# NILPOTENT SUPERGRAVITY, INFLATION AND MODULI STABILIZATION

Based on :

- I.Antoniadis, E.D., S.Ferrara and A. Sagnotti, Phys.Lett.B733 (2014) 32 [arXiv:1403.3269 [hep-th]].
- E.D., S.Ferrara, A.Kehagias and A.Sagnotti, JHEP 1509 (2015) 217 [arXiv:1507.07842 [hep-th]].
- G. Dall'Agata, E.D., F.Farakos, arXiv:1603.03416 [hep-th].

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# Outline

1) Nonlinear SUSY realizations
 - constrained superfields
2) Nonlinear supergravities
- Minimal nonlinear supergravity
3) Simplest models of inflation in supergravity
- Chaotic inflation
- Starobinsky models
4) Inflation with nilpotent superfields
5) Moduli stabilization with nilpotent uplift
5) Conclusions and perspectives

Large literature on SUSY non-linear realizations and low-energy goldstino interactions

- Volkov-Akulov, Ivanov-Kapustnikov, Siegel, Samuel-Wess, Clark and Love...
- Casalbuoni, Dominicis, de Curtis, Feruglio, Gatto
- Luty, Ponton
- Brignole, Feruglio, Zwirner; Brignole
- Casas, Espinosa, Navarro
- Komargodski and Seiberg (KS)...

Most pheno studies based on a component formalism, tedious computations. With constrained superfield formalism, easier computations.

Today (some) applications to inflation.

Application to (MS)SM: Antoniadis,E.D.,Ghilencea,Tziveloglou: E.D.,Gersdorff, Ghilencea,Lavignac,Parmentier; Petersson,Romagnoni…

recent review: D. Ghilencea

# 1) Non-linear SUSY realizations

- In supergravity, the gravitino $\Psi$ becomes massive by absorbing a spin ½ fermion: the goldstino G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

Goldstino is part of a multiplet $X = (x, G, F_X)$

The gravitino mass is $m_{3/2} \sim \frac{f}{M_P}$ , where

$F_X = f + \cdots$ is the scale of SUSY breaking

In the decoupling limit $f << M_P$ , the partner of the goldstino (sgoldstino x) decouples.
Goldstino couplings to matter scale as $1/f$

The first written SUSY lagrangian had nonlinearly realized SUSY, Volkov-Akulov (VA), 1973.

$$\mathcal{L}_X = \int d^4\theta \; X^\dagger X + \left\{ \int d^2\theta \, f \; X + h.c. \right\}$$

$$= det \; (E_\mu^a) \;, \quad \text{where} \quad E_\mu^a = e_\mu^a + (\frac{i}{2f^2} G \sigma^a \partial_\mu \bar{G} + h.c.)$$

is the VA "vierbein". In the standard VA prescription, couplings to matter proceed as in gravity

$$G^{\mu\nu} \; T_{\mu\nu,M} \;=\; g^{\mu\nu} \; T_{\mu\nu,M} \;+\; (\frac{i}{2f^2} G \sigma^\mu \partial^\nu \bar{G} + h.c.) \; T_{\mu\nu,M}$$

# - Constrained superfields

- VA action can be constructed in superspace (Rocek,78) introducing a constrained, nilpotent superfield

$$X^2 = 0$$

whose solution is

no fundamental scalar

$$X = \frac{GG}{2F_X} + \sqrt{2}\,\theta\, G + \theta^2 F_X$$

The full VA action is $\mathcal{L}_{VA} = \left[X\,\overline{X}\right]_D + \left[f X + h.c.\right]_F$

Analogy with the sigma model :

- O(N) linear sigma model

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a - \lambda(\phi_a \phi_a - v^2)^2$$

has 1 massive (« Higgs ») and N-1 goldstone bosons,

versus the

- O(N)/ O(N-1) nonlinear sigma model ( $\lambda \to \infty$ limit)

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a$$

plus the constraint $\phi_a \phi_a = v^2$ , which describes self-interactions of the N-1 goldstone's.

O(N) symmetry is nonlinearly realized.

There are two different cases to consider:

i) non-SUSY matter spectrum

$$E << m_{sparticles} \; , \; \sqrt{f}$$

$\Rightarrow$ non-linear SUSY in the matter sector

ii) SUSY matter multiplets : $(\tilde{q}, q)$ , etc

$$m_{sparticles} \leq E << \sqrt{f}$$

$\Rightarrow$ linear SUSY in the matter sector

Case i) (non-linear matter) $\Rightarrow$ additional constraints (KS)

- light fermions : X Q = 0 eliminates complex scalars

$$Q = \frac{1}{F_X}(\Psi_q - \frac{F_q G}{2F_X})G + \sqrt{2}\theta\Psi_q + \theta^2 F_q$$

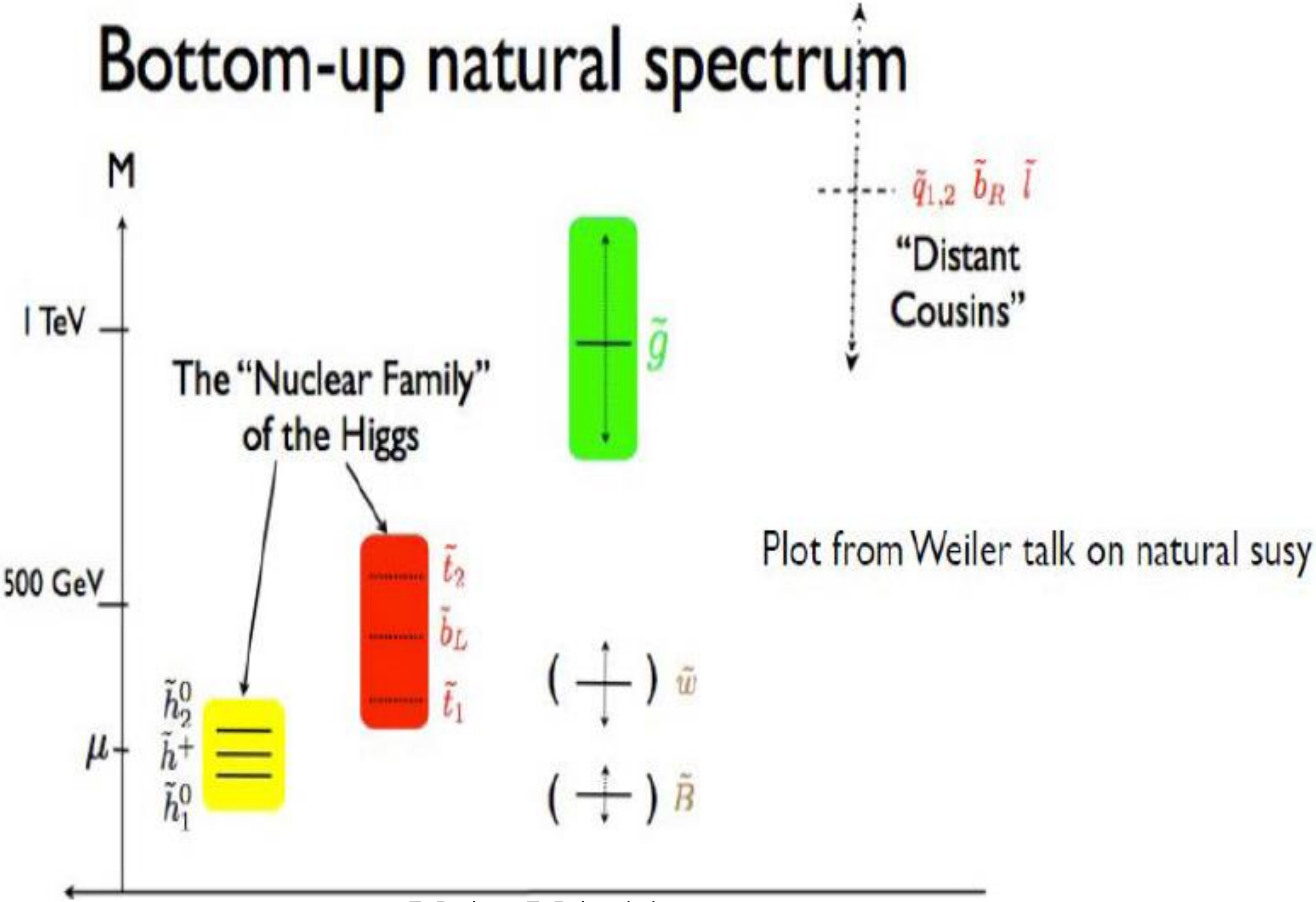
- light complex scalars : $X\bar{H} =$ chiral, eliminates fermions

$$H = h + i\sqrt{2}\theta\sigma^m\partial_m h\frac{\bar{G}}{\bar{F}_X} + \theta^2[-\partial_n(\frac{\bar{G}}{\bar{F}_X})\bar{\sigma}^m\sigma^n\partial_m h\frac{\bar{G}}{\bar{F}_X} + \frac{1}{2\bar{F}_X^2}\bar{G}^2\partial^2 h]$$

In this case, there is no more auxiliary field.

- light real scalar (inflaton ?) : $X(\Phi - \bar{\Phi}) = 0$ eliminates a scalar (sinflaton ?) and the fermion (inflatino ?)

Why more constraints ? first two generation squarks could be heavy ⇒ impose X Q_1,2=X L_1,2 = 0



- Recent (field theory) progress in the possible UV origin of constrained superfields (d'ADF).
- Ex: decoupling a fermion can be obtained with an

UV operator

$$-\frac{m_\chi}{2f^2}\int d^4\theta\left[|X|^2 D^\alpha Y D_\alpha Y + c.c.\right]$$

which decouples only the fermion ($m_\chi \to \infty$) and produces the constraint

$$|X|^2 D_\alpha Y = 0,$$

The KS constraint needs two operators

$$-\frac{m_h}{2f^2}\int d^4\theta\left[|X|^2 D^\alpha \mathcal{H} D_\alpha \mathcal{H} + c.c.\right] - \frac{g_{FH}}{f^2}\int d^4\theta\left[|X|^2 D^2 \mathcal{H} \overline{D}^2 \overline{\mathcal{H}}\right]$$

and eliminates also the auxiliary field

# 2) Non-linear supergravities

In SUGRA, the most general couplings of the nilpotent X are described by (ADFS)

$$K = -3\log(1 - X\overline{X}) \equiv 3X\overline{X}\,, \qquad W = fX + W_0$$

and as a result

$$\mathcal{L}_{mass} = -m_{3/2}\left(\psi_m + \frac{i}{\sqrt{6}}\sigma_m\overline{G}\right)\sigma^{mn}\left(\psi_n + \frac{i}{\sqrt{6}}\sigma_n\overline{G}\right) + \text{h.c.}$$

The SUGRA lagrangian contains the proper goldstino couplings and

$$V = \frac{1}{3}|f|^2 - 3|W_0|^2\,, \qquad m_{3/2}^2 = |W_0|^2$$

Recently the complete lagrangian was written by
Bergshoeff, Freedman,Kallosh, Van Proeyen and Hasegawa,Yamada (2015)

Consider the gravity multiplet,

$$\left(e_m^a, \psi_m^\alpha, u, A_m\right)$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

vierbein gravitino auxiliary fields

coupled to the goldstino multiplet X, in the decoupling limit. The theory contains actually just

the graviton + one massive gravitino

It seems logical to anticipate that there should be a purely gravitational description, with a

modified/constrained gravity multiplet.

# - Minimal nonlinear supergravity

(DFKS ; Antoniadis,Markou, 2015)

- This is described by

$$\mathcal{L} = \left[-S_0 \overline{S}_0\right]_D + \left[W_0 S_0^3\right]_F \qquad (1)$$

with the constraint

$$\left(\frac{\mathcal{R}}{S_0} - \lambda\right)^2 = 0 \qquad (2)$$

where :

- $\mathcal{R}$ is the chiral curvature multiplet,
- $S_0$ is the chiral compensator field
- $\lambda$ is related to the cosmological constant.

whereas the goldstino is

$$G \;=\; -\,\frac{3}{2\,\lambda}\left(\gamma^{\mu\nu}\,\partial_\mu\,\psi_\nu \;-\; \frac{\lambda}{2}\,\gamma^\mu\,\psi_\mu\right)$$

This describes just the gravitational multiplet, with nonlinear SUSY. One can show this is exactly dual to the simplest Volkov-Akulov SUGRA .

Introduce two lagrange multipliers $X, \Lambda_1$ that « linearize » the lagrangian

$$\mathcal{L} \;=\; \left[-\;S_0\,\overline{S}_0\right]_D \;+\; \left[\left\{X\left(\lambda \;-\; \frac{\mathcal{R}}{S_0}\right) \;-\; \frac{1}{4\Lambda_1}\,X^2 \;+\; W_0\right\}S_0^3\right]_F$$

which leads to

$$\mathcal{L} = \Big[ - (1 + X + \bar{X}) \; S_0 \overline{S}_0 \Big]_D + \Big[ \Big\{ \lambda X - \frac{1}{4\Lambda_1} X^2 + W_0 \Big\} S_0^3 \Big]_F$$

One finally finds a standard SUGRA with

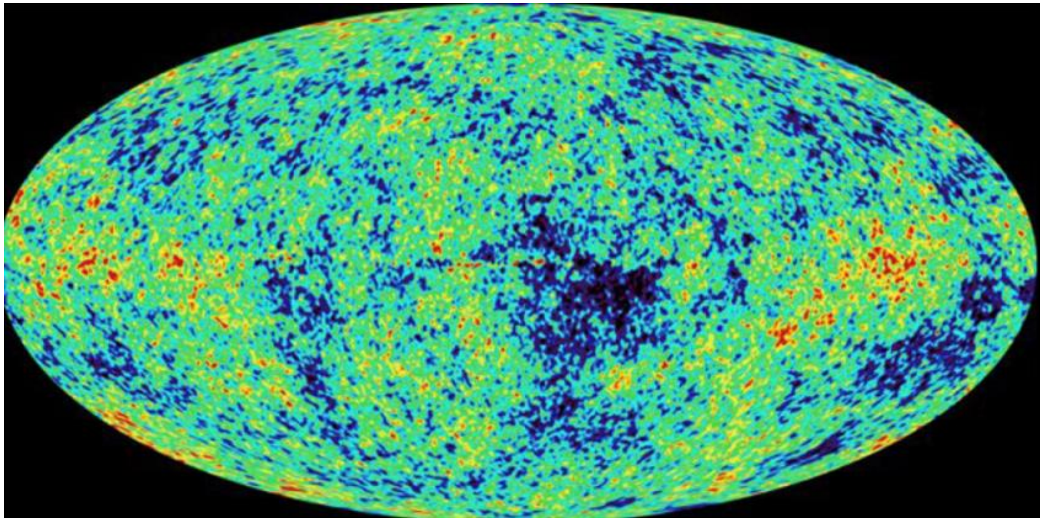
$$K = -3 \ln \left(1 + X + \overline{X}\right) , \qquad W = W_0 + \lambda X$$

plus the constraint $X^2 = 0$ , or equivalently

$$K = 3 |X|^2 , \qquad W = W_0 + (\lambda - 3 W_0) X$$

# 3) Simplest models of inflation in supergravity

## Why Supergravity for early cosmology ?



- Inflation with super-Planckian field variations needs an UV completion ⇒ String Theory
- Supersymmetry crucial ingredient in String Theory, supergravity its low-energy effective action

# - Chaotic inflation in supergravity

- Large-field chaotic inflation (Linde,1983) is an attractive scenario for explaining the initial conditions of the early universe.

Simplest realization: free massive scalar field

$$V = \frac{1}{2} m^2 \varphi^2$$

Chaotic inflation needs transplanckian values and

small inflaton mass

$$\varphi \sim 10 - 15 \ M_P \qquad\qquad m \sim 10^{-5} \ M_P$$

Best explanation of smallness of inflaton mass ( the $\eta$ problem) is an approximate shift symmetry $\varphi \to \varphi + \alpha$

Realistic model contain a « stabilizer » field S, with no shift symmetry (Kawasaki,Yamaguchi,Yanagida,2000)

where $\varphi = \sqrt{2}\,\mathrm{Im}\,\phi$ and

$$W = mS\phi \quad , \qquad K = \frac{1}{2}(\phi + \bar{\phi})^2 + |S|^2 - \xi |S|^4$$

The term in $\xi$ is needed in order to give a large mass to S during inflation.

# - Starobinsky model

- First model of inflation, based on a higher-deriv gravity action (1980)

$$\mathcal{S}_2 = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \alpha R^2\right]$$

It can be rewritten with a help of a lagrange multiplier scalar

$$\mathcal{S}_1 = \int d^4x\sqrt{-g}\left[\frac{1}{2}(1+2\chi)R - \frac{1}{4\alpha}\chi^2\right]$$

Going in the Einstein frame and defining $1+2\chi = e^{\sqrt{\frac{2}{3}}\phi}$

one finds a standard dual two-derivative action

$$S = \int d^4x\sqrt{-g_E}\left[\frac{1}{2}R_E - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

with $$V(\phi) = \frac{1}{16\alpha}\left(1-e^{-\sqrt{\frac{2}{3}}\phi}\right)^2$$

The embedding of the model in SUGRA is (Cecotti,1987) :

real function

$$\mathcal{L} = \left[ - S_0 \, \overline{S}_0 \; + \; h \left( \frac{\mathcal{R}}{S_0} , \frac{\overline{\mathcal{R}}}{\overline{S}_0} \right) S_0 \, \overline{S}_0 \right]_D + \left[ W \left( \frac{\mathcal{R}}{S_0} \right) S_0^3 \right]_F \; ,$$

where :

- D and F subscripts are superspace densities
- $\mathcal{R}$ is the chiral scalar curvature superfield $\mathcal{R} = \frac{\Sigma(\bar{S}_0)}{S_0}$,

defined with the curved chiral projector $\Sigma$ , of components

$$\mathcal{R} = \left( \overline{u} \equiv S + iP \, , \; \gamma^{mn} \mathcal{D}_m \psi_n \, , \; - \frac{1}{2} \, R - \frac{1}{3} \, A_m^2 + i \, D^m A_m - \frac{1}{3} \, u \, \overline{u} \right) ,$$

and :

- u and $A_m$ are « old minimal » auxiliary fields of N=1 SUGRA
- $\psi_n$ is the gravitino

This action can be recast into a two-derivative dual form, using two chiral multiplets (Cecotti;Ferrara,Kallosh,Van Proeyen) :

$$\mathcal{L} = \left[ - \; S_0 \, \overline{S}_0 + h(C,\overline{C}) \, S_0 \, \overline{S}_0 \right]_D + \left[ \left\{ \Lambda \left( C - \frac{\mathcal{R}}{S_0} \right) + W(C) \right\} S_0^3 \right]_F$$

where $\Lambda$ implements $\mathcal{R} = S_0 C$ .

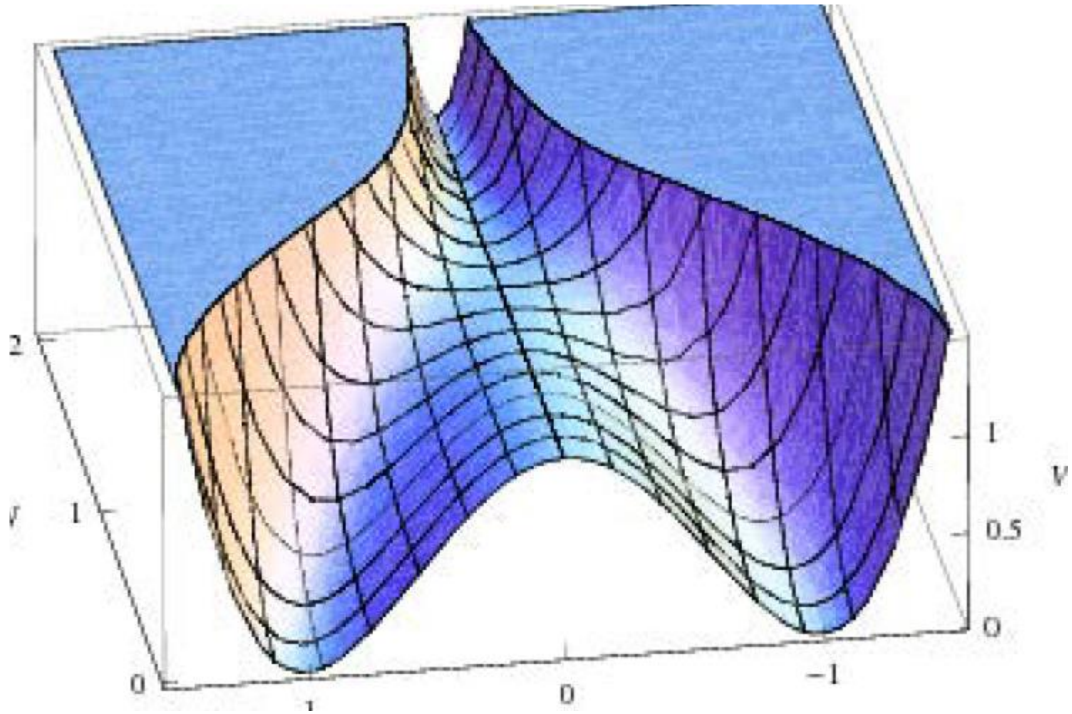
- $W(C)$ can be shifted away by a field redefinition.

- Defining $\Lambda' = T - \frac{1}{2}$ , one finally finds a standard N=1 SUGRA with

$$K = -3 \ln \left[ T + \overline{T} - h(C,\overline{C}) \right] \; , \qquad W = C \left( T - \frac{1}{2} \right) + W_0$$

The inflaton is $Re(T) = \exp \left( \sqrt{2/3} \; \phi \right)$

# Cosmological data favors models with one light inflation




Simplest SUGRA models contain 2 complex scalars for inflation and 3 complex scalars if SUSY breaking. Desirable to have simpler models.

# 4) Inflation with nilpotent fields
## - minimal Starobinsky model (ADFS)

- Consider the chiral formulation of Starobinsky

$$K = -3 \ln \left[ T + \overline{T} - X \overline{X} \right] , \qquad W = M X T + f X + W_0$$

and impose $X^2 = 0$ . There is only one scalar and one axion bosonic fields

$$T = e^{\phi \sqrt{\frac{2}{3}}} + i a \sqrt{\frac{2}{3}}$$

$$\mathcal{L} = \frac{R}{2} - \frac{3}{(T + \overline{T})^2} |\partial T|^2 - \frac{|M T + f|^2}{3 (T + \overline{T})^2}$$

$$= \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} e^{-2 \phi \sqrt{\frac{2}{3}}} (\partial a)^2 - \frac{M^2}{12} \left( 1 - e^{-\sqrt{\frac{2}{3}} \phi} \right)^2 - \frac{M^2}{18} e^{-2 \phi \sqrt{\frac{2}{3}}} a^2$$

This is exactly Starobinsky + a (much) heavier axion.

# - Dual gravitational formulation

- The previous lagrangian is

$$\mathcal{L} = -\left[(T+\overline{T}-|X|^2)S_0\,\overline{S}_0\right]_D + \left[(MXT+fX+W_0)S_0^3 + \text{h.c}\right]_F$$

and can be rewritten

$$\mathcal{L} = \left[|X|^2\,S_0\,\overline{S}_0\right]_D + \left[\left(T(-\frac{\mathcal{R}}{S_0}+M\,X)+f\,X+W_0\right)S_0^3 + \text{h.c}\right]_F$$

using the identity $\left[(T+\overline{T})S_0\,\overline{S}_0\right]_D = \left[T\,\mathcal{R}\,S_0^2\right]_F + \text{h.c.}$

T enters now as a Lagrange multiplier, which imposes the constraint

$$X = \frac{1}{M}\frac{\mathcal{R}}{S_0} \Longrightarrow \mathcal{R}^2 = 0$$

Nonlinear Supergravity

There are consistency conditions to be satisfied (Dall'Agata-Zwirner,2014) :

- $F_X \neq 0$ during inflation and afterwards
- unitarity. In flat background, unitarity valid for

$$E^2 < m_{3/2} M_P$$

During inflation, this should be replaced by

$$E_q^2 < m_{3/2}(\varphi) M_P$$

where $E_q \sim V_{\text{inf}}^{1/2}(\varphi)/M_P$ is the typical energy scale of quantum fluctuations during inflation.

# Several inflationary models constructed afterwards

((Ferrara) Kallosh,Linde, Dall'Agata-Zwirner)

Simple models constructed based on

$$K = \tfrac{1}{2}(\Phi + \bar{\Phi})^2 + |X|^2 \quad , \quad W = f(\Phi)(1 + \sqrt{3}X)$$

with $X^2 = 0$ , $f(0) \neq 0, f'(0) = 0$ and $\overline{f(x)} = f(-\bar{x})$

The inflationary potential is $V = \left| f'\left( i\,\frac{\varphi}{\sqrt{2}} \right) \right|^2$

Inflationary energy is decoupled from the gravitino mass: possible to have GUT vacuum energy with low (TeV) gravitino mass.

# Explicit examples :

$$f(\Phi) = f_0 - \frac{m}{2}\Phi^2 \qquad \text{(chaotic inflation)}$$

$$f(\Phi) = f_0 - i\sqrt{V_0}(\Phi + i\frac{\sqrt{3}}{2}e^{\frac{2i\Phi}{3}}) \qquad \text{(Starobinsky)}$$

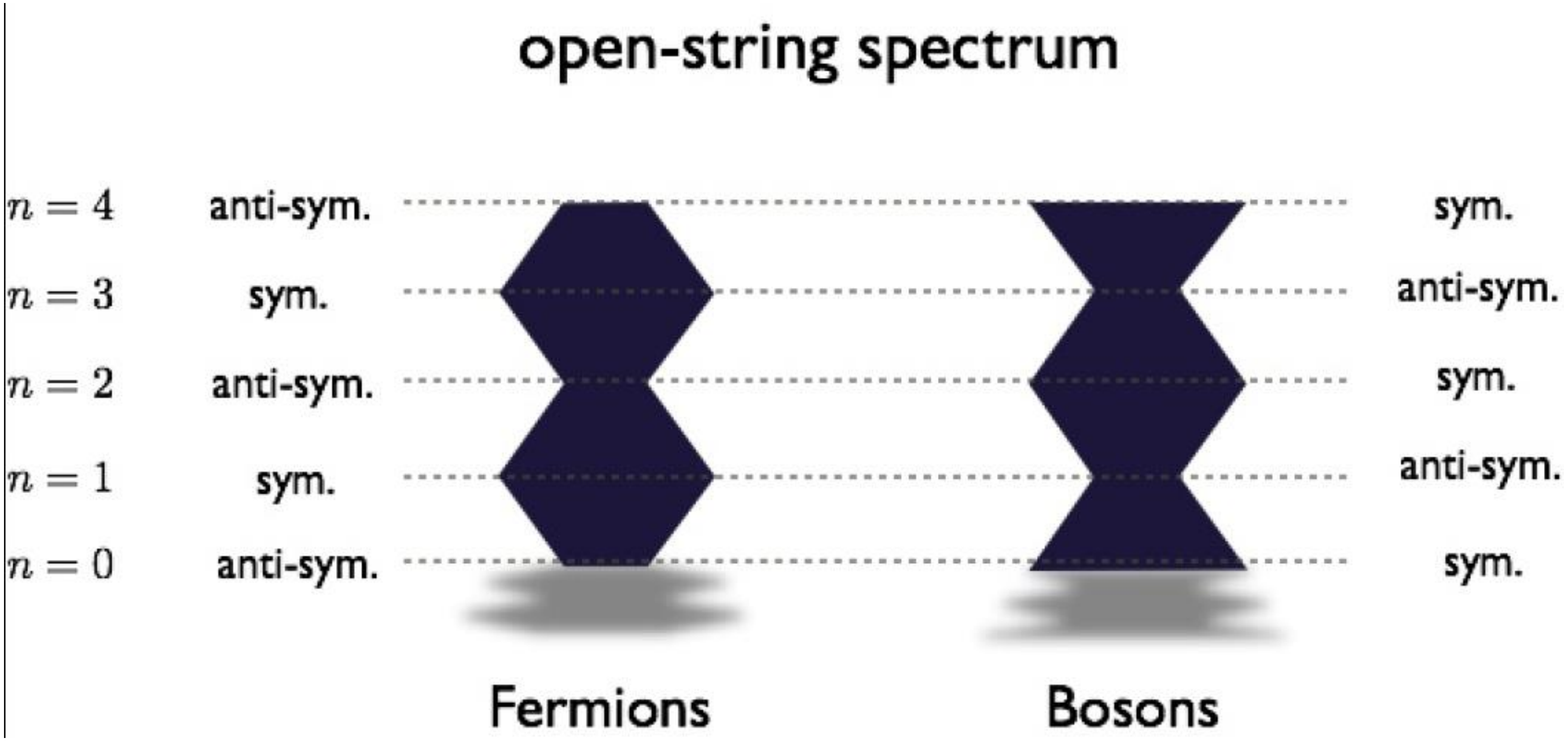
Perturbative unitarity OK if

$$|f'(i\frac{\varphi}{\sqrt{2}})| < |f(i\frac{\varphi}{\sqrt{2}})|$$

This is satisfied in both examples and is essentially insensitive on the scale of SUSY breaking in the vacuum.

# 5) Moduli stabilization with nilpotent uplift

- The string theory realization of the nilpotent superfield is by the $\overline{D}3/O3$ system (Kallosh, Quevedo,Uranga, 2015). Explicit similar string vacua known (Brane SUSY breaking: (Sugimoto;Antoniadis,E.D.,Sagnotti, 1999; $\overline{D}p/Op_+$ )



-SUSY is non-linearly realized on the antibranes (E.D.,Mourad, 2000).

-For one $\overline{D}3$ on top of an $O3_-$ the only degree of freedom is the goldstino.

-The $\overline{D}3$ action can be written with the goldstino nilpotent superfield (Bandos,Martucci,Sorokin,Tonin,2015.)

$\overline{D}3/O3$ also used in the KKLT moduli stabilization. Its effective SUGRA description is (Kallosh-Linde,2014)

$$W = W_0 + Ae^{-a\rho} - \mu^2 S \, , \qquad K = -3\ln(\rho + \overline{\rho}) + S\bar{S} \qquad \text{at} \qquad S^2 = 0$$

- The scalar potential is

$$V_{New\,O'KKLT} = V_{KKLT}(\rho,\bar{\rho}) + \frac{\mu^4}{(\rho+\bar{\rho})^3}$$

- The uplift term is actually as the $\overline{D}3$ tension in flat space.

- More detailed effective action analysis reveals the emergence of the constraint

$$\left(\frac{\mathcal{R}}{S_0} - \lambda\right)^2 = 0$$ (Bandos,Martucci,Sorokin,Tonin,2015.)

# Conclusions and perspectives

- The standard models of inflation in SUGRA contain (at least) three more scalars in addition to the inflaton (more if we include SUSY breaking).
- Using constrained superfields one can now construct simpler models (only the inflaton !) ; SUSY is automatically broken.
- Recent progress in the UV origin of constrained superfields. Several constraints possible to eliminate fermions, with different low-energy physics.
- There are consistency conditions to be satisfied in nilpotent SUGRA during inflation. Are they really sufficient ?
- 

- Interesting to analyze further the gravity dual of Volkov-Akulov nonlinear SUSY ⟹ nonlinear supergravity. Constraints on couplings to matter. Component action ?

  - Right framework to include moduli stabilization, reheating.

Thank you

# Backup slides

Setting to zero the other three fields one recovers, for $W_0 = 0$ exactly the Starobinsky model.
However, for the minimal choice

$$h(C,\overline{C}) = C\overline{C}$$

the field direction C is unstable during inflation (Kallosh-Linde).
Non-minimal Kahler potential is needed, for ex:

$$h(C,\overline{C}) = C\,\overline{C} - \zeta\,(C\,\overline{C})^2$$

Coupling to matter can lead to positive definite potentials and inflation (DFKS)

$$K_0 \; = \; - \, 3 \;\; \ln \left( 1 \; - \; \frac{1}{2} \, (\Phi + \bar{\Phi})^2 \; - \; |Q|^2 \right)$$

$$W \; = \; W_0(\Phi, Q_i) \; + \; f(\Phi, Q_i) \, X$$

where $$W_0 \; = \; \alpha(\Phi) \; + \; \frac{1}{2} \, b_{ij} \, Q_i \, Q_j \; + \; \frac{1}{6} \, \lambda_{ijk} \, Q_i \, Q_j \, Q_k$$

$$f \; = \; 6 \, \alpha(\Phi) \; + \; b_{ij} \, Q_i \, Q_j \;\; ,$$

The scalar potential is a generalization of Dall'Agata-Zwirner

$$V \; = \; \frac{1}{3Y^2} \left\{ \sum_i \left| \frac{\partial W_0}{\partial Q_i} \right|^2 + \left| \frac{\partial W_0}{\partial \Phi} \right|^2 + \frac{1}{2} \, (\Phi + \bar{\Phi})^2 \, |f|^2 \, + \, (\Phi + \bar{\Phi}) \left( \bar{f} \, \frac{\partial W_0}{\partial \Phi} \; + \; \mathrm{h.c} \right) \right\}$$

Recent ex. of chaotic inflation with $V = \frac{1}{2} m^2 \varphi^2$; $m_{3/2} \sim m^2 \sim 10^5 TeV$ and $m_{\mathrm{soft}} \sim m^3 \sim TeV$ MSSM soft masses (Hasegawa, Yamada)

Coupling to matter (chiral multiplets Q) can be implemented starting from

$$\mathcal{L} = \left[ - e^{-\frac{1}{3} K_0(Q_i, \bar{Q}_{\bar{i}})} \, S_0 \, \overline{S}_0 \right]_D + \left[ W_0(Q_i) \, S_0^3 \right]_F$$

supplemented by the nilpotency constraint

$$\left( \frac{\mathcal{R}}{S_0} - f(Q_i) \right)^2 = 0$$

It can be shown that there is a consistency condition

$$K_{0,\bar{i}} \, K_0^{\bar{i}j} \, K_{0,j} < 3$$