SUSY breaking, (s)goldstino and the constrained superfields formalism.

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- Outline:

- Non-linear Susy multiplets and constrained superfields.
- Results for general Kahler potential.
- The conjecture(s) of Seiberg and Komargodski.
- Some phenomenology.
- Conclusions.

- Some history.....

- Large literature on SUSY non-linear realisations:

- Akulov-Volkov, Ivanov-Kapustinov, Siegel, Samuel-Wess, Clark and Love, ...
- Casalbuoni, Dominici, de Curtis, Feruglio, Gatto;
- Luty, Ponton; Brignole, Feruglio, Zwriner.
- Brignole, Casas, Espinosa, Navarro.
- Komargodski and Seiberg.
- Antoniadis and Tuckmantel.

This talk based on work of: DG, Antoniadis, Dudas, von Gersdorf:

e-Print: arXiv:1210.8336 [hep-th], arXiv:1110.5939 [hep-th], arXiv:1106.5792 [hep-th]

- 1) Most phenomenological studies based on the component-fields formalism, tedious computations.
- 2) Alternative: use superfields endowed with (IR) constraints.

- what are these constraints? UV (in)sensitive? independent of dynamics? The equivalence of these two methods was never checked in general. Purpose of this talk: address some of these questions.

- Non-linear Supersymmetry

In supergravity: gravitino Ψ_μ becomes massive by "absorbing" the spin 1/2 fermion (goldstino)

$$\Psi_\mu(\pm 3/2) + \Psi_\mu(\pm 1/2) \rightarrow \Psi_\mu(\pm 3/2, \pm 1/2).$$

Gravitino mass (nearly massless if low vev for F)

$$m_{3/2} \sim \frac{F}{M_{Planck}}$$

Sgoldstino mass: depends on the details of UV completion (microscopic model, see later).

- In Susy, for energy $E \ll \sqrt{f} \equiv \sqrt{F}$

$\Rightarrow$ massive sgoldstino decouples; the (very light) goldstino $m_\psi = f/M_{\rm Planck}$ remains in spectrum:
- spin 1/2 goldstino couplings to matter: $\sim 1/f$,
- spin 3/2-gravitino components have couplings $\sim 1/M^2_{Planck}$, thus strongly suppressed.

If $\sqrt{f}$ is low enough ($< 10^{10}$ GeV), one can work in the decoupling limit for most purposes.
- goldstino multiplet non-linearly realised.

- There are two cases of goldstino couplings to matter:

i) Non-linear SUSY in the matter sector (ex: SM...), non-linear goldstino multiplet:

$$E \ll m_{spartners}, m_s, \sqrt{f}, \qquad m_s : \text{sgoldstino mass}$$

This regime: from component fields Lagrangian after integrating out massive d.o.f.

ii) linear SUSY in the matter sector, non-linear goldstino multiplet:

$$m_{spartners} \leq E \ll m_s, \sqrt{f}$$

⇒ linear SUSY matter sector coupled to the (non-linear) goldstino superfield:

⇒ new MSSM couplings, correction to the higgs potential: "non-linear" MSSM (see later).

[I. Antoniadis, E. Dudas, D.G., P. Tziveloglou]

• Formalisms for non-linear SUSY. $\Rightarrow$ (a) Akulov-Volkov formalism:

A SUSY transformation (supertranslation)

$$x^{\mu\,\prime} = x^\mu + i\,(\theta\sigma^\mu\overline{\xi} - \xi\sigma^\mu\overline{\theta}); \qquad \theta' = \theta + \xi; \qquad \overline{\theta}' = \overline{\theta} + \overline{\xi}$$

$\theta \to \psi/(f\sqrt{2})$ (the Weyl goldstino spinor ψ). The analogy gives

$$\psi'(x') = \psi(x) + f\sqrt{2}\,\xi, \qquad \to \qquad \delta\psi = f\,\sqrt{2}\,\xi + \frac{1}{\sqrt{2}f}\,i\,(\psi\sigma^\mu\overline{\xi} - \xi\sigma^\mu\overline{\psi})\,\partial_\mu\psi$$

(no superpartner). Introduce:

$$E_\mu^\nu \;=\; \delta_\mu^\nu + i/(2\,f^2)\;(\partial_\mu\,\psi\sigma^\nu\,\overline{\psi} - \psi\,\sigma^\nu\,\partial_\mu\overline{\psi}\,) \qquad \text{(vierbein)}$$

$$\delta(\det E) \;=\; i/(\sqrt{2}f)\;\partial_\mu\,[\,(\psi\sigma^\mu\overline{\xi} - \xi\overline{\sigma}^\mu\psi)\,\det E]$$

$\Rightarrow$ susy-invariant A-V non-linear action for a goldstino:

$$L_{AV} = -f^2\,\det E = f^2\left[1 - \frac{i}{2\,f^2}\,(\psi\,\sigma^\mu\partial_\mu\overline{\psi} - \partial_\mu\psi\,\sigma^\mu\overline{\psi}) + \cdots\right]$$

• can be extended to interaction with matter fields (geometric method).

$\delta\phi = (1/f)\,i(\psi\sigma^\mu\overline{\xi} - \xi\sigma^\mu\overline{\psi})\partial_\mu\phi$, similar for $\delta(\mathcal{D}_\mu\phi)$ if $\mathcal{D}_\mu\phi = (E^{-1})_\mu^\nu\,\mathcal{D}_\nu\phi$; similarly: $\delta(\mathcal{F}_{\mu\nu})$, then so does $\delta(L)$. $\Rightarrow$ $\mathcal{L}_{eff} = \det E\;\mathcal{L}$ inv. up to total ∂_μ....

• Formalisms for non-linear SUSY. $\Rightarrow$ (b) Lagrangians with constrained superfields.

Goldstino ψ described by a chiral superfield Φ, with a constraint

$$\Phi^2 = 0 \qquad\qquad (*)$$

(*) removes the sgoldstino partner (not F...), and is solved by [Siegel, Casalbuoni et al.]

$$\Phi = \frac{\psi\psi}{2\,F} + \sqrt{2}\,\theta\,\psi + \theta\theta\,F \qquad (**)$$

The Lagrangian

$$L = \int d^4\theta\,\Phi^\dagger\,\Phi + \left\{ \int d^2\theta\, f\,\Phi + h.c. \right\}\Big|_{\Phi^2=0} = f^2\left[1 - \frac{i}{2\,f^2}\,(\psi\,\sigma^\mu\partial_\mu\overline{\psi} - \partial_\mu\psi\,\sigma^\mu\overline{\psi}) + \cdots\right]$$

$$= f^2\,\det E = L_{AV},$$

recovers previous Akulov-Volkov L_{AV} of goldstino, after replacing (**) in lhs of last eq.

$\Rightarrow$ Superfield description of V-A lagrangian. One can non-linearise over more fields.

• non-linear realisation via constrained superfields: (linear) superfields also add aux fields;

$\Rightarrow$ need to eliminate them: via eqs motion; susy algebra only onshell.

$\Rightarrow$ A-V L_{AV} and constrained superfield L are not equiv if coupling to other fields (couplings via F).

- Ferrara-Zumino multiplet, goldstino superfield and the field X:

$$\overline{D}^{\dot\alpha} J_{\alpha\dot\alpha} = D_\alpha X, \qquad \text{where} \qquad J_{\alpha\dot\alpha} = -2\sigma^\mu_{\alpha\dot\alpha} J_\mu, \quad X = (\phi_X, \psi_X, F_X)$$

$$J_\mu = j_\mu + \Big\{\theta^\alpha \Big[S_{\mu\alpha} + \frac{1}{3}(\sigma^\mu \overline{\sigma}^\rho S_\rho)_\alpha\Big] + h.c.\Big\} + (\theta\sigma^\mu \overline{\theta})\Big[2\,T_{\nu\mu} - \frac{2}{3}\eta_{\mu\nu}\,T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho} j^{\sigma]}\Big] + \frac{i}{2}\theta\theta\,\partial_\mu \phi_x^2 + h.$$

$$X = \phi_X(y) + \sqrt{2}\theta\,\psi_X(y) + \theta\theta\,F_X(y), \qquad \psi_{X\alpha} = \frac{\sqrt{2}}{3}\sigma^\mu_{\alpha\dot\alpha}\,\overline{S}^{\dot\alpha}_\mu, \quad F_X = \frac{2}{3}T + i\,\partial_\mu j^\mu$$

- In a superconformal theory: $X = 0$ ($T = 0, \quad \partial_\mu j^\mu = 0, \quad \overline{\sigma}^{\mu\,\alpha\dot\alpha} S_{\mu\,\alpha} = 0$)
- In a general theory of Kahler K and superpotential W [Clark, Love]

$$X = 4\,W(\Phi^j) - \frac{1}{3}\overline{D}^2 K(\Phi^j, \Phi_j^\dagger) - \frac{1}{2}\overline{D}^2 Y^\dagger(\Phi_j^\dagger) \qquad [*]$$

- It was conjectured (Seiberg, Komargodski) that X flows in IR to Goldstino superfield (Φ).
- Let us see if this is true, for when matter fields are 1) absent (VA) or 2) present.

• Case 1: one field present: $\Phi:(\phi,\psi,F)$, ϕ: sgoldstino, ψ: goldstino:

$$L=\int d^4\theta\, K(\Phi,\Phi^\dagger)+\Big\{\int d^2\theta\, f\,\Phi+h.c.\Big\},\qquad K=\Phi^\dagger\Phi+\mathcal{O}(1/\Lambda)$$

Then

$$V(\phi)=f^2/K_\phi^\phi$$

Expand about ground state $\phi=0$: $K(\phi,\phi^\dagger)=\phi\,k_\phi+\phi^\dagger\,k^\phi+\mathcal{O}(1/\Lambda)$ so $m_{1,2}^2=-f^2\,(k_{\phi\phi}^{\phi\phi}\pm|\,k_{\phi\phi\phi}^{\phi}|\,)$

Integrate out ϕ via eqs motion:

$$\phi=\frac{\psi\psi}{2\,F}+\mathcal{O}(1/\Lambda)\quad\Rightarrow\quad\Phi=\frac{\psi\psi}{2F}+\sqrt{2}\theta\psi+F\,\theta\theta,\quad\Rightarrow\quad\Phi^2=0\ \ \text{(offshell-susy, IR)}$$

Then

$$[*]\quad\Rightarrow\quad X=(4\,f+4/3F^\dagger)\Big[\frac{\psi\psi}{2\,F}+\sqrt{2}\,\theta\psi+F\,\theta\theta\Big],\qquad\Rightarrow\qquad X^2=0\ \text{(in IR, offshell)}$$

$\Rightarrow$ X indeed flows (onshell) to goldstino superfield with $\Phi^2=0$ in IR..... $X=(8/3)\,f\,\Phi$.

$\Rightarrow$ also V-A action recovered in $\mathcal{O}(1/\Lambda)$ after going onshell susy $F\to-f$.

• A simple example:

$$K = \Phi^\dagger \Phi - \frac{c}{\Lambda^2}\,\Phi^2\,\Phi^{\dagger 2} - \frac{\tilde{c}}{\Lambda^2}\,(\Phi^3\,\Phi^\dagger + \Phi\,\Phi^{\dagger 3}\,) + \mathcal{O}(1/\Lambda^3), \quad W = f\,\Phi$$

giving

$$V = f^2\left[1 + 4\,c/\Lambda^2\ \phi^\dagger\phi + 3/\Lambda^2\ (\tilde{c}\,\phi^2 + h.c.) + \mathcal{O}(1/\Lambda^3)\right]$$

One finds $m_{1,2}^2 = f^2\,(4\,c \pm 6\tilde{c})/\Lambda^2$, so goldstino massive via D-terms. $m_{1,2}^2 > 0$ with suitable $c, \tilde{c}$.

Using eqs of motion to integrate out massive ϕ, one then finds

$$\phi = \frac{\psi\psi}{2F} + \mathcal{O}(1/\Lambda) \qquad \Rightarrow \qquad \Phi = \frac{\psi\psi}{2F} + \sqrt{2}\theta\psi + F\,\theta\theta \qquad \Phi^2 = 0,\ \ V - A.....$$

• Q: how to generate mass terms in a ren model? O'Raifeartaigh with small susy breaking $f\,h \ll |m_s|^2$:

$$K \;=\; \Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3, \qquad W = \frac{1}{2}\,h\,\Phi_1\,\Phi_2^2 + m_s\,\Phi_2\Phi_3 + f\,\Phi_1$$

$$\Rightarrow \quad K_{eff} \;=\; \Phi_1^\dagger\Phi_1 - \epsilon\,(\Phi_1^\dagger\Phi_1)^2 + \mathcal{O}(\epsilon^2), \qquad W_{eff} = f\,\Phi_1, \qquad \text{with} \qquad \epsilon = \frac{1}{12}\left(\frac{h^2}{4\,\pi}\right)^2 \frac{1}{|m_s|^2}$$

after 1-loop and int out $\Phi_{2,3}$ one finds a K similar to that above.

For reliable effective theory approach: $m_1^2 = 4\,\epsilon\,f^2 \sim f$, i.e. $h^2 \sim \mathcal{O}(4\pi)$ i.e. strong dynamics.

• Case 2: more fields: Goldstino and matter superfields.

Claim (SK) $X^2 = 0$ still defines goldstino superfield in IR in presence of matter fields? just compute it!

$$L = \int d^4\theta \, K(\Phi^i, \Phi_j^\dagger) + \int d^2\theta \, W(\Phi^i) + h.c. = K_i{}^j \Big[\partial_\mu \phi^i \, \partial^\mu \phi_j^\dagger + \frac{i}{2} \left(\psi^i \, \sigma^\mu \mathcal{D}_\mu \overline{\psi}_j - \mathcal{D}_\mu \psi^i \sigma^\mu \overline{\psi}_j\right) + F^i F_j^\dagger\Big]$$
$$+ \frac{K_{ij}^{kl}}{4} \psi^i \psi^j \, \overline{\psi}_k \overline{\psi}_l + \Big[\Big(W_k - \frac{K_k^{ij}}{2} \overline{\psi}_i \overline{\psi}_j\Big) F^k - \frac{W_{ij}}{2} \psi^i \psi^j + h.c.\Big]$$

i,j=1,2. Also take $W = f \, \Phi^1$, so Φ^1 =goldstino superfield. Massive ϕ^1, ϕ^2, integrate them out:

$$K_i^{jm} = k_i^{jm} + \phi^1 \, k_{i1}^{jm} + \phi^2 \, k_{i2}^{jm} + \mathcal{O}(1/\Lambda^3), \qquad K_{il}^{jm} = k_{il}^{jm} + \mathcal{O}(1/\Lambda^3), \qquad K_k^{ijl} = k_k^{ijl} + \mathcal{O}(1/\Lambda^3)$$

giving offshell:

$$\Rightarrow \quad \phi^1 = \frac{\psi^1 \psi^1}{2F^1} - \frac{c_1}{2F^1}(F^2 \psi^1 - F^1 \psi^2)^2 + \mathcal{O}(1/\Lambda), \qquad c_1 = \frac{\det \big[\, R_{2\,m}^{k\,n} \, F_k^\dagger \,\big]}{\det \big[\, R_{i\,l}^{j\,p} \, F^i \, F_j^\dagger \,\big]}$$

$$\Rightarrow \quad \phi^2 = \frac{\psi^2 \psi^2}{2F^2} - \frac{c_2}{2F^2}(F^2 \psi^1 - F^1 \psi^2)^2 + \mathcal{O}(1/\Lambda), \qquad c_2 = \frac{\det \big[\, R_{1\,m}^{k\,n} \, F_k^\dagger \,\big]}{\det \big[\, R_{i\,l}^{j\,p} \, F_i \, F_j^\dagger \,\big]}$$

$$\Rightarrow \quad \Phi_1^2 \neq 0, \;\; \Phi_1 \Phi_2 \neq 0, \quad \Phi_1^3 = \Phi_1^2 \, \Phi_2 = \Phi_1 \, \Phi_2^2 = \Phi_2^3 = 0,$$

$\Rightarrow$ UV details dependence, see $c_{1,2}$!

• What about X field? recall that: $X = 4\,W(\Phi^i) - (1/3)\,\overline{D}^2\,K(\Phi^i,\Phi_j^\dagger)$. So:

$$X = (\phi_X, \psi_X, F_X),\ W = f\Phi_1$$

$$\begin{aligned}\phi_X &= 4\,W(\phi^i) + \frac{4}{3}\Big[K^j\,F_j^\dagger - \frac{1}{2}K^{ij}\,\overline{\psi}_i\overline{\psi}_j\Big],\\ \psi_X &= \psi^k\,\frac{\partial\phi_X}{\partial\phi^k} + \mathcal{O}(\partial_\mu),\\ F_X &= F^i\,\frac{\partial\phi_X}{\partial\phi^i} - \frac{1}{2}\,\psi^i\psi^j\,\frac{\partial^2\phi_X}{\partial\phi^i\partial\phi^j} + \mathcal{O}(\partial_\mu)\end{aligned}$$

gives:

$$X^2 \neq 0\ \text{(offshell)}; \qquad X^2\Big|_{\text{onshell}} = -\frac{64}{9}\left[\frac{\det R^{1l}_{2m}}{\det R^{1l}_{1m}}\right] f\ (\psi^2\psi^2)\left[\frac{\psi^1\psi^1}{2\,f} - \sqrt{2}\,(\theta\psi^1) + (\theta\theta)\,f\right] + \mathcal{O}(1/\Lambda)$$

for 2 fields case: i=1 goldstino; i=2 matter.

• Note that X^2 depends on the UV completion (curvature terms), $R^{nl}_{ij} = k^{nl}_{ij}$.

• However, it can be shown by direct computation, that independent of the UV completion:

$$X^3 = 0,\ \ \text{offshell, in IR},\ \ X^2 \neq 0.$$

• The conjecture $X^2 = 0$ is not valid offshell or onshell.

$\Rightarrow$ Thus results derived from it cannot be correct in general, exceptions apply: extra discrete R symmetries, sgoldstino mass v.large - pertubrative expansion?.

• An example: consider

$$K = \Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 - \epsilon_1(\Phi_1^\dagger\Phi_1)^2 - \epsilon_2(\Phi_2^\dagger\Phi_2)^2$$
$$- \epsilon_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - \epsilon_4\left[(\Phi_1^\dagger)^2\Phi_2^2 + h.c.\right] + \mathcal{O}(1/\Lambda^3)$$

and superpotential

$$W = f\,\Phi_1, \qquad \epsilon_i = \mathcal{O}(1/\Lambda^2)$$

$$V = (K^{-1})_\rho^k\, W_k\, W^\rho = (K^{-1})_1^1 f^2 = f^2\left(1 + \epsilon_3\,|\phi^2|^2 + 4\,\epsilon_1\,|\phi^1|^2\right)$$

therefore $m_{\phi_1}^2 = \epsilon_1\,f^2$, $m_{\phi_2}^2 = 4\,\epsilon_3\,f^2$ for the sgoldstino (ϕ^1) and scalar matter field (ϕ^2), respectively (we choose $\epsilon_{1,3} > 0$) and the ground state is indeed at $\phi_{1,2} = 0$.

The onshell result is then

$$\phi^1 = -\frac{\psi^1\psi^1}{2\,f} - \frac{\epsilon_4}{\epsilon_1}\,\frac{\psi^2\psi^2}{2\,f} + \mathcal{O}(1/\Lambda)$$
$$\phi^2 = -\frac{\psi^1\psi^2}{f} + \mathcal{O}(1/\Lambda)$$

⇒ We also have that $\Phi_1^2 \neq 0$ and $X^2|_{onshell} \propto c_1 \neq 0$, contrary to SK.

However: if $\epsilon_1 \gg 1$ sgoldstino mass v. large, decouples, $\Phi_1^2 = 0$, or R-symmetries.

• Some phenomenology: non-linear goldstino, linear matter multiplets. $m_{spartners} \leq E \ll m_s, \sqrt{f}$ (ii)

- if only goldstino non-linear: $\Phi_1^2 = 0$ then coupling to MSSM:

$\Phi_1 = \psi_1\psi_1/(2F_1) + \sqrt{2}\theta\psi_1 + F_1\theta\theta;$ Analogy: $S = \theta\theta\, m_0 \to (\Phi_1/f)\, m_0$.

$$\mathcal{L} \supset \sum_{i=1,2} \frac{-m_i^2}{f^2} \int d^4\theta\ \Phi_1^\dagger \Phi_1\ H_i^\dagger\, e^{V_i}\, H_i + \sum_M \frac{-m_M^2}{f^2} \int d^4\theta\ \Phi_1^\dagger \Phi_1\, M^\dagger e^V\, M$$
$$+\ \frac{B}{f} \int d^2\theta\, \Phi_1\, H_1\, H_2 + \frac{A_u}{f} \int d^2\theta\, \Phi_1\, H_2\, Q\, U^c + \frac{A_d}{f} \int d^2\theta\, \Phi_1\, Q\, D^c\, H_1 + \frac{A_e}{f} \int d^2\theta\, \Phi_1\, L\, E^c\, H_1 + h.c.$$
$$+\ \sum_{i=1}^{3} \frac{1}{16\, g_i^2} \frac{2\, m_{\lambda_i}}{f} \int d^2\theta\, \Phi_1\, \mathrm{Tr}\, [W^\alpha\, W_\alpha]_i + h.c.$$

$\Rightarrow$ the potential (integrate the dynamics of Φ_1)

$$V_F = |\mu|^2 \left[|h_1|^2 + |h_2|^2 \right] + \frac{|f + (B/f)\, h_1.h_2|^2}{1 + c_1\, |h_1|^2 + c_2\, |h_2|^2} + \mathcal{O}(1/f^3) \qquad c_i = -\frac{m_i^2}{f^2}$$

$$\Rightarrow \quad V = f^2 + \left(|\mu|^2 + m_1^2\right) |h_1|^2 + \left(|\mu|^2 + m_2^2\right) |h_2|^2 + \left(B\, h_1.h_2 + h.c.\right)$$
$$+ \frac{1}{f^2} \left| m_1^2\, |h_1|^2 + m_2^2\, |h_2|^2 + B\, h_1.h_2 \right|^2 + \frac{g_1^2 + g_2^2}{8} \left[|h_1|^2 - |h_2|^2 \right]^2 + \frac{g_2^2}{2}\, |h_1^\dagger\, h_2|^2 + \mathcal{O}(1/f^3)$$

$\Rightarrow$ new quartic terms $\propto m_i^2$. Relevant for low scale susy breaking: $\sqrt{f} \sim \mathcal{O}(10)$ TeV or so.

- impact on the MSSM Higgs sector (top figures: m_A parameter; bottom: μ parameter):

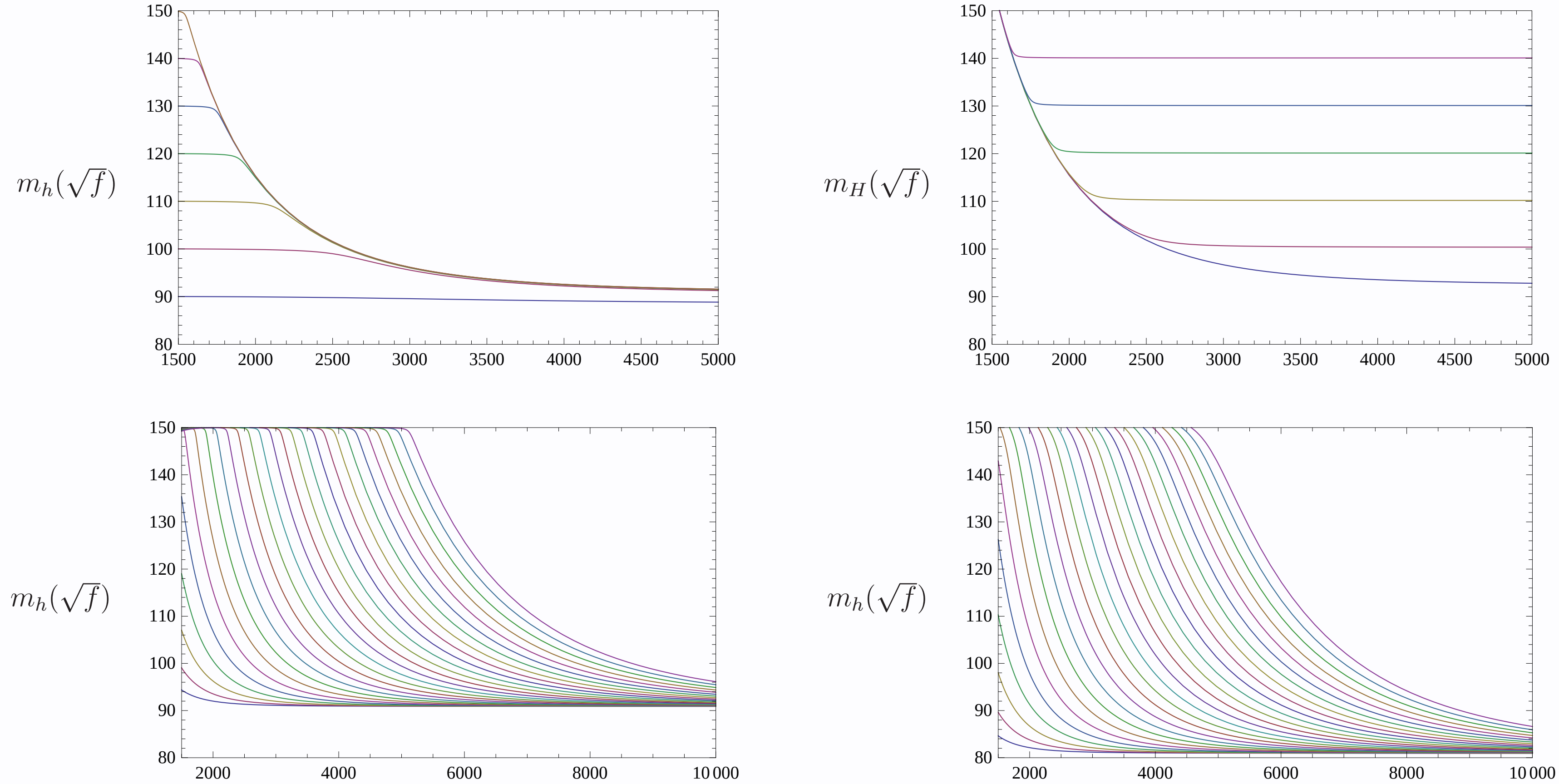

Figure 1:

$$m_h^2 = \Big[m_Z^2 + \mathcal{O}(1/u^2)\Big] + \frac{v^2}{2\,f^2}\Big[(2\,\mu^2 + m_Z^2)^2 + \mathcal{O}(1/u^2)\Big] + \mathcal{O}(1/f^3)$$

$$m_H^2 = \Big[m_A^2 + \mathcal{O}(1/u^2)\Big] + \frac{1}{f^2}\,\mathcal{O}(1/u^2) + \mathcal{O}(1/f^3) \qquad\qquad u \equiv \tan\beta$$

$\Rightarrow m_h \sim 120$ GeV for $\sqrt{f} \sim 2$ to 7 TeV, already at classical level. Reduced EW fine tuning.

• Conclusions:

• investigated the link X superfield versus goldstino superfield.

• In IR and for large Λ, X converges to the goldstino superfield.
But $X^2 \neq 0$ and $\Phi_1^2 \neq 0$. unless
a) the sgoldstino mass very large (perturbative convergence?) or
b) some extra (discrete) symmetries.

• The result remains true for arbitrary W, K and any number of fields.

• if only goldstino non-linear, matter fields linear and low $\sqrt{f} \sim \mathcal{O}(10)$ TeV:

$\Rightarrow$ low susy scale models (gauge mediation?), corrections to m_h: $m_h > 120$ GeV at classical level.

• invisible Higgs decay (neutralino+goldstino): BR similar to $h \to \gamma\gamma$.