections can lead to dS

What to do next?

1.Consider dS in Heterotic and see if things work.

2.Non-Kahler compactification Cosmology compactifications